British Journal of Mathematics & Computer Science
12(6): 1-13, 2016, Article no.BJMCS.21932
ISSN: 2231-0851

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On Solving Three Level Fractional Programming Problem with Rough Coefficient in Constraints

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Article Information

DOI: 10.9734/BJMCS/2016/21932
Editor(s):
(1) Dijana Mosic, Department of Mathematics, University of Nis, Serbia.
Reviewers:
(1) Kavita Gupta, University of Delhi, India.
(2) Shafi Ullah, Aligarh Muslim University, Aligarh, India.
(3) José Alejandro González Campos, Playa Ancha University, Chile.
Complete Peer review History: http://sciencedomain.org/review-history/12180

Original Research Article

Received: 09 September 2015
Accepted: 05 October 2015
Published: 09 November 2015

Abstract

A three level fractional programming problem is presented in this paper with a random rough coefficient in constraints at the first phase of the solution approach and to avoid the complexity of this problem we begin with converting fractional programming problem into linear problem using Charnes & Cooper method, Then interval technique is used to convert the rough nature in constraints into equivalent crisp model. At the final phase, a membership function is constructed to develop a fuzzy model for obtaining a compromised solution of the three level programming problem. Finally results are illustrated by a numerical example.

Keywords: Linear programming problem; fractional programming problem; rough in constraints; interval coefficient.

1 Introduction

The multilevel programming (MLP) problem is a hierarchical optimization problem where a subset of variables is constrained to be a solution of a given optimization problem parameterized by the remaining variables ,they are formulated in order to solve decentralized planning problems involving several decision makers (DMs) in a hierarchical organization based on the concept of Stackelberg game theory [1].

Multilevel programming has been applied to decentralized planning problems involving a decision process with a hierarchical structure as it involves optimization problems where the constraint region of the first

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level problem is implicitly determined by the second level problem and the constrained region of the second level problem is determined by the third level problem, and so on.

Optimization of one or several ratios of functions is called a fractional programming problem [2]. These models arise naturally in decision making when several rates need to be optimized simultaneously such as production planning, financial and corporate planning, health care and hospital planning. Several methods were suggested for solving this problem such as the variable transformation method [3], Several new methods are proposed in [4,5].

Almost all concepts which we are using in natural language are vague. Perhaps some people think that they are subjective probability or fuzzy. However, a lot of investigations have been shown that those imprecise quantities behave neither like randomness nor like fuzziness. In modern times, scholars are often faced with complex decision making problems concerning uncertainty. These uncertainties are stated by interval data, roughness or their hybrid with fuzziness and randomness [6,7].

Interval programming based on the interval analysis has been developed as a useful and simple method to deal with uncertainty, the rough intervals (RIs) [8] are used to deal with partially unknown or ill-defined parameters and variables. RI is introduced to adapt the rough set principles to model continuous variables.

Ali Hamzehee et al. [8] presented a method to solve a linear problem (LP) with rough interval coefficient in the objective function and/or constraints by constructing two linear problems with interval coefficients. One of these problems is an LP where all of its coefficients are upper approximations of rough intervals and the other is an LP where all of its coefficients are lower approximations of rough intervals.

Omar M. Saad et al. [9] presented a method of solving a three-level quadratic programming (QP) problem where some or all of its coefficients in the objective function are rough intervals. The solution based on formulating two problems with interval coefficient one of these problems all of its coefficients are upper approximation of rough intervals and the other problem all of its coefficients are lower approximations of rough intervals then a membership function is constructed to develop a fuzzy model for obtaining the optimal solution.

O. E. Emam et al. [10] presented three-level quadratic programming problem with random rough coefficient in constraints, solution concept based on converting the rough nature in constraints into equivalent crisp using intervals technique and a membership function is constructed to develop a fuzzy model for obtaining a compromise solution of the problem.

In [11] authors proposed two fuzzy goal programming (FGP) models to multi-level linear fractional programming problem (MLFPP) with single decision maker (DM) at each level, The linear fractional membership functions for the fuzzily described linear fractional objective functions of level decision makers are developed first then First order Taylor polynomial series is employed in order to transform the linear fractional membership functions into equivalent linear membership functions, Then the FGP technique is utilized to achieve compromise optimal solution of the multi-level system by minimizing negative deviational variables two fuzzy goal programming (FGP) models to multi-level linear fractional programming problem (MLFPP) with single decision maker (DM) at each level.

In this paper, first we introduce a multilevel linear fractional programming problem (MLFPP) with rough coefficient in constraints. Then converting the fractional Programming problem to a linear programming problem is essential to solve the problem for each level. Also the transformation of random rough coefficient in the constraints into crisp is presented finally fuzzy approach of an equivalent crisp three-level linear programming problem is suggested , an example is presented to illustrate the developed results.

2 Problem Formulation and Solution Concept

Considering a three-level programming problem (TLPP) of maximization-type with random rough coefficient in the constraints can be written as:

$$\underset{\overline{X}_1}{\mathrm{Max}}\ f_1\,(x) = \frac{a_1 x + \alpha_1}{b_1 x + \beta_1} \tag{1}$$

$$\underset{\overline{X}_2}{\mathrm{Max}}\ f_2\,(x) = \frac{a_2 x + \alpha_2}{b_2 x + \beta_2} \tag{2}$$

$$\underset{\overline{X}_3}{\mathrm{Max}}\ f_3\,(x) = \frac{a_3 x + \alpha_3}{b_3 x + \beta_3} \tag{3}$$

Subject to

$$x \in s = \left\{\sum_{j=1}^{n}\left([a_j, b_j], [c_j, d_j]\right)x_j \le l, x_j \ge 0, j = 1, \dots, n\right\} \tag{4}$$

Where f_1, f_1 and f_3 are the objective functions of the first level decision maker (FLDM), second level decision maker (SDLM) and third level decision maker (TLDM), $[a_j, b_j], [c_j, d_j]$ are rough intervals coefficient of the constraints for the three levels.

To tackle problem (1)-(4) and to deal with rough nature in (TLPP) the problem is transformed using the Intervals method to transform the rough coefficient in constraints into crisp number presented in the following section.

Definition 1 [8]:

Consider all of the corresponding (TLLFPR) Problem (1)-(4)

a) The interval $[Z_*^L, Z_*^U]([Z^{*L}, Z^{*U}])$ is called the surely optimal range of problem (1)-(4), if the optimal range of each LPIC problem is a superset of $[Z_*^L, Z_*^U]([Z^{*L}, Z^{*U}])$.
b) Let $[Z_*^L, Z_*^U]([Z^{*L}, Z^{*U}])$ be surely optimal range of problem (1)-(4).Then the rough interval $([Z_*^L, Z_*^U]([Z^{*L}, Z^{*U}])$ is called the rough optimal range of problem (1)-(4).
c) The optimal solution of each corresponding LP problem of the problem (1)-(4).Which its optimal value belongs to $[Z_*^l, Z_*^u]([Z^{*l}, Z^{*u}])$ is called a completely satisfactory solution of the problem (1)-(4).

Definition 2: In problem (1)-(4), we define the following sets:

$$P_1 = \left\{x \in R^2 | \sum_{j=1}^{n} d_j x_j \le l; x_j \ge 0, j = 1, \dots, n\right. \qquad P_2 = \left\{x \in R^2 | \sum_{j=1}^{n} b_j x_j \le l; x_j \ge 0, j = 1, \dots, n\right.$$

$$P_3 = \left\{x \in R^2 | \sum_{j=1}^{n} c_j x_j \le l; x_j \ge 0, j = 1, \dots, n\right. \qquad P_4 = \left\{x \in R^2 | \sum_{j=1}^{n} a_j x_j \le l; x_j \ge 0, j = 1, \dots, n\right.$$

Here, the interval [P1, P2] is the possibly optimal range (upper), and the interval [P3, P4] is the surely optimal range (lower).

2.1 The Equivalent Crisp Model for Three-level Linear Problem with Rough in Constraints

The equivalent problem of three-level fractional programming problem with rough in constraints by using Intervals method can be written as:

P1	**P2**
$\underset{\bar{x}_1}{\text{Max}}\ f_1\,(x) = \dfrac{a_1 x + \alpha_1}{b_1 x + \beta_1}$,	$\underset{\bar{x}_1}{\text{Max}}\ f_1\,(x) = \dfrac{a_1 x + \alpha_1}{b_1 x + \beta_1}$,
$\underset{\bar{x}_2}{\text{Max}}\ f_2\,(x) = \dfrac{a_2 x + \alpha_2}{b_2 x + \beta_2}$,	$\underset{\bar{x}_2}{\text{Max}}\ f_2\,(x) = \dfrac{a_2 x + \alpha_2}{b_2 x + \beta_2}$,
$\underset{\bar{x}_3}{\text{Max}}\ f_3\,(x) = \dfrac{a_3 x + \alpha_3}{b_3 x + \beta_3}$,	$\underset{\bar{x}_3}{\text{Max}}\ f_3\,(x) = \dfrac{a_3 x + \alpha_3}{b_3 x + \beta_3}$,
Subject to	Subject to
$s = \{\ \sum d_j x_j \le l_j, x_j \ge 0\ \}$.	$s = \{\ \sum b_j x_j \le l_j, x_j \ge 0\ \}$.
P3	**P4**
$\underset{\bar{x}_1}{\text{Max}}\ f_1\,(x) = \dfrac{a_1 x + \alpha_1}{b_1 x + \beta_1}$,	$\underset{\bar{x}_1}{\text{Max}}\ f_1\,(x) = \dfrac{a_1 x + \alpha_1}{b_1 x + \beta_1}$,
$\underset{\bar{x}_2}{\text{Max}}\ f_2\,(x) = \dfrac{a_2 x + \alpha_2}{b_2 x + \beta_2}$,	$\underset{\bar{x}_2}{\text{Max}}\ f_2\,(x) = \dfrac{a_2 x + \alpha_2}{b_2 x + \beta_2}$,
$\underset{\bar{x}_3}{\text{Max}}\ f_3\,(x) = \dfrac{a_3 x + \alpha_3}{b_3 x + \beta_3}$,	$\underset{\bar{x}_3}{\text{Max}}\ f_3\,(x) = \dfrac{a_3 x + \alpha_3}{b_3 x + \beta_3}$,
Subject to	Subject to
$s = \{\ \sum c_j x_j \le l_j, x_j \ge 0\ \}$.	$s = \{\ \sum a_j x_j \le l_j, x_j \ge 0\ \}$.

Interval method has been used to convert problem (1)-(4) from rough nature to crisp that resulted in four multi-level fractional programming problem, each level has his optimal solution.

To deal with the conflict between solutions the fuzzy approach is used.

3 Fuzzy Approach of Three Level Linear Fractional Programming Problems [12]

To solve Three-level linear programming problem, by using fuzzy approach combine with a computer oriented technique, the fuzzy approach split the problem into three separated problems.

3.1 FLDM Problem

First, The FLDM solves his Problem and find the individual best and the worst solution $F_1^* = \left(\bar{f}_1^L, \bar{f}_1^U, \underline{f}_1^L, \underline{f}_1^U\right)$, $F_1^- = \left(\bar{f}_1^L, \bar{f}_1^U, \underline{f}_1^L, \underline{f}_1^U\right)$ where $F_1^* = \max_{x \in G} F_1(x)$, $F_1^- = \min_{\,x \in G} F_1(x)$

This data can then be formulated as the following membership function:

$$\mu[F_1(x)] = \begin{cases} 1 & \text{if} \quad F_1(x) > F_1^*, \\ \dfrac{F_1(x) - \bar{F}_1}{F_1^* - \bar{F}_1} & \text{if} \quad \bar{F}_1 \le F_1(x) \le F_1^*, \\ 0 & \text{if} \quad \bar{F}_1 \ge F_1(x). \end{cases} \tag{5}$$

Solution of the FLDM can be obtained by solving the Tchebycheff Problem:

$$\max \lambda \tag{6}$$

Subject to

$x \in G$,

$\mu[F_1(x)] \geq \lambda$,

$\lambda \in [0,1]$.

Whose solution is assumed to be $[x_1^F, x_2^F, x_3^F, F_1^F, \lambda^F]$ where λ^F is satisfactory level.

3.2 SLDM Problem

The SLDM do the same action like the FLDM till solution had been obtained to be $[x_1^S, x_2^S, x_3^S, F_2^S, \lambda^S]$ where λ^S is satisfactory level.

3.3 TLDM Problem

The TLDM do the same action like the SLDM till solution had been obtained to be $[x_1^T, x_2^T, x_3^T, F_3^T, \lambda^T]$ where λ^T is satisfactory level.

FLDM, SLDM, and TLDM solution are now disclosed, and due the difference in the objective function these solutions are usually different, and a conflict arise when FLDM uses his optimal decision x_1^F as a control factors for the SLDM, and TLDM, as a solution there must be a tolerance that gives the SLDM, TLDM a possibility to search for their optimal solution by extending their a feasible region.

Again, the same problem arise when SLDM using his optimal decisions x_1^S as a control factor for TLDM ,as a practical solution there must be a tolerance that gives the TLDM a possibility to search for his optimal solution by extending his feasible region.

In this way, the range of decision variable x_1, x_2 should be around x_1^F, x_2^S with maximum tolerance t_1, t_2 and the following membership function specifies x_1, x_2 as:

$$\mu(x_1) = \begin{cases} \frac{x_1-(x_1^F-t_1)}{t_1} & x_1^F - t_1 \leq x_1 \leq x_1^F, \\ \frac{-x_1+(x_1^F+t_1)}{t_1} & x_1^F \leq x_1 \leq x_1^F - t_1, \end{cases} \tag{7}$$

$$\mu(x_2) = \begin{cases} \dfrac{x_2-(x_2^S-t_2)}{t_2} x_2^S - t_2 \leq x_2 \leq x_2^S, \\ \dfrac{-x_2+(x_2^S+t_2)}{t_2} x_2^S \leq x_2 \leq x_2^S - t_2. \end{cases} \tag{8}$$

The FLDM goals are absolutely acceptable if $F_1 \geq F_1^F$ and absolutely unacceptable if $F_1 \leq F_1^F$, and that the preference with $[\grave{F_1}, F_1^F]$ is linearly increasing. This due to the fact that the SLDM obtained the optimum at $\left(x_1^S, x_2^S, x_3^S\right)$, which in turn provides the FLDM the objective function values $\grave{F_1}$, make any $F_1 \geq \grave{F_1} = F_1(x_1^S, x_2^S, x_3^S)$ unattractive in practice.

The following membership functions of the FLDM can be stated as:

$$\grave{\mu}[F_1(x)] = \begin{cases} 1 & F_1(x) > F_1^F, \\ \dfrac{F_1(x) - \grave{F}_1}{F_1^F - \grave{F}_1} & \grave{F}_1 \le F_1(x) \le F_1^F, \\ 0 & \grave{F}_1 \ge F_1(x). \end{cases} \tag{9}$$

Second, the SLDM goals may reasonably consider that $F_2 \ge F_2^S$ is absolutely acceptable and $F_2 \le F_2^S$ is absolutely unacceptable, and that the preference with $[\grave{F}_2, F_2^S]$ is linearly increasing. In this way, the SLDM has the following membership functions for his/her goal as (9).

Third, the TLDM goals may reasonably consider that $F_3 \ge F_3^T$ is absolutely acceptable and $F_3 \le F_3^T$ is absolutely unacceptable, and that the preference with $[\grave{F}_3, F_3^T]$ is linearly increasing. In this way, the TLDM has the following membership functions for his/her goal as (9).

Finally, in order to generate the satisfactory solution, which is also a Pareto optimal solution with overall satisfaction for all decision-makers, we can solve the following Tchebycheff problem.

$$\max \beta, \tag{10}$$

Subject to

$$\grave{\mu}[F_1(x)] \ge \beta,$$

$$\grave{\mu}[F_2(x)] \ge \beta,$$

$$\grave{\mu}[F_3(x)] \ge \beta,$$

$$\frac{[x_1 - (x_1^F - t_1)]}{t_1} \ge \beta,$$

$$\frac{-x_1 + (x_1^F + t_1)}{t_1} \ge \beta,$$

$$\frac{[x_2 - \left(x_2^S - t_2\right)]}{t_2} \ge \beta,$$

$$\frac{-x_2 + \left(x_2^S + t_2\right)}{t_2} \ge \beta,$$

$$x \in G,$$

$$t_i > 0\,,$$

$$\beta \in [0,1].$$

4 Numerical Example

The three-level Fractional Linear programming problem with rough in objective function can be formulated as:

$$\max_{x_1} Z_1 = \frac{2x_1 + x_2 + 3x_3}{x_1 + x_2 + x_3}$$

$$\max_{x_2} Z_2 = \frac{x_1 + 4x_2 - 2x_3}{2x_1 + 2x_2 + x_3 + 3}$$

$$\max_{x_3} Z_3 = \frac{3x_1 + x_2 - x_3}{4x_1 + 3x_2 - x_3}$$

Subject to

$$([7,9][6,10])x_1 + ([10,11][8,12])x_2 + ([11,13][10,12])x_3 \le 7,$$

$$([6,8][5,9])x_1 - ([9,10][7,11])x_2 + ([5,7][4,8])x_3 \le 4,$$

$$-([8,10][7,11])x_1 + ([13,15][12,16])x_2 + ([8,10][7,11])x_3 \ge 1,$$

$$([5,7][4,8])x_1 + ([6,8][5,9])x_2 \ge 3,$$

$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.$$

4.1 FLDM Problem

By converting the FLPP into its equivalent LPP using Charnes and Cooper transforms the FLPP into the following linear programming problem:

max $2y_1 + y_2 + 3y_3$

Subject to

$$y_1 + y_2 + y_3 = 1,$$

$$([7,9][6,10])y_1 + ([10,11][8,12])y_2 + ([11,13][10,12])y_3 - 7\rho_1 \le 0,$$

$$([6,8][5,9])y_1 - ([9,10][7,11])y_2 + ([5,7][4,8])y_3 - 4\rho_1 \le 0,$$

$$-([8,10][7,11])y_1 + ([13,15][12,16])y_2 + ([8,10][7,11])y_3 - \rho_1 \ge 0,$$

$$([5,7][4,8])y_1 + ([5,7][4,8])y_2 - 3\rho_1 \ge 0,$$

$$y_1 \ge 0, y_2 \ge 0, y_3 \ge 0, and\ \rho_1 > 0.$$

The equivalent problem of the first level programming problem with rough in objective function by using Intervals method can be written as

Table 1. The upper and lower FLDM problems

Upper	**Lower**
$\boldsymbol{P1}:=\textbf{max}\ \ \boldsymbol{2y_1 + y_2 + 3y_3}$	$\boldsymbol{P3}:\textbf{max}\ \ \boldsymbol{2y_1 + y_2 + 3y_3}$
Subject to	Subject to
$y_1 + y_2 + y_3 = 1,$	$y_1 + y_2 + y_3 = 1,$
$6y_1 + 8y_2 + 10y_3 - 7\rho_1 \le 0,$	$7y_1 + 10y_2 + 11y_3 - 7\rho_1 \le 0,$
$5y_1 - 7y_2 + 4y_3 - 4\rho_1 \le 0,$	$6y_1 - 9y_2 + 5y_3 - 4\rho_1 \le 0,$
$-7y_1 + 12y_2 + 7y_3 - \rho_1 \ge 0,$	$-8y_1 + 13y_2 + 8y_3 - \rho_1 \ge 0,$
$4y_1 + 4y_2 - 3\rho_1 \ge 0,$	$5y_1 + 5y_2 - 3\rho_1 \ge 0,$

Upper	**Lower**
$\boldsymbol{P2}$: **max** $\boldsymbol{2y_1 + y_2 + 3y_3}$	$\boldsymbol{P4}$: **max** $\boldsymbol{2y_1 + y_2 + 3y_3}$
Subject to	Subject to
$y_1 + y_2 + y_3 = 1,$	$y_1 + y_2 + y_3 = 1,$
$10y_1 + 12y_2 + 12y_3 - 7\rho_1 \le 0,$	$9y_1 + 11y_2 + 13y_3 - 7\rho_1 \le 0,$
$9y_1 - 11y_2 + 8y_3 - 4\rho_1 \le 0,$	$8y_1 - 10y_2 + 7y_3 - 4\rho_1 \le 0,$
$-11y_1 + 16y_2 + 11y_3 - \rho_1 \ge 0,$	$-10y_1 + 15y_2 + 10y_3 - \rho_1 \ge 0,$
$8y_1 + 8y_2 - 3\rho_1 \ge 0,$	$7y_1 + 7y_2 - 3\rho_1 \ge 0,$

The FLDM build the membership functions $\mu(\bar{f}_1^L, \bar{f}_1^U, \underline{f}_1^L, \underline{f}_1^U)(x)$ from Table 1 then solve problem (6) as follows:

Table 2. The upper and lower Tchebycheff problems

P1	***P3***
max λ , subject to $2y_1 + y_2 + 3y_3 \le 1.94\lambda,$ $x \in G$, $\lambda \in [0,1]$. Whose solution is $(y_1^F, y_2^F, y_3^F, \rho_1^F) = (0,1,0,1.14)$, $(\bar{\boldsymbol{f}}_1^l)^F = \mathbf{1}$, $\lambda^F = 0.9$.	max λ , subject to $2y_1 + y_2 + 3y_3 \le 2\lambda,$ $x \in G$, $\lambda \in [0,1]$. Whose solution is $(y_1^F, y_2^F, y_3^F, \rho_1^F) = (0,1,0,1.4)$, $(\underline{\boldsymbol{f}}_1^l)^F = \mathbf{1}$, $\lambda^F = 0.9$.
P2	***P4***
max λ , subject to $2y_1 + y_2 + 3y_3 \le 2.271\,\lambda,$ $x \in G$, $\lambda \in [0,1]$. Whose solution is $(y_1^F, y_2^F, y_3^F, \rho_1^F) = (0,1,0,1.7)$, $(\bar{\boldsymbol{f}}_1^u)^F = \mathbf{1}$, $\lambda^F = 0.9$.	max λ , subject to $2y_1 + y_2 + 3y_3 \le 2.15\lambda,$ $x \in G$, $\lambda \in [0,1]$. Whose solution is $(y_1^F, y_2^F, y_3^F, \rho_1^F) =$ (0,1,0,1.57), $(\underline{\boldsymbol{f}}_1^u)^F = \mathbf{1}$, $\lambda^F = 0.9$.

4.2 SLDM Problem

By converting the SLDM into its equivalent LPP using Charnes and Cooper transforms the SLDM into the following linear programming problem:

$$\max \ y_1 + 4y_2 - 2y_3$$

Subject to

$$2y_1 + 2y_2 + y_3 + 3\rho_1 = 1,$$

$$([7,9][6,10])y_1 + ([10,11][8,12])y_2 + ([11,13][10,12])y_3 - 7\rho_1 \le 0,$$

$$([6,8][5,9])y_1 - ([9,10][7,11])y_2 + ([5,7][4,8])y_3 - 4\rho_1 \le 0,$$

$$-([8,10][7,11])y_1 + ([13,15][12,16])y_2 + ([8,10][7,11])y_3 - \rho_1 \ge 0,$$

$$([5,7][4,8])y_1 + ([5,7][4,8])y_2 - 3\rho_1 \ge 0,$$

$$y_1 \ge 0, y_2 \ge 0, y_3 \ge 0, and\ \rho_1 > 0.$$

The equivalent problem of the first level programming problem with rough in objective function by using Intervals method can be written as

Upper	**Lower**
$\boldsymbol{P1}$: $=$ **max** $y_1 + 4y_2 - 2y_3$ Subject to $2y_1 + 2y_2 + y_3 + 3\rho_1 = 1,$ $6y_1 + 8y_2 + 10y_3 - 7\rho_1 \le 0,$ $5y_1 - 7y_2 + 4y_3 - 4\rho_1 \le 0,$ $-7y_1 + 12y_2 + 7y_3 - \rho_1 \ge 0,$ $4y_1 + 4y_2 - 3\rho_1 \ge 0,$	$\boldsymbol{P3}$: **max** $y_1 + 4y_2 - 2y_3$ Subject to $2y_1 + 2y_2 + y_3 + 3\rho_1 = 1,$ $7y_1 + 10y_2 + 11y_3 - 7\rho_1 \le 0,$ $6y_1 - 9y_2 + 5y_3 - 4\rho_1 \le 0,$ $-8y_1 + 13y_2 + 8y_3 - \rho_1 \ge 0,$ $5y_1 + 5y_2 - 3\rho_1 \ge 0,$
$\boldsymbol{P2}$: **max** $y_1 + 4y_2 - 2y_3$ Subject to $2y_1 + 2y_2 + y_3 + 3\rho_1 = 1,$ $10y_1 + 12y_2 + 12y_3 - 7\rho_1 \le 0,$ $9y_1 - 11y_2 + 8y_3 - 4\rho_1 \le 0,$ $-11y_1 + 16y_2 + 11y_3 - \rho_1 \ge 0,$ $8y_1 + 8y_2 - 3\rho_1 \ge 0,$	$\boldsymbol{P4}$: **max** $y_1 + 4y_2 - 2y_3$ Subject to $2y_1 + 2y_2 + y_3 + 3\rho_1 = 1,$ $9y_1 + 11y_2 + 13y_3 - 7\rho_1 \le 0,$ $8y_1 - 10y_2 + 7y_3 - 4\rho_1 \le 0,$ $-10y_1 + 15y_2 + 10y_3 - \rho_1 \ge 0,$ $7y_1 + 7y_2 - 3\rho_1 \ge 0,$

The SLDM build the membership functions $\mu(\bar{f}_2^L, \bar{f}_2^U, \underline{f}_2^L, \underline{f}_2^U)(x)$ from Table 1 then solve problem (6) as follows:

P1	***P3***
$\max \lambda$, subject to $y_1 + 4y_2 - 2y_3 \le .7368\lambda,$ $x \in G$, $\lambda \in [0,1]$. Whose solution is $(y_1^F, y_2^F, y_3^F, \rho_1^F) =$ $(.10, .5, .4, .21)$, $(\bar{\boldsymbol{f}}_2^{\boldsymbol{l}})^{\boldsymbol{F}} = \mathbf{.2382}$, $\lambda^F = 0.9$.	$\max \lambda$, subject to $y_1 + 4y_2 - 2y_3 \le .6363\lambda,$ $x \in G$, $\lambda \in [0,1]$. Whose solution is $(y_1^F, y_2^F, y_3^F, \rho_1^F) = (.90, .45, .45, .227)$, $(\underline{\boldsymbol{f}}_2^{\boldsymbol{l}})^{\boldsymbol{F}} = \mathbf{.1818}$, $\lambda^F = 0.9$.
P2	***P4***
$\max \lambda$, subject to $y_1 + 4y_2 - 2y_3 \le .5600\lambda,$ $x \in G$, $\lambda \in [0,1]$. Whose solution is $(y_1^F, y_2^F, y_3^F, \rho_1^F) =$ $(.72, .21, .64, .249)$, $(\bar{\boldsymbol{f}}_2^{\boldsymbol{u}})^{\boldsymbol{F}} = \mathbf{.29332}$, $\lambda^F = 0.9$.	$\max \lambda$, subject to $y_1 + 4y_2 - 2y_3 \le .5957\lambda,$ $x \in G$, $\lambda \in [0,1]$. Whose solution is $(y_1^F, y_2^F, y_3^F, \rho_1^F) = (.75, .3, .54, .24)$, $(\underline{\boldsymbol{f}}_2^{\boldsymbol{u}})^{\boldsymbol{F}} = \mathbf{.8563}$, $\lambda^F = 0.9$.

4.3 TLDM Problem

By converting the TLDM into its equivalent LPP using Charnes and Cooper transforms the TLDM into the following linear programming problem:

$$\max \ 3y_1 + y_2 - y_3$$

Subject to

$$4y_1 + 3y_2 - y_3 = 1,$$

$$([7,9][6,10])y_1 + ([10,11][8,12])y_2 + ([11,13][10,12])y_3 - 7\rho_1 \le 0,$$

$$([6,8][5,9])y_1 - ([9,10][7,11])y_2 + ([5,7][4,8])y_3 - 4\rho_1 \le 0,$$

$$-([8,10][7,11])y_1 + ([13,15][12,16])y_2 + ([8,10][7,11])y_3 - \rho_1 \ge 0,$$

$$([5,7][4,8])y_1 + ([5,7][4,8])y_2 - 3\rho_1 \ge 0,$$

$$y_1 \ge 0, y_2 \ge 0, y_3 \ge 0, and\ \rho_1 > 0.$$

The equivalent problem of the first level programming problem with rough in objective function by using Intervals method can be written as

Upper	**Lower**
$\boldsymbol{P1}$: $=$ **max** $3y_1 + y_2 - y_3$ Subject to $4y_1 + 3y_2 - y_3 = 1,$ $6y_1 + 8y_2 + 10y_3 - 7\rho_1 \le 0,$ $5y_1 - 7y_2 + 4y_3 - 4\rho_1 \le 0,$ $-7y_1 + 12y_2 + 7y_3 - \rho_1 \ge 0,$ $4y_1 + 4y_2 - 3\rho_1 \ge 0,$	$\boldsymbol{P3}$: **max** $3y_1 + y_2 - y_3$ Subject to $4y_1 + 3y_2 - y_3 = 1,$ $7y_1 + 10y_2 + 11y_3 - 7\rho_1 \le 0,$ $6y_1 - 9y_2 + 5y_3 - 4\rho_1 \le 0,$ $-8y_1 + 13y_2 + 8y_3 - \rho_1 \ge 0,$ $5y_1 + 5y_2 - 3\rho_1 \ge 0,$
$\boldsymbol{P2}$: **max** $3y_1 + y_2 - y_3$ Subject to $4y_1 + 3y_2 - y_3 = 1,$ $10y_1 + 12y_2 + 12y_3 - 7\rho_1 \le 0,$ $9y_1 - 11y_2 + 8y_3 - 4\rho_1 \le 0,$ $-11y_1 + 16y_2 + 11y_3 - \rho_1 \ge 0,$ $8y_1 + 8y_2 - 3\rho_1 \ge 0,$	$\boldsymbol{P4}$: **max** $3y_1 + y_2 - y_3$ Subject to $4y_1 + 3y_2 - y_3 = 1,$ $9y_1 + 11y_2 + 13y_3 - 7\rho_1 \le 0,$ $8y_1 - 10y_2 + 7y_3 - 4\rho_1 \le 0,$ $-10y_1 + 15y_2 + 10y_3 - \rho_1 \ge 0,$ $7y_1 + 7y_2 - 3\rho_1 \ge 0,$

The TLDM build the membership functions $\mu(\bar{f}_2^L, \bar{f}_2^U, \underline{f}_2^L, \underline{f}_2^U)(x)$ from Table 1 then solve problem (6) as follows:

P1	***P3***
max λ, subject to $3y_1 + y_2 - y_3 \le .6049\lambda,$ $x \in G$, $\lambda \in [0,1].$ Whose solution is $(y_1^F, y_2^F, y_3^F, \rho_1^F) = (0, .34, .46, .46),$ $(\bar{\boldsymbol{f}}_2^{\boldsymbol{l}})^{\boldsymbol{F}} = \mathbf{.3023},$ $\lambda^F = 0.9$.	max λ, subject to $3y_1 + y_2 - y_3 \le .6000\lambda,$ $x \in G$, $\lambda \in [0,1].$ Whose solution is $(y_1^F, y_2^F, y_{3,}^F, \rho_1^F) =$ $(0, .35, .53, .58),$ $(\underline{\boldsymbol{f}}_2^{\boldsymbol{l}})^{\boldsymbol{F}} = \mathbf{.2978},$ $\lambda^F = 0.9$.
P2	***P4***
max λ, subject to $3y_1 + y_2 - y_3 \le .6025\lambda,$	max λ, subject to $3y_1 + y_2 - y_3 \le .5976\lambda,$

x ∈ G , λ ∈ [0,1]. Whose solution is $(y_1^F, y_2^F, y_{3,}^F, \rho_1^F) = (0, .40, .22, 1.09)$, $(\bar{\boldsymbol{f}}_{\boldsymbol{2}}^{\boldsymbol{u}})^{\boldsymbol{F}} = \mathbf{.1818}$, $\lambda^F = 0.9$.	$x \in G$, $\lambda \in [0,1]$. Whose solution is $(y_1^F, y_2^F, y_3^F, \rho_1^F) =$ $(0, .38, .15, .90)$, $(\underline{\boldsymbol{f}}_{\boldsymbol{2}}^{\boldsymbol{u}})^{\boldsymbol{F}} = \mathbf{.2277}$, $\lambda^F = 0.9$.

Finally, in order to generate the satisfactory solution, which is also a Pareto optimal solution with overall satisfaction for all decision-makers, by using (10):

1- We assume the FLDM'S control decision$x_1^F$with the tolerance 1, and assume the SLDM'S control decision x_2^S with the tolerance 1.2.

2- By using [(5)–(7)] calculating membership functionsμ̀, then solves the Tchebycheff problem for equivalent crisp model for each level.

$\boldsymbol{Max\omega_1}$, *Subject to* $2y_1 + y_2 + 3y_3 - 1.9 \geq -.9\omega_1$, $y_1 + 4y_2 - 2y_3 - 4 \geq -3.7618\omega_1$, $3y_1 + y_2 - y_3 - .4 \geq -.09\omega_1$, $x_1 + 1 \geq \omega_1$, $-x_1 + 1 \geq \omega_1$, $x_2 + .7 \geq 1.2\omega_1$, $-x_2 + 1.7 \geq 1.2\omega_1$, $x \in G$, $t_i > 0, (i = 1,2)$, $\omega_1 \in [0,1]$. $(y_1, y_2, y_3) = (0, .50, .166)$. *Objective value 1*	$\boldsymbol{Max\omega_3}$, *Subject to* $2y_1 + y_2 + 3y_3 - 3.6 \geq -2.6\omega_1$, $y_1 + 4y_2 - 2y_3 - 4 \geq -3.8182\omega_1$, $3y_1 + y_2 - y_3 - 2.7 \geq -2.4\omega_1$, $x_1 + 1 \geq \omega_3$, $-x_1 + 1 \geq \omega_3$, $x_2 + .75 \geq 1.2\omega_3$, $-x_2 + 1.65 \geq 1.2\omega_3$, $x \in G$, $t_i > 0, (i = 1,2)$, $\omega_3 \in [0,1]$. $(y_1, y_2, y_3) = (.1666, .4700, .1800)$.. *Objective value .9833*
$\boldsymbol{Max\omega_2}$, *Subject to* $2y_1 + y_2 + 3y_3 - 3.57 \geq -2.57\omega_1$, $y_1 + 4y_2 - 2y_3 - 4 \geq -3.70668\omega_1$, $3y_1 + y_2 - y_3 - 1.73 \geq -1.548\omega_1$, $x_1 + 1 \geq \omega_2$, $-x_1 + 1 \geq \omega_2$, $x_2 + 1 \geq 1.2\omega_2$, $-x_2 + 1.4 \geq 1.2\omega_2$, $x \in G$, $t_i > 0, (i = 1,2)$, $\omega_2 \in [0,1]$. $(y_1, y_2, y_3) = (.8688, .3042, .2484)$. *Objective value .913*	$\boldsymbol{Max\omega_4}$, *Subject to* $2y_1 + y_2 + 3y_3 - 3.42 \geq -2.42\omega_1$, $y_1 + 4y_2 - 2y_3 - 4 \geq -3.1437\omega_1$, $3y_1 + y_2 - y_3 - 2.01 \geq -1.7823\omega_1$, $x_1 + 1 \geq \omega_4$, $-x_1 + 1 \geq \omega_4$, $x_2 + .9 \geq 1.2\omega_4$, $-x_2 + 1.5 \geq 1.2\omega_4$, $x \in G$, $t_i > 0, (i = 1,2)$, $\omega_4 \in [0,1]$. $(y_1, y_2, y_3) = (.6013, .3721, 2176)$. *Objective value .9398*

After using fuzzy approach to solve multi-level programming problem we get the final intervals:

The possibly range	***The surely range***
FLDM: [1, 2.8]	FLDM: [1.5, 2.2].
SLDM: [1.7, 1.6].	SLDM: [1.7, 1.6].
TLDM: [.3, 2.7].	TLDM: [.8, 2].

5 Conclusion

A three level linear fractional programming problem was considered where some or all of its coefficient in the constraints are rough intervals, at the first phase of the solution converting the fractional programming problem into linear programming problem for each level is necessary then two problems with interval coefficient will be constructed one of these problems all of its coefficient are upper approximation and other problem all of its coefficients are lower approximation of rough intervals. As a final phase a membership function was constructed to develop a fuzzy model for obtaining the optimal solution of the three level programming problem.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Lai YJ. Hierarchical optimization: A satisfactory solution. Fuzzy Sets and Systems. 1996;77:3: 321-335.

[2] Sohrab Effati, Morteza Pakdaman. Solving the interval-valued linear fractional programming problem. American Journal of Computational Mathematics. 2012;2:51-55.

[3] Charnes A, Cooper WW. Programs with linear fractional functions. Naval Research Logistics Quarterly. 1962;9(3-4):181-196.

[4] Tantawy SF. A new procedure for solving linear fractional programming problems. Mathematical and Computer Modelling. 2008;48(5-6):969-973.

[5] Pandey P, Punnen AP. A simplex algorithm for piecewise-linear fractional programming problems. European Journal of Operational Research. 2007;178(2):343-358.

[6] De Korvin A, McKeegan C, Kleyle R. Knowledge acquisition using rough sets when membership values are fuzzy sets. Journal of Intelligent and Fuzzy Systems. 1998;6:237–244.

[7] Liu B. Theory and practice of uncertain programming. Physica Verlag, New York; 2002.

[8] Ali Hamzehee, Mohammad Ali Yaghoobia, Mashaalah Mashinchib. Linear programming with rough interval coefficients. Journal of Intelligent & Fuzzy Systems. 2014;26:1179–1189.

[9] Omar M. Saad, Emam OE, Marwa M. Sleem. On the solution of a rough interval three-level quadratic programming problem. British Journal of Mathematics & Computer Science. 2015;5:3.
ISSN: 2231-0851.

[10] Emam OE, Nasr SA. A three-level quadratic programming problem with random rough coefficient in constraints. International Journal of Mathematical Archive. 2014;5(6):11-18.

[11] Partha Pratim Dey, Surapati Pramanik, Bibhas C. Giri. MULTI-level linear fractional programming problem based on fuzzy goal programming approach. International Journal of Innovative Research in Technology & Science (IJIRTS); 2014.

[12] Osman M, Abo-Sinna M, Amer A, Emam O. A multi-level non-linear multi-objective decision-making under fuzziness. Applied Mathematics and Computation. 2004;153:239-252.

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