

# A Model of Animals Phenotype with Superior Growth

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## Abstract

The aim of this study is to model and analyse a dynamic of feed conversion coefficient in pigs' ontogeny. Feed conversion is a process that couples feed intake, and growth. While there has been much research into the problem, a quantitative relation between the traits has not been revealed. The study considers feed as nutrient weight rather than its energy or a separate feedstuff. A main task of the research is to find out an analytical function between the traits. It is expected that the study will provide a new insight into the problem.

Animals are open systems; they need feed to sustain life, to grow and develop. It is plausible to suppose that growth of animal is a function of feed conversion and body weight. To find out and analyse this function a deterministic model of growth was built. The model was built as a dynamic system that describes the growth of individual animals. Both continuum and discrete-time modelling techniques are employed. The model is based on a data set obtained in experiments and field observations. Theoretical notions about the growth have been used for the model analyses.

It is shown that in ontogeny feed conversion and growth rate are functionally related traits. Between the traits, there is a nonlinear relation that concerns the growth rate, and feed conversion coefficient. In the model, the feed conversion coefficient is the variable that determines the dynamic of growth.

**Keywords:** Ontogenetic Growth, Feed Conversion, Weight Trajectory, Dynamic System, Hybrid Model, Local Extremum

## 1. Introduction

This study is a mathematical model of a relation between the growth, and feed conversion in pigs. The aim of the research is to find out a quantitative relation between the studied traits. Results of the model are expected to be useful to understand growth in other mammals as well. Growth dynamic of the pig provides a biological model of the growth in mammals including humans. In research on cancer, pigs maybe are the best model due to their size, physiological, genetic, and biochemical similarities to humans (Watson et al. 2016). In some instances, the findings can be translated to man, in others they apply to animal. The research is a model of growth of the animals; it is based on a relation between variables that follows from the analyses of a data set. Both the data set and records obtained in the field observations are empirical evidence for finding a quantitative relation between the traits. The theoretical notions about the growth have been used for the model analyses. From the heuristic notions, it follows that feed conversion should be explicitly included in the model. This follows from the physiology; feed intake, and growth are related through the feed conversion. In the model it is shown that dynamic of the studied traits is nonlinear and, in some aspects, it is counter-intuitive.

### 1.1 Growth of Animals

The species in general differ greatly in their growth kinetics. One general rule, found across the animal kingdom, is that the rate of growth declines as development progresses (Vollmer et al. 2017). This progressive decline in growth results from a genetic programme that occurs in multiple organs and involves the down-regulation of a large set of growth-promoting genes. This genetic programme does not appear to be driven simply by time, but rather depends on growth itself, suggesting that the limit on adult body size is imposed by a negative feedback loop (Lui and Baron, 2011). In biological systems, the nonlinearities enter through feedback mechanisms (Coveney and Fowler, 2005). Current findings suggest that in mammals, body growth is limited, at least in part, by a negative feedback loop (Lui and Baron, 2011). This reasoning implies that the growth of mammals is a nonlinear process.

## 1.2 Feed Conversion and Growth

Living organisms are open systems that function far from equilibrium (Skår, 2003). To sustain life the organisms need energy and material to grow and develop. Living beings can grow and develop by consuming feed. The conversion of feed into animals' weight is an essential process that is related to growth. It follows that feed conversion is relevant to the growth and development. The process is species-specific and to some extent understood. While much is known about nutrient utilization and tissue metabolism, merging these subjects into a discussion on feed efficiency has proven to be difficult. It was clarified that feed efficiency is a function of body weight (Patience et al., 2015). This result is supported by the correlation analyses. Still, this is insufficient; a functional relation between the studied traits remain undecided. While modelling the growth of animals, the feed conversion coefficient should be counted in as a focal variable. It follows from the logic of the process: feed consumed has been transformed into animal weight. This transformation of feed is reasonably model by the feed conversion coefficient. In the model,  $Z$  denotes the feed conversion coefficient and it is explicitly included in the model. Feed conversion efficiency modelled by  $Z$  is an intricate trait. The biological basis of feed efficiency is poorly understood. Feed efficiency is also highly complex in nature, because it is affected by much more than diet composition (Patience et al., 2015). In the model, the feed conversion coefficient  $Z$  characterises the total efficiency with which animals convert feed into their body weight.

## 2. Materials and Methods

The data set was obtained in the experiments on growing domestic pigs, fed from  $30 \pm 6$  kg up to  $96 \pm 4$  kg live weight. The pigs were housed and fed under non-industrial conditions, either in a pig testing station or in research facilities. The experiments were performed in compliance with Declaration of Helsinki, National legislation, and institutional rules.

### 2.1 The Model's Variables

In the model, we use the following variables.

Let  $M$  denote an individual animal current live weight, measured in kilograms.  $M = \{M \in \mathbb{R}_+ \mid 30 \leq M \leq 600\}$ , an animal individual maximum weight,  $M_x = 600$  kg.  $m$  denotes an animal initial considered weight, measured in kilograms,  $m \leq M$ ,  $m_0 = 30$  kg.

Let  $t$  denote the chronological discrete current time, measured in days from animals' birth.  $t = \{t \in \mathbb{N} \mid 0 \leq t < \infty\}$ ,  $\Delta t = 1, 2, 3, \dots, n. n \in \mathbb{N}$ .  $t_0$  denotes time related to  $m_0$ ,  $t_0 = 90$  days.

Let  $K$  denote the invariant of growth, nondimensional.  $K = \{K \in \mathbb{R}_+ \mid 1 \leq K < 11\}$ ,  $K_0 = 1$ .

Let  $Z$  denote the current feed conversion coefficient, nondimensional.  $Z = \{Z \in \mathbb{R}_+ \mid Z_0 \leq Z \leq \infty\}$ ,  $(Z = \infty) \rightarrow (M = M_x)$ .

Let  $F$  denote consumed feed, measured in kilograms a day.  $F = \{F \in \mathbb{R}_+ \mid 0 \leq F < \infty\}$ .

### 2.2 Analytical Methods

In the life sciences, modelling of biological systems by a single paradigm often is a difficult task due to intrinsic complexity of the systems. In many cases, the modelling paradigms are coupled in order to create hybrid methods (Smith & Yates, 2017). The model was built by applying both continuum and discrete-time techniques. A combination of both approaches proved efficient in this study. The model is formulated as a dynamic system. It is worthwhile to point out that processes in biology and medicine, due to their intrinsic nonlinearity, is rational to model by hybrid dynamic systems (Aihara & Suzuki, 2010). A basic relation between the studied traits had been obtained in the experiments and field observations. In the model, it has been established that between the studied traits there is a functional relation. Theoretical notions about the growth have been used to analyse the model, and to explain results.

## 3. Results

We consider growth of animals as a function of consumed feed  $F$ , feed conversion coefficient  $Z$ , and the current live weight  $M$ . In this section theoretical results obtained by applying abstract constructions, and results obtained by modelling the growth by applying basic functional relations have been analysed. In the model, a theoretical relation between the traits was found out by applying a mathematical concept. As this theoretical form does not entail the species-specific facts, we can call it a general formulation.

### 3.1 A Theoretical Relation Between the Traits

We start by writing a well-known formula, given below

$$\frac{M}{Z} = \int \frac{1}{Z} dM - \int \frac{M}{Z^2} dz, Z > 0, Z \neq \infty, \quad (1)$$

and we can use the following formula

$$\bar{Z} = \frac{M}{\int \frac{1}{Z} dM}, Z > 0, \quad (2)$$

where  $\bar{Z}$  denotes the average feed conversion coefficient. Since  $\bar{Z} = \frac{F}{M}$ , where  $F$  denotes consumed feed, we have

$$\int \frac{1}{Z} dM = \frac{M^2}{F} \quad (3)$$

Substituting (3) into (1) we get

$$\frac{M}{Z} = \frac{M^2}{F} - \int \frac{M}{Z^2} dZ, Z > 0, Z \neq \infty. \quad (4)$$

In differential form, equation (4) is given by

$$\frac{1}{Z} = \frac{2M}{F} - \frac{M^2}{F^2} \cdot \frac{dF}{dM}, Z > 0, Z \neq \infty. \quad (5)$$

One can see that equations (4) and (5) specify a relation between animal's weight  $M$ , consumed feed  $F$ , and the feed conversion coefficient  $Z$ . Both equations possess the biological content; for example, if in (5) we let  $\frac{dF}{dM} = 1$ , then this means that  $M = F$  and  $Z = 1$ , and the equation (5) holds. It follows that (4) and (5) are biologically meaningful equations. The equations (4) and (5) are nonlinear, it follows that analytical solution to the equations is hardly possible. As the equations specify a theoretical relation between the traits, they are not species-specific. To make them species-specific there is a long way to go. The task is to find out the species-specific relation between the traits and substitute it into (5). It is worth to notice that such a function could be both phenotype-dependant and nonlinear.

### 3.2 Growth of Pigs

A detailed analysis of the data set is summarised in the following system

$$\begin{cases} K = \frac{Mt}{m_o(2t-t_o)} \\ \frac{M}{m_o} = 2K - 1 + \frac{t-Kt_o}{t} \\ \frac{t-Kt_o}{t} = \frac{(Z-2K)(K-1)}{ZK} \end{cases} \quad (6)$$

In the system (6), basic relations between variables are in the form as they follow from the data set analyses. Parameter  $K$  is an invariant of growth; it is the same value for the same weight animals.

From the system (6) one can derive the following equation

$$\frac{1}{m_o} \cdot \frac{\Delta M}{\Delta t} = \frac{1}{t} \cdot \frac{ZK(2K+1)-2K^2}{Z(K+1)-2K}, m_o \leq M \leq M_x, \quad (7)$$

where  $M_x$  denotes maximum weight of individual animals. Under the model conditions this weight is 600 kg. One more equation that follows from the system (6) is given by

$$\frac{1}{m_o} \cdot \frac{\Delta M}{\Delta K} = \frac{2K+1}{K} - \frac{2}{Z}, m_o \leq M \leq M_x. \quad (8)$$

While the feed conversion coefficient  $Z$  is included in the system (6), and in other equations, (7) and (8) are inconvenient for the further analyses.

### 3.3 Dynamic of Feed Conversion Coefficient $Z$

From the system (6) one can deduce the following equation

$$\frac{M}{m_o} = K + \frac{K^2-1}{K} - \frac{4\Delta K}{3Z^2} - \frac{3\Delta Z}{KZ}, \quad m_o \leq M \leq M_x. \quad (9)$$

Equation (9) can be rewritten as

$$\frac{1}{m_o} \cdot \frac{\Delta M}{\Delta K} = \frac{2K+1}{K} - \frac{4}{3Z^2} - \frac{3}{KZ} \cdot \frac{\Delta Z}{\Delta K} \quad (10)$$

From (8) and (10) we have a general relation between  $Z$  and  $K$ , given below

$$\frac{\Delta Z}{\Delta K} = \frac{2K(3Z-2)}{9Z}, \quad m_o \leq M \leq M_x. \quad (11)$$

Under condition  $\frac{\Delta Z}{\Delta K} = 0$ , and minimum  $K=1$ , we have minimum  $Z_o = 2/3$ . The same result one can obtain by analysing the second derivative of (11), given by

$$\frac{\Delta^2 Z}{\Delta K^2} = \frac{2(3Z-2)(9ZZ_o+4K)}{81Z^2Z_o} \quad (12)$$

However, the most motivating trajectory of growth is in the weight range from 30 kg up to 100 kg. In this weight range maximum growth rate has been reported in most pigs. In this weight range, the growth in pigs has been modelled well by

$$\frac{M}{m_o} = 2K - 1, \quad m_o \leq M \leq 96. \quad (13)$$

From (9) and (13) we obtain

$$\frac{\Delta Z}{\Delta K} = \frac{3Z^2-4K}{9Z}, \quad Z \geq \frac{2}{\sqrt{3}}. \quad m_o \leq M \leq 96. \quad (14)$$

In (14), under condition  $\frac{\Delta Z}{\Delta K} = 0$ , we can find minimum  $Z$ , let  $Z_e$  denote minimum  $Z$ , it follows

$$Z_e = \frac{2\sqrt{K}}{\sqrt{3}}, \quad m_o \leq M \leq 96. \quad (15)$$

Equation (15) will serve us in the further analyses. This equation is the main tool for calculating the rate of growth in animals with superior performance. Equation (15) specifies the minimum value of feed conversion coefficient for animals in weight range between 30 and 96 kg.

Substituting (14) into (9) we have a functional relation between the current weight  $M$ , and the current feed conversion coefficient  $Z$ , given below

$$\frac{1}{m_o} \cdot \frac{\Delta M}{\Delta Z} = \frac{18Z}{3Z^2-4K}, \quad Z > \frac{2}{\sqrt{3}}. \quad m_o \leq M \leq 96. \quad (16)$$

Substituting (11) into (9) we have a general relation between  $M$  and  $Z$ , given by

$$\frac{1}{m_o} \cdot \frac{\Delta M}{\Delta Z} = \frac{9}{2K^2} \cdot \frac{Z(2K+1)-2K}{3Z-2}, \quad Z > \frac{2}{3}. \quad m_o \leq M \leq M_x. \quad (17)$$

We can add two more equations, given below.

$$\frac{\Delta Z}{\Delta t} = \frac{1}{t} \cdot \frac{2K^3(3Z-2)}{9[Z(K+1)-2K]}, \quad Z > 1. \quad m_o \leq M \leq M_x. \quad (18)$$

$$\frac{\Delta Z}{\Delta t} = \frac{t_o}{t^2} \cdot \frac{K^2(3Z^2-4K)}{9Z}, \quad Z \geq \frac{2}{\sqrt{3}}. \quad m_o \leq M \leq 96. \quad (19)$$

### 3.4 Animals with Superior Growth Rate

The study raises the question as to how to describe animals that perform superior growth rate. There are a few possible options. One option is to assume that the animals fit in a set with a distinct phenotype. However, the model says that there are only three phenotypes of growth (Stass, 2019a). In this section we carry out an analysis from the model position. The premise is that animals with superior growth rate have both best feed conversion and greatest feed intake. Under the model conditions, the best feed conversion coefficient is  $Z_c$ , given by (15). In this section we will model superior growth performance in three phenotypes: BB, Bb, bb (Stass, 2019a).

Let  $EM$  denote expected growth rate, measured in kilograms a day. The formula for growth rate of animals with the minimum feed conversion coefficient  $Z_c$  is given below

$$EM = F/Z_c \quad (20)$$

In complete form  $EM$  is given by

$$EM = \frac{F \cdot \sqrt{3}}{2 \cdot \sqrt{K}}, \quad m_o \leq M \leq 96, \quad (21)$$

where  $F$  denotes consumed feed.

#### 3.4.1 Superior Growth Rate in Three Phenotypes {BB; Bb; bb}

In this section we deal with the problem of which phenotype of the three identified, BB, Bb, and bb can perform superior growth. I can remind the interested reader that under the model conditions, maximum average growth rate in the weight range between 30 kg and 96 kg is 1,885 kg a day, and maximum growth rate at  $m_o = 30$  kg is 1,333 kg a day (Stass, 2019b). Calculation will be carried out by using (20). For all animals with superior performance growth rate at  $m_o = 30$  kg is the same value 1,333 kg a day. In this case,  $M = m_o$ ,  $K = 1$ . If  $F = 1,54$  kg then  $EM = 1,333$  kg a day. Consumed feed 1,54 kg a day is a typical quantity for 30 kg heavy animals. This means that for animals 30 kg in weight, the main factor to perform superior growth is minimum value of the feed conversion coefficient, and not extra quantity of consumed feed.

Growth phenotype BB. This phenotype has maximum growth rate at  $K = 1$ , 686 in weight about 70 kg. At first, we will calculate the growth rate at  $M = 96$  kg,  $K = 2,1$ . In this weight, animals with superior growth can consume about 3,0 kg of feed a day. It follows  $EM = 1,792$  kg a day. The main influence on average growth rate has maximum growth rate. We can calculate expected growth rate at  $K = 1,686$  to meet average 1,885 kg a day. It is  $EM = 2,530$  kg a day. From (20) it follows that to perform this growth rate animals must consume  $F = 3,79$  kg of feed a day over a period of the maximum growth rate. For a 70 kg rapidly growing pig it is possible. We can infer that animals with BB phenotype can perform superior growth.

Growth phenotype Bb. This phenotype performs maximum growth rate at  $K = 1,581$  in weight about 60 kg. Most calculation is the same as for phenotype BB, and we will not repeat them. The main point is, to meet average growth rate 1,885 kg a day, animals at the maximum growth rate that corresponds to  $K = 1,581$  must consume 3,67 kg of feed a day over a period of the maximum growth rate. For a 60 kg heavy pig consume this quantity of feed is problematic; however, for some animals it is possible. We can infer that some animals with Bb phenotype could perform superior growth.

Growth phenotype bb. Animals with this phenotype have no chance to perform superior growth.

## 4. Discussion

In animals, consumed feed is contingent on many factors such as foraging ability, body weight, health conditions, time of year, and many others. Feed conversion is contingent on many factors as well, such as health conditions, body weight, quality of feed, regularity of feed intake, and many others (Patience et al. 2015). In this respect and in this sense the rate of the growth of animals should be considered as unstable process. Even on a daily basis fluctuation of the growth rate could be substantial. Modelling this process by a system of differential equations is not an easy task; it is a problematic, and hardly tenable task.

### 4.1 Modelling Feed Conversion

Both traits, consumed feed  $F$  and the feed conversion coefficient  $Z$ , are contingent on many factors as well as environmental influences. The intricate dependence on many factors makes the traits difficult for modelling.

Besides, the traits are interdependent; as a result, a relation between the traits is nonlinear. It has the consequence that at this stage and with this model we can look into simple situations. It follows from the model that functional form between current weight  $M$ , and feed conversion coefficient  $Z$  is as follows  $M = F(Z, K)$ . It is possible to express this form as  $M = Q(Z(K))$ . However, it does not make the functional relation simple. If  $M = Q(Z(K))$  substitute into (5) then the resulting equation turns into a more complicated one. In this stage, a different modelling technique can be applied.

#### 4.2 Phenotype with Superior Growth

It is important to note that all animals with BB phenotype cannot perform superior growth. Actual expression of a quantitative trait, for example growth rate, mediated by gene B, is associated with other traits. The actual growth rate is a result of an association  $BB \cdot X_1 \cdot X_2 \cdot X_3 \dots X_n$ , where  $X_n$  denotes sets with genes that influence growth rate. It follows that only phenotype BB in association with certain genes  $X_n$ , it is  $BB \cdot X_n$ , and under optimum conditions can perform superior growth. The biological meaning of this result is clear. Animals with phenotype BB have growth maximum at about 70 kg,  $K=1,686$ . In this weight pigs can have greatest growth rate on condition that feed conversion coefficient is minimum, and the necessary quantity of extra feed is available. Factors that can diminish superior growth of phenotype BB are sets with associated genes  $X_n$ , quantity and quality of available feed, and environmental conditions.

In this section we will extrapolate formulation (20) to 600 kg heavy boars. Under the model conditions, 600 kg heavy boars do not grow. However, in this thought experiment we extend the capacity to grow typical for a 96 kg heavy pig to a 600 kg heavy boar. So,  $M = 600$  kg,  $K = 10,196152$ , and  $F = 8,5$  kg a day. We have  $EM = 2,305$  kg a day.

#### 4.3 Feed Conversion Coefficient

The feed conversion coefficient is considered as a function of consumed feed and body weight; it is a complicated, intricate trait. The trait is highly complex in nature and it is poorly understood (Patience et al. 2015). In animals' ontogeny, the dynamic of the trait is closely related to the growth and development. Animals can grow if  $Z > 2/3$ . However, the growth trajectory between  $Z = 2/3$ , and  $Z = 1$  is unclear. At  $Z = 1$  maximum growth rate is expected. But, in this case, the growth rate is hardly equal to consumed feed. In this range of values  $Z$ , the growth is likely nonlinear and growth trajectory is not continuous. A simple speculation is that there should be a symmetry between the growth rate with  $Z$  in the range  $2/3 < Z \leq 1$ , and  $Z > 1$ . The symmetry can emerge as a result of the growth trajectory bifurcation at  $Z = 1$ .

### 5. Conclusions

- Animals with the superior growth performance do not form a separate set with distinct phenotypes; they fit in phenotype BB.
- Animals with the growth phenotypes Bb and bb have little or no chance to perform superior growth.
- Two conditions are to have been met to recognise phenotype BB with the superior performance. The necessary condition is the growth rate 1,333 kg a day in weight 30 kg. The sufficient condition is the maximum growth rate at  $K = 1,686$  and the average growth rate 1,885 kg a day over the period of growth.
- Under optimum conditions, there are two factors that determine superior growth of animals. One factor is the ability to consume extra quantity of feed at the growth rate maximum. Second factor is a quality to keep feed conversion coefficient as close to  $Z_c$  as possible.
- Animals can grow if feed conversion coefficient  $Z > 2/3$ . However, the growth trajectory in the range  $2/3 < Z \leq 1$  is poorly understood.

### Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

### References

- Aihara, K., & Suzuki, H. (2010). Theory of hybrid dynamical systems and its applications to biological and medical systems. *Phil. Trans. R. Soc., A368*, 4893-4914.
- Coveney, P. V., & Fowler, P. W. (2005). Modelling biological complexity: A physical scientist's perspective. *J. R. Soc. Interface*, 2, 267-280.
- Lui, J. C., & Baron, J. (2011). Mechanisms Limiting Body Growth in Mammals. *Endocrine Reviews*, 32, 422-440.

- Patience, J. F., Rossoni-Serão, M. C., & Gutiérrez, N. A. (2015). A review of feed efficiency in swine: Biology and application. *Journal of Animal Science and Biotechnology*, 6, 33.
- Skår, J. (2003). Introduction: Self-organization as an actual theme. *Phil. Trans. R. Soc. Lond.*, A361, 1049-1056.
- Smith, C. A., & Yates, C. A. (2018). Spatially extended hybrid methods: A review. *J. R. Soc. Interface*, 15, 20170931.
- Stass, V. L. (2019a). Quantitative phenomics of growth and size in animals. *International Journal of Biology*, 11(3), 16-24.
- Stass, V. L. (2019b). Genetic potential for growth, feed conversion and longevity. *International Journal of Biology*, 11(1), 66-75.
- Vollmer, J., Casares, F., & Iber, D. (2017). Growth and size control during development. *Open Biol.*, 7, 170190.
- Watson, A. L., Carlson, D. F., Largaespada, D. A., Hackett, P. B., & Fahrenkrug, S. C. (2016). Engineered Swine Models of Cancer. *Front. Genet.*, 7, 78.

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