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Extended (G'/G)-Expansion Method for Abundant Traveling Wave Solutions of Nonlinear Evolution CDG Equation

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Abstract

In the present paper, extended (G'/G)-expansion method is used to find new precise solutions of nonlinear partial differential equations with the aid of symbolic computation. To test the validity of the algorithm, the fifth order CDG equation has been used. Plentiful traveling wave solutions including; exponential, hyperbolic and trigonometric functions are successfully accomplished by the proposed method with capricious parameters. It is revealed that the proposed method is straightforward, constructive and many nonlinear evolution equations in mathematical physics are solved by this method.

Keywords: Extended (G'/G)-expansion method; fifth order Caudrey-Dodd-Gibbon equation; auxiliary equation; travelling wave solutions.

1 Introduction

The nonlinear evolution equations (NLEEs) are widely used as models to describe complex physical phenomena in various fields of sciences, especially in fluid mechanics, solid state physics, plasma physics, plasma waves and biology. One of the basic physical problems for those models is to obtain their traveling wave solutions. Particularly, various methods have been utilized to explore different kinds of solutions of physical models depicted by nonlinear PDE's.

Several effective analytical and semi analytical methods including Homotopy Perturbation method [1], Variational Iteration method [2], Parameter-expansion method [3], Exp-function method [4], Sine–Cosine method [5], Tanh method [6,7], Simple equation method [8,9] and other methods [10,11] have been developed considerably to tackle nonlinear partial differential equations.

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A straightforward and brief method called (G'/G)-expansion was introduced by Wang et al. [12] to obtain exact traveling wave solutions of nonlinear evolution equations, whereas the differential equation $G'' + \lambda G' + \mu G = 0$ is used as an auxiliary equation. The better perceptive of (G'/G)-expansion method are given in [13-17]. In order to instigate the efficiency and trustworthiness of the (G'/G)-expansion method and to enlarge the possibility of its application, further research has been conceded out by many authors. For instance, an improvement of (G'/G)-expansion method was made by Zhang et al. [18] to search for further general traveling wave solutions. Another extension of (G'/G)-expansion method was given by Zayed [19] to attain new exact solutions, whereas Jacobi elliptic equation $[G'(\xi)]^2 = e_2 G^4(\xi) + e_1 G^2(\xi) + e_0$, is used as an auxiliary equation and e_2, e_1, e_0 are random constants. Other improvements of (G'/G)-expansion method are given in [20-28]. The (G'/G)-expansion method and the transformed rational function method used by Ma [29] have a common idea. That is, we firstly put the given nonlinear evolution equation into the corresponding ordinary differential equation (ODE), then the ordinary differential equation can be transformed into systems of algebraic polynomials with the determining constants. By the solutions of the ordinary differential equation, we can obtain the exact traveling solutions and rational solution of the nonlinear evolution equation. However, to get the *N*-soliton and *N*-wave solution of the partial differential equation, we may consider the linear superposition principle [30] and multiple exp-function method [31]; these methods can be applied to the Hirota bilinear equations and others. Moreover, many encouraging solution formulas are obtained using the variation of parameters method by Ma and You [32] for solving the concerned non-homogeneous partial differential equations. They obtained many new solutions including rational solutions, solitons, positions, negatons, breathers, complextions and interaction solutions of the KdV equations. Lately, Ma et al. [33] presented a much more general idea to yield exact solutions to nonlinear wave equations by searching for the so-called Frobenius transformations. More recently, Liu et al. [34] introduced an improved (G'/G)-expansion method and find more general traveling wave solutions of two nonlinear evolution equations. The traveling wave solutions are obtained of the nonlinear Zakharov equations by a good-looking method called $(G'/G, 1/G)$-expansion method introduced by Li et al. [35] also.

In this paper, we will form more general and precise solutions of nonlinear evolution equations by using the extended (G'/G)-expansion method. For illustration, we restrict our attention to the study of the fifth order Caudrey-Dodd-Gibbon equation and successfully construct numerous novel and broader exact solutions.

2 The Methodology

Consider the following nonlinear differential equation in the form,

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \cdots) = 0. \tag{1}$$

The key steps of the extended (G'/G)-expansion method are:

Step 1: The wave transformation

$$u\,(x,t) = u\,(\xi),\ \xi = x - V\,t, \tag{2}$$

where V is the speed of the traveling wave, that change Eq. (1) to an ordinary differential equation in the form:

$$F(u, u', u'', u''', \cdots) = 0. \tag{3}$$

Step 2: Imagine the solution of the Eq. (3) can be written as follows:

$$u(\xi) = \sum_{i=0}^{N} \alpha_i \left(\frac{G'}{G} \right)^i, \tag{4}$$

where α_i ($i = 1, 2, 3, \cdots, N$) are constants provided $\alpha_N \neq 0$ and $G(\xi)$ satisfies the following auxiliary equation

$$GG'' = AG^2 + BGG' + C(G')^2, \tag{5}$$

where *A*, *B* and *C* are arbitrary constants which are calculated later. We have the following four different solutions of (G'/G) from the wide-ranging solutions of Eq. (5) with the help of Maple:

Case 1: When $B \neq 0$ and $\Delta = B^2 + 4A - 4AC \geq 0,$ then

$$\left(\frac{G'}{G} \right) = \frac{B}{2(1-C)} + \frac{\sqrt{\Delta}}{2(1-C)} \frac{\alpha e^{\frac{\sqrt{\Delta}}{2}\xi} + \beta e^{-\frac{\sqrt{\Delta}}{2}\xi}}{\alpha e^{\frac{\sqrt{\Delta}}{2}\xi} - \beta e^{-\frac{\sqrt{\Delta}}{2}\xi}}. \tag{6}$$

Case 2: When $B \neq 0$ and $\Delta = B^2 + 4A - 4AC < 0,$ then

$$\left(\frac{G'}{G} \right) = \frac{B}{2(1-C)} + \frac{\sqrt{-\Delta}}{2(1-C)} \frac{i\alpha \cos \frac{\sqrt{-\Delta}}{2}\xi - \beta \sin \frac{\sqrt{-\Delta}}{2}\xi}{i\alpha \sin \frac{\sqrt{-\Delta}}{2}\xi + \beta \cos \frac{\sqrt{-\Delta}}{2}\xi}. \tag{7}$$

Case 3: When $B = 0$ and $\Delta = A(C-1) \geq 0,$ then

$$\left(\frac{G'}{G} \right) = \frac{\sqrt{\Delta}}{(1-C)} \frac{\alpha \cos \sqrt{\Delta}\xi + \beta \sin \sqrt{\Delta}\xi}{\alpha \sin \sqrt{\Delta}\xi - \beta \cos \sqrt{\Delta}\xi}. \tag{8}$$

Case 4: When $B = 0$ and $\Delta = A(C-1) < 0,$ then

$$\left(\frac{G'}{G} \right) = \frac{\sqrt{-\Delta}}{(1-C)} \frac{i\alpha \cosh \sqrt{-\Delta}\xi - \beta \sinh \sqrt{-\Delta}\xi}{i\alpha \sinh \sqrt{-\Delta}\xi - \beta \cosh \sqrt{-\Delta}\,\xi}, \tag{9}$$

where $\xi = x - Vt$, *V* is the wave velocity. *A*, *B*, *C* and α, β are real parameters.

Step 3: The positive integer N in Eq. (4) can be calculated by comparing the highest order nonlinear term with the highest order linear term appearing in Eq. (3).

Step 4: Inserting Eq. (4) into Eq. (3) along with Eq. (5), an algebraic structure of equations in powers of $(G'/G)^i$ will lead to the calculation of the arbitrary constants α_N and V with Maple.

3 Applications

In this section, extended (G'/G)-expansion method will be applied to generate abundant traveling wave solution of the Caudrey-Dodd-Gibbon equation and some of the solutions are shown in graphs.

3.1 The Fifth Order CDG Equation

We start with the fifth order Caudrey–Dodd–Gibbon (CDG) equation [36]

$$u_t + u_{xxxxx} + 30uu_{xxx} + 30u_x u_{xx} + 180u^2 u_x = 0. \tag{10}$$

This equation play a significant role in many scientific applications such as nonlinear optics, dislocations in crystals, kink dynamics and chemical kinetics and quantum field theory [37].

Using Eq. (2), that carries Eq. (10) into an ordinary differential equation as

$$-Vu' + u^{(5)} + 30uu''' + 30u'u'' + 180u^2u' = 0, \tag{11}$$

where primes denote the derivative with respect to ξ. Integrating Eq. (11) once with respect to ξ and neglecting the constant of integration we obtain

$$-Vu + u^{(4)} + 30uu'' + 60u^3 = 0. \tag{12}$$

Now, balancing the highest order partial derivative $u^{(4)}$ and the highest order nonlinear term u^3, we get

$N = 2$. Therefore, the solution of Eq. (4) turns out to be:

$$u(\xi) = \alpha_0 + \alpha_1\left(\frac{G'}{G}\right) + \alpha_2\left(\frac{G'}{G}\right)^2, \quad \alpha_2 \neq 0, \tag{13}$$

where α_0, α_1 and α_2 are constants and to be determined later. Substituting Eq. (13) along with Eq. (5) into Eq. (12), yields a system of algebraic equations in $(G'/G)^i$, setting each coefficient of $(G'/G)^i$ in the obtained system of equations to zero, we can obtain the following set of algebraic equations (which are not shown here for the sake of simplicity) with respect to unknowns α_0, α_1 and α_2 and V.

Solving the above system of equations with the assist of Maple 11, we get the following solution.

$$\begin{aligned} &V = 16C^2A^2 - 8CB^2A - 32CA^2 + B^4 + 8B^2A + 16A^2, \\ &\alpha_0 = A(1-C), \qquad \alpha_1 = B(1-C), \qquad \alpha_2 = -(1-C)^2. \end{aligned} \tag{14}$$

Substituting Eq. (14) into Eq. (13) and according to Eqs. (6)- (9), we obtain the following exponential function, hyperbolic function and triangular function solutions of Eq. (10). These solutions are:

(1). When we choose $B \neq 0$ and $\Delta = B^2 + 4A - 4AC \geq 0$, then the exponential function solutions can be found as

$$u\ (x,t)=\frac{\Delta}{4}\left[1-\left(\frac{\alpha e^{\frac{\sqrt{\Delta}}{2}\xi}+\beta e^{-\frac{\sqrt{\Delta}}{2}\xi}}{\alpha e^{\frac{\sqrt{\Delta}}{2}\xi}-\beta e^{-\frac{\sqrt{\Delta}}{2}\xi}}\right)^2\right]. \tag{15}$$

(2). When we choose $B\neq 0$ and $\Delta=B^2+4A-4AC<0,$ then the triangular function solutions will be

$$u\ (x,t)=\frac{\Delta}{4}\left[1+\left(\frac{i\alpha\cos\frac{\sqrt{-\Delta}}{2}\xi-\beta\sin\frac{\sqrt{-\Delta}}{2}\xi}{i\alpha\sin\frac{\sqrt{-\Delta}}{2}\xi+\beta\cos\frac{\sqrt{-\Delta}}{2}\xi}\right)^2\right]. \tag{16}$$

(3). If we choose $B=0$ and $\Delta=A(C-1)\geq 0$, then the triangular function solutions are

$$u\ (x,t)=-\Delta\left[1+\left(\frac{\alpha\cos\sqrt{\Delta}\xi+\beta\sin\sqrt{\Delta}\xi}{\alpha\sin\sqrt{\Delta}\xi-\beta\cos\sqrt{\Delta}\xi}\right)^2\right]. \tag{17}$$

(4). Again, if we choose $B=0$ and $\Delta=A(C-1)<0$, then the hyperbolic function solutions are

$$u\ (x,t)=-\Delta\left[1-\left(\frac{i\alpha\cosh\sqrt{-\Delta}\xi-\beta\sinh\sqrt{-\Delta}\xi}{i\alpha\sinh\sqrt{-\Delta}\xi-\beta\cosh\sqrt{-\Delta}\xi}\right)^2\right], \tag{18}$$

where $\xi=x-\left(16C^2A^2-8CB^2A-32CA^2+B^4+8B^2A+16A^2\right)t;$ and *A*, *B*, *C* and α,β are real parameters. In particular, if we take $\alpha=\beta$ in Eq. (15), we obtain:

$$u(x,t)=\frac{\Delta}{4}\left[1-\coth^2\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]. \tag{19}$$

Again, if we take $\alpha=-\beta$ in Eq. (15), we obtain:

$$u(x,t)=\frac{\Delta}{4}\left[1+\tanh^2\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]. \tag{20}$$

Similarly, if we take $\alpha=0,\ \beta\neq 0$ in Eq. (16), we obtain:

$$u(x,t)=\frac{\Delta}{4}\left[1-\tan^2\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]. \tag{21}$$

And, if we take $\alpha \neq 0, \quad \beta = 0$ in Eq. (16), we obtain:

$$u(x,t) = \frac{\Delta}{4}\left[1+\cot^2\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]. \tag{22}$$

Also, if we take $\alpha = 0, \ \beta \neq 0$ and $\alpha \neq 0, \quad \beta = 0$ in Eq. (17), respectively and setting $A = k^2, C = 0,$ we obtain

$$u\ (x,t) = -k^2 \sec^2\left(k\left(x-16k^4\, t\right)\right), \tag{23}$$

and

$$u(x,t) = -k^2 \csc^2\left(k\left(x-16k^4\, t\right)\right). \tag{24}$$

Similarly, if we take $\alpha = 0, \ \beta \neq 0$ and $\alpha \neq 0, \quad \beta = 0$ in Eq. (18), respectively and setting $A = -k^2,$ $C = 0$, we obtain

$$u(x,t) = k^2 \sec h^2\left(k\left(x-16k^4\, t\right)\right), \tag{25}$$

and

$$u(x,t) = -k^2 \csc h^2\left(k\left(x-16k^4\, t\right)\right), \tag{26}$$

where k is a free parameter.

On comparing our results (23), (24) and results (17) obtained by using (G'/G)-expansion method, in [37] and the results (13), (14) by using the Exp-function method in [36] and (25), (26) by using the Hirota's method in [38] and (5.13) by using the Sine-Cosine method in [39], then it can be seen that the results are the same. Also, the solutions (25) and (26) are new solutions of the fifth order Caudrey-Dodd-Gibbon equation.

4 Graphical Representations of the Solutions

The graphs of some solutions are shown in Figs. 1–4.

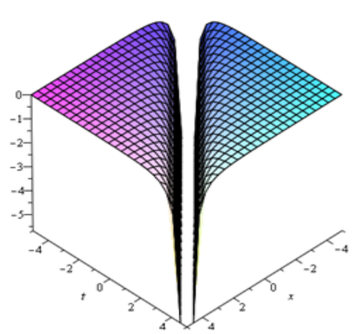

Fig. 1. Solitary wave obtained from solution of (19) for $A = B = C = 1$

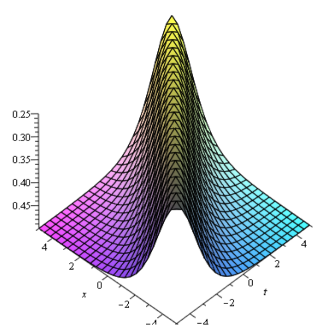

Fig. 2. Bell-shaped $\sec h^2$ **solitary solution of (20) for** $A = B = C = 1$

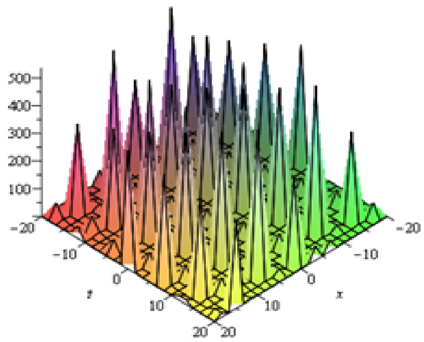

Fig. 3. Periodic wave solution of (21) for $A=1, B=1, C=2.$

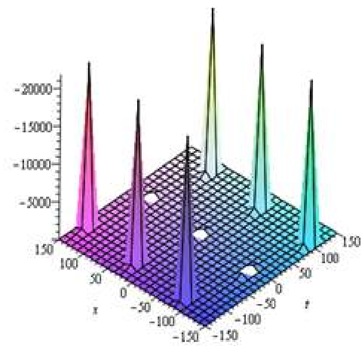

Fig. 4. Solitary wave obtained solution of (22) for $A=1, B=1, C=2.$

5 Conclusion

In this work, three types of traveling wave solutions, such as, the exponential, the hyperbolic and trigonometric functions of the Caudrey-Dodd-Gibbon equation are successfully obtained by using the extended (G'/G)-expansion method. On comparing the results obtained by using the extended (G'/G)-expansion method and the other methods such as the modified Tanh-Coth method, Exp-function method, we conclude that the extended (G'/G)- expansion method is more commanding, useful and expedient. The recital of this method is trustworthy, straightforward and gives many new precise solutions. Some of these results are in full agreement with the results reported by others in the literature by giving particular values to the arbitrary constants.

Competing Interests

Authors have declared that no competing interests exist.

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