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Common Fixed Point Results for the (F, L)-Weak Contraction on Complete Weak Partial Metric Spaces

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Authors' contributions

This work was carried out in collaboration between all authors. Authors read and approved the final manuscript.

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Abstract

In this paper, we define the concepts of (F, L)-contraction and (F, L)-weak contraction in weak partial metric space which is generalized metric space. Using these concepts we prove some common fixed point theorems for two self mappings and we give some fixed point results for a single mapping in weak partial metric spaces. Also, we give some examples to support our new results. The theorems obtained here extend and generalize many results in the literature.

Keywords: *(F, L)-contraction; (F, L)-weak contraction; fixed point; common fixed point; weak partial metric space.*

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1 Introduction

Partial metric space was introduced by Matthews [1] in 1994 as a generalization of metric space, replacing the equality $d(x,x) = 0$ in the definition of metric with the inequality $d(x,x) \leq d(x,y)$ for all $x, y \in X$. It is widely recognized that partial metric space plays an important role in constructing models in the theory of computation. Matthews [1] proved the contraction principle of Banach in this new framework and also discussed some properties of convergence of sequences. Then, Valero [2], Oltra and Valero [3] and Altun et al. [4], [5] gave some generalizations of the results of Matthews. Recently, Romaguera [6] proved the Caristi type fixed point theorem on this space. Thereafter, by omitting the small self-distance axiom of partial metric, Heckmann [7] defined the weak partial metric space. By using the notion of F-contraction Wardowski [8] has proved a fixed point theorem which generalized Banach contraction principle. Later, Wardowski and Van Dung [9] introduced the notion of F-weak contraction and established fixed point theorems for such mappings in complete metric spaces. They extend and generalize the presented theorems and improve many existing results.

We recall some definitions in partial metric space and some properties of them, see [1], [2], [3], [6], [7], [10].

The main purpose of this paper is to define the concepts of F-contraction and F-weak contraction, that were given for metric spaces by Wardowski and Van Dung [9], for weak partial metric spaces and to generalize these new concepts to the concepts of (F, L)-contraction and (F, L)-weak contraction in weak partial metric spaces. By using these concepts, we prove some common fixed point results for self mappings T and S and some fixed point results for a single mapping T on a weak partial metric space. The paper is organized as follows. In Section 2, we give some basic definitions and concepts which are necessary in the next section. We present main theorems and corollaries which extend and improve many results in the literature, in Section 3.

2 Preliminaries and Definitions

Now, we mention briefly some fundamental definitions and results.

Definition 2.1. [1] A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}^+$ (nonnegative reals) such that, for all $x, y, z \in X$:

p1. $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$ (T_0-separation axiom),

p2. $p(x,x) \leq p(x,y)$ (small self-distance axiom),

p3. $p(x,y) = p(y,x)$ (symmetry),

p4. $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$ (modified triangular inequality).

A partial metric space is a pair (X, p) such that X is a nonempty set and p is a partial metric on X.

It is clear that if $p(x,y) = 0$, then, from (p1) and (p2), $x = y$. But if $x = y$, $p(x,y)$ may not be 0. A basic example of a partial metric space is the pair $(\mathbb{R}^+, p)$, where $p(x,y) = \max\{x,y\}$ for all $x, y \in \mathbb{R}^+$. For another example, let I denote the set of all closed intervals $[a,b]$ for any real numbers $a \leq b$. Let $p : I \times I \to \mathbb{R}^+$ be the function such that $p([a,b],[c,d]) = \max\{b,d\} - \min\{a,c\}$. Then (I,p) is a partial metric space. Other examples of partial metric space which are interesting from a computational point of view may be found in [1] and [11].

Each partial metric p on X generates a T_0 topology τ_p on X which has a base as the family open p-balls

$$\{B_p(x,\epsilon) : x \in X, \epsilon > 0\},$$

where

$$B_p(x, \epsilon) = \{y \in X : p(x, y) < p(x, x) + \epsilon\},$$

for all $x \in X$ and $\epsilon > 0$.

Definition 2.2. [1] Let (X, p) be a partial metric space. Then,

i. a sequence $\{x_n\}$ in a partial metric space (X, p) converges with respect to τ_p to a point $x \in X$ if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$,

ii. a sequence $\{x_n\}$ in a partial metric space (X, p) is called a Cauchy sequence if there exists $\lim_{n,m\to\infty} p(x_n, x_m)$ (and is finite),

iii. a partial metric space (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges to a point $x \in X$; that is, $p(x, x) = \lim_{n,m\to\infty} p(x_n, x_m)$.

If p is a partial metric on X, then the functions $d_p, d_w : X \times X \to \mathbb{R}^+$ given by

$$d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y) \tag{2.1}$$

and

$$\begin{aligned} d_w(x, y) &= \max\{p(x, y) - p(x, x), p(x, y) - p(y, y)\} \\ &= p(x, y) - \min\{p(x, x), p(y, y)\} \end{aligned} \tag{2.2}$$

are ordinary metrics on X. It is easy to see that d_p and d_w are equivalent metrics on X.

Lemma 2.1. [1] *Let (X, p) be a partial metric space. Then,*

i. $\{x_n\}$ is a Cauchy sequence in (X, p) if and only if it is Cauchy sequence in the metric space (X, d_p),

ii. (X, p) is complete if and only if the metric space (X, d_p) is complete. Furthermore,

$$\lim_{n\to\infty} d_p(x_n, x) = 0 \Leftrightarrow p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m).$$

Definition 2.3. [12] Suppose that (X, p) is a partial metric space. A mapping $T : X \to X$ is said to be continuous at $x \in X$ if for every $\epsilon > 0$, there exist $\delta > 0$ such that $T(B_p(x, \delta)) \subseteq B_p(Tx, \epsilon)$. We say that T is continuous if T is continuous at all $x \in X$.

It is easy to see that if (X, p) is a partial metric space, $T : X \to X$ is continuous, $\{x_n\}$ is a sequence in X, $x \in X$ and

$$\lim_{n\to+\infty} p(x_n, x) = p(x, x), \quad \text{then} \quad \lim_{n\to+\infty} p(Tx_n, Tx) = p(Tx, Tx).$$

Remark 2.1. Since d_p and d_w are equivalent, we can take d_w instead of d_p in Lemma 2.1.

Heckmann [7] introduced the concept of weak partial metric space, which is a generalized version of Matthews' partial metric space by omitting the small self-distance axiom. That is, the function $p : X \times X \to \mathbb{R}^+$ is called weak partial metric on X if the conditions (p1), (p3) and (p4) satisfied. Also, Heckmann showed that, if p is a weak partial metric on X then for all $x, y \in X$, we have the following weak small self-distance property

$$p(x, y) \geq \frac{p(x, x) + p(y, y)}{2}.$$

Weak small self-distance property shows that weak partial metric spaces are not far from small-self distance axiom. It is clear that every partial metric space is a weak partial metric space, but the converse may not be true. A basic example of a weak partial metric space but not a partial metric space is the pair $(\mathbb{R}^+, p)$, where $p(x, y) = \dfrac{x + y}{2}$ for all $x, y \in \mathbb{R}^+$. For another example, let I denote the set of all intervals $[a, b]$ for any real numbers $a \leq b$. Let $p : I \times I \to \mathbb{R}^+$ be the function such that $p([a, b], [c, d]) = \dfrac{b + d - a - c}{2}$. Then (I, p) is a weak partial metric space but not a partial metric space. Again, for $x, y \in \mathbb{R}^+$ the function $p(x, y) = \dfrac{e^x + e^y}{2}$ is a non partial metric but is a weak partial metric on $\mathbb{R}^+$.

Remark 2.2. As mentioned in [13], if (X, p) be a weak partial metric space, but not partial metric space, then the function d_p as in (2.1) may not be an ordinary metric on X, but d_w as in (2.2) is still an ordinary metric on X.

In a weak partial metric space, the convergence of a sequence, Cauchy sequence, completeness and continuity of a function are defined as partial metric space.

To prove some fixed point results on a weak partial metric space we need to give following lemma, that was proved by omitting the small-self distance axiom.

Lemma 2.2. [13] *Let (X, p) be a weak partial metric space.*

i. *$\{x_n\}$ is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in the metric space (X, d_w).*

ii. *(X, p) is complete if and only if (X, d_w) is complete. Furthermore*

$$\lim_{n\to\infty} d_w(x_n, x) = 0 \Leftrightarrow p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m).$$

Definition 2.4. [14] Let T and S be self-mappings of a set X (i.e., $T, S : X \to X$). If $z = Tx = Sx$ for some $x \in X$, then x is called a coincidence point of T and S, and z is called a point of coincidence of T and S. Self-mappings T and S are said to be weakly compatible if they commute at their coincidence point, that is, if $Tx = Sx$ for some $x \in X$, then $TSx = STx$.

Remark 2.3. [14] Let T and S be weakly compatible self-mappings of a set X. If T and S have a unique point of coincidence $z = Tx = Sx$, then z is the unique common fixed point of T and S.

Definition 2.5. [8] Let $\mathcal{F}$ be the family of all functions $F : (0, +\infty) \longrightarrow \mathbb{R}$ such that

F1. F is strictly increasing, that is, for all $\alpha, \beta \in (0, +\infty)$ if $\alpha < \beta$ then $F(\alpha) < F(\beta)$,

F2. for each sequence $\{\alpha_n\}$ of positive numbers, the following holds:

$$\lim_{n\to\infty} \alpha_n = 0 \Leftrightarrow \lim_{n\to\infty} F(\alpha_n) = -\infty,$$

F3. there exists $k \in (0, 1)$ such that $\lim_{\alpha\to 0^+} (\alpha^k F(\alpha)) = 0$.

Example 2.3. [8] *The following functions $F : (0, +\infty) \longrightarrow \mathbb{R}$ are the elements of $\mathcal{F}$:*

i. $F\alpha = \ln \alpha$,

ii. $F\alpha = \ln \alpha + \alpha$,

iii. $F\alpha = -\frac{1}{\sqrt{\alpha}}$,

iv. $F\alpha = (\ln \alpha^2 + \alpha)$.

Definition 2.6. [9] **1.** Let (X,d) be a metric space. A mapping $T : X \to X$ is said to be an F-weak contraction on (X,d) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x,y \in X$ satisfying $d(Tx,Ty) > 0$, the following holds:

$$\tau + F(d(Tx,Ty)) \leq F\left(\max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2}\right\}\right). \tag{2.3}$$

2. Let (X,d) be a metric space. A mapping $T : X \to X$ is said to be an F-contraction on (X,d) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x,y \in X$ satisfying $d(Tx,Ty) > 0$, the following holds:

$$\tau + F(d(Tx,Ty)) \leq F(d(x,y)).$$

i. Every F-contraction is an F-weak contraction.

ii. Let T be an F-weak contraction. From (2.3) we have, for all $x,y \in X$, $Tx \neq Ty$

$$F(d(Tx,Ty)) < \tau + F(d(Tx,Ty)) \leq F\left(\max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2}\right\}\right).$$

Then by (F1), we get

$$d(Tx,Ty) < \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2}\right\},$$

for all $x,y \in X$, $Tx \neq Ty$.

Lemma 2.4. [15] *Let X be a nonempty set and $T : X \to X$ be a mapping. Then there exists a subset $Y \subseteq X$ such that $T(Y) = T(X)$ and $T : Y \to X$ is one-to-one.*

3 Main Results

We start our work by introducing the following two concepts.

Definition 3.1. 1. Let (X,p) be a weak partial metric space. A mapping $T : X \to X$ is said to be an F-weak contraction on (X,p) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x,y \in X$ satisfying $p(Tx,Ty) > 0$, the following holds:

$$\tau + F(p(Tx,Ty)) \leq F\left(\max\left\{p(x,y), p(x,Tx), p(y,Ty), \frac{p(x,Ty)+p(y,Tx)}{2}\right\}\right). \tag{3.1}$$

2. Let (X,p) be a weak partial metric space. A mapping $T : X \to X$ is said to be an F-contraction on (X,p) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x,y \in X$ satisfying $p(Tx,Ty) > 0$, the following holds:

$$\tau + F(p(Tx,Ty)) \leq F(p(x,y)).$$

Remark 3.1. i. Every F-contraction is an F-weak contraction.

ii. Let T be an F-weak contraction. From (3.1) we have, for all $x,y \in X$, $p(Tx,Ty) > 0$

$$F(p(Tx,Ty)) < \tau + F(p(Tx,Ty)) \leq F\left(\max\left\{p(x,y), p(x,Tx), p(y,Ty), \frac{p(x,Ty)+p(y,Tx)}{2}\right\}\right).$$

Then by (F1), we get

$$p(Tx,Ty) < \max\left\{p(x,y), p(x,Tx), p(y,Ty), \frac{p(x,Ty)+p(y,Tx)}{2}\right\},$$

for all $x,y \in X$ with $p(Tx,Ty) > 0$.

Now, we give a new definition of (F, L)-contraction that will be used later.

Definition 3.2. Let (X, p) be a weak partial metric space. A mapping $T : X \to X$ is said to be the (F, L)-contraction if $F \in \mathcal{F}$ and there exist $\tau > 0$ and $L \geq 0$ such that, for all $x, y \in X$ satisfying $p(Tx, Ty) > 0$, the following holds:

$$\tau + F(p(Tx, Ty)) \quad \leq \quad F(p(x, y)) + Ld_w(y, Tx). \tag{3.2}$$

Because of the symmetry of the distance, the (F, L)-contraction condition implicity includes the following dual one:

$$\tau + F(p(Tx, Ty)) \quad \leq \quad F(p(x, y)) + Ld_w(x, Ty). \tag{3.3}$$

Thus by (3.2) and (3.3), the (F, L)-contraction condition can be replaced by the following condition: $p(Tx, Ty) > 0$, then

$$\tau + F(p(Tx, Ty)) \leq F(p(x, y)) + L \min\{d_w(x, Ty), d_w(y, Tx)\}. \tag{3.4}$$

Now, let us state and prove our first theorem for self mapping T which satisfies inequality (3.4).

Theorem 3.1. *Let (X, p) be a complete weak partial metric space and $T : X \to X$ be the (F, L)-contraction. If F is continuous, then T has a fixed point in X.*

Proof. Let $x_0 \in X$ be an arbitrary point. We define a sequence $\{x_n\}$ in X by $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$. If $x_n = x_{n+1}$ for some $n \in \mathbb{N}$, then x_n is a fixed point of T and the proof is completed. Thus, assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. By (3.4), we have

$$\begin{aligned} F(p(x_n, x_{n+1})) &= F(p(Tx_{n-1}, Tx_n)) \\ &\leq F(p(x_{n-1}, x_n)) + L \min\{d_w(x_{n-1}, Tx_n), d_w(x_n, Tx_{n-1})\} - \tau \\ &= F(p(x_{n-1}, x_n)) + L \min\{d_w(x_{n-1}, x_{n+1}), d_w(x_n, x_n)\} - \tau \\ &= F(p(x_{n-1}, x_n)) - \tau. \end{aligned} \tag{3.5}$$

Repeating (3.5) n-times, we get that

$$F(p(x_n, x_{n+1})) \leq F(p(x_0, x_1)) - n\tau, \tag{3.6}$$

for all $n \in \mathbb{N}$. Taking the limit as $n \to \infty$ in (3.6), we get

$$\lim_{n\to\infty} F(p(x_n, x_{n+1})) = -\infty,$$

that together with (F2) gives

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = 0. \tag{3.7}$$

As $F \in \mathcal{F}$, there exists $k \in (0, 1)$ such that

$$\lim_{n\to\infty} (p(x_n, x_{n+1}))^k F(p(x_n, x_{n+1})) = 0. \tag{3.8}$$

It follows from (3.6) that

$$(p(x_n, x_{n+1}))^k [F(p(x_n, x_{n+1})) - F(p(x_0, x_1))] \leq -(p(x_n, x_{n+1}))^k n\tau \leq 0, \tag{3.9}$$

for all $n \in \mathbb{N}$. By using (3.7) and (3.8) and taking the limit as $n \to \infty$ in (3.9), we get

$$\lim_{n\to\infty} (p(x_n, x_{n+1}))^k n = 0.$$

Then there exists $n_1 \in \mathbb{N}$ such that $(p(x_n, x_{n+1}))^k n \leq 1$ for all $n \geq n_1$, that is,

$$p(x_n, x_{n+1}) \leq \frac{1}{n^{\frac{1}{k}}}, \tag{3.10}$$

for all $n \geq n_1$.

In order to show that $\{x_n\}$ is a Cauchy sequence consider $m, n \in \mathbb{N}$ such that $m > n \geq n_1$. Using the triangular inequality for the weak partial metric and from (3.10), we obtain

$$\begin{aligned} p(x_n, x_m) &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + ... + p(x_{m-1}, x_m) \\ &\quad -[p(x_{n+1}, x_{n+1}) + p(x_{n+2}, x_{n+2}) + ... + p(x_{m-1}, x_{m-1})] \\ &\leq \sum_{i=n}^{m-1} p(x_i, x_{i+1}) \\ &\leq \sum_{i=n}^{\infty} p(x_i, x_{i+1}) \\ &\leq \sum_{i=n}^{\infty} \frac{1}{i^{\frac{1}{k}}}. \end{aligned}$$

As $k \in (0,1)$, the series $\sum_{i=1}^{\infty} \frac{1}{i^{\frac{1}{k}}}$ is convergent, so it follows from the above inequality that $\lim_{n,m\to\infty} p(x_n, x_m) = 0$, that is, $\{x_n\}$ is a Cauchy sequence in (X, p). Then, from Lemma 2.2, $\{x_n\}$ is a Cauchy sequence in the metric space (X, d_w). Also, the completeness of the weak partial metric space (X, p) yields that the metric space (X, d_w) is complete. Then, there exists $x^* \in X$ such that

$$\lim_{n\to\infty} d_w(x_n, x^*) = 0. \tag{3.11}$$

Again from Lemma 2.2, we have

$$p(x^*, x^*) = \lim_{n\to\infty} p(x_n, x^*) = \lim_{n,m\to\infty} p(x_n, x_m) = 0. \tag{3.12}$$

Suppose that F is continuous. In this case, we consider the following two cases.

Case 1. For each $n \in \mathbb{N}$, there exists $i_n \in \mathbb{N}$ such that $p(x_{i_n+1}, Tx^*) = 0$ and $i_n > i_{n-1}$ where $i_0 = 1$. Then we have

$$p(x^*, Tx^*) = \lim_{n\to\infty} p(x_{i_n+1}, Tx^*) = 0.$$

Therefore, $x^* = Tx^*$, that is, x^* is a fixed point of T.

Case 2. There exists $n_2 \in \mathbb{N}$ such that $p(x_{n+1}, Tx^*) > 0$ for all $n \geq n_2$. It follows from (3.4) that

$$\begin{aligned} \tau + F(p(x_{n+1}, Tx^*)) &= \tau + F(p(Tx_n, Tx^*)) \\ &\leq F(p(x_n, x^*)) + L \min\{d_w(x_n, Tx^*), d_w(x^*, Tx_n)\} \\ &= F(p(x_n, x^*)) + L \min\{d_w(x_n, Tx^*), d_w(x^*, x_{n+1})\}. \end{aligned}$$

Also, from (3.12), there exists $n_3 \in \mathbb{N}$ such that, for all $n \geq n_3$ we get $p(x_n, x^*) < p(x^*, Tx^*)$. So, for all $n \geq \max\{n_2, n_3\}$ we obtain

$$\tau + F(p(x_{n+1}, Tx^*)) \leq F(p(x^*, Tx^*)) + L \min\{d_w(x_n, Tx^*), d_w(x^*, x_{n+1})\}.$$

As F is continuous, letting $n \to \infty$ in the above inequality and using (3.11) and (3.12), we have

$$\tau + F(p(x^*, Tx^*)) \leq F(p(x^*, Tx^*)),$$

which is a contradiction as $\tau > 0$. Thus, we must have $p(x^*, Tx^*) = 0$, that is, $x^* = Tx^*$. Therefore, x^* is a fixed point of T. □

The fixed point of T in Theorem 3.1 is unique if we replace $\min\{d_w(x,Ty), d_w(y,Tx)\}$ by $\min\{d_w(x,Tx), d_w(x,Ty), d_w(y,Tx)\}$ in (3.4). So, we have the following result.

Theorem 3.2. *Let (X,p) be a complete weak partial metric space and $T : X \to X$ be a mapping. Suppose that $F \in \mathcal{F}$ and there exist $\tau > 0$ and $L \geq 0$ such that, for all $x, y \in X$ satisfying $p(Tx,Ty) > 0$, the following holds:*

$$\tau + F(p(Tx,Ty)) \leq F(p(x,y)) + L\min\{d_w(x,Tx), d_w(x,Ty), d_w(y,Tx)\}. \tag{3.13}$$

If F is continuous, then T has a unique fixed point in X.

Proof. The existence of the fixed point of T follows from Theorem 3.1. For the uniqueness of the fixed point of T, let us suppose that x_1^* and x_2^* be two fixed points of T and $x_1^* \neq x_2^*$. Then $p(Tx_1^*, Tx_2^*) > 0$. By (3.13), we have

$$\begin{aligned}\tau + F(p(x_1^*, x_2^*)) &= \tau + F(p(Tx_1^*, Tx_2^*)) \\ &\leq F(p(x_1^*, x_2^*)) + L\min\{d_w(x_1^*, Tx_1^*), d_w(x_1^*, Tx_2^*), d_w(x_2^*, Tx_1^*)\} \\ &= F(p(x_1^*, x_2^*)).\end{aligned}$$

It is a contradiction. Then, we have $x_1^* = x_2^*$. This proves that the fixed point of T is unique. □

By the aid of Lemma 2.4, we obtain the following result of Theorem 3.2.

Corollary 3.3. *Let (X,p) be a weak partial metric space and $T, S : X \to X$ be two mappings. Suppose that $F \in \mathcal{F}$ and there exist $\tau > 0$ and $L \geq 0$ such that, for all $x, y \in X$ satisfying $p(Tx,Ty) > 0$, the following holds:*

$$\tau + F(p(Tx,Ty)) \leq F(p(Sx,Sy)) + L\min\{d_w(Sx,Tx), d_w(Sx,Ty), d_w(Sy,Tx)\}.$$

Also, assume that

i. $TX \subseteq SX$.

ii. *SX is a complete subspace of the weak partial metric space X.*

If F is continuous, then T and S have a unique point of coincidence in X. Moreover, if T and S are weakly compatible, then T and S have a unique common fixed point in X.

We see that the following corollary becomes a direct result if we take $L = 0$ in Theorem 3.1. Note that, F need not be continuous in the following result.

Corollary 3.4. *Let (X,p) be a complete weak partial metric space and $T : X \to X$ be an F-contraction. Then T has a unique fixed point in X.*

Now, we give an example for illustration.

Example 3.5. *Let $X = [0,1]$. Define a weak partial metric $p : X \times X \to [0,+\infty)$ by the formula $p(x,y) = \max\{x,y\}$, then $d_w(x,y) = |x-y|$. Since (X,d_w) is complete, by Lemma 2.2, (X,p) is complete weak partial metric space. Also, define $T : X \to X$ by $Tx = \dfrac{x}{1+x}$. Without loss of generality, we may assume that $0 \leq y < x$. Then, we have*

$$p(Tx,Ty) = p\left(\frac{x}{1+x}, \frac{y}{1+y}\right) = \max\left\{\frac{x}{1+x}, \frac{y}{1+y}\right\} = \frac{x}{1+x} > 0$$

and

$$\begin{aligned}F(p(x,y)) + L\min\{d_w(x,Tx), d_w(x,Ty), d_w(y,Tx)\} &= F(x) + L\min\{d_w(x,Tx), d_w(x,Ty), d_w(y,Tx)\} \\ &\geq F(x).\end{aligned}$$

Therefore, by choosing $F\alpha = \ln\alpha$, $\alpha \in (0, +\infty)$ *and* $\tau = \ln(1+x)$, *for any* $L \geq 0$, *we obtain*

$$\begin{aligned}
\tau + F(p(Tx, Ty)) &= \tau + F\left(\frac{x}{1+x}\right) \\
&= \ln(x) \\
&= F(x) \\
&\leq F(p(x,y)) + L\min\{d_w(x,Tx), d_w(x,Ty), d_w(y,Tx)\}.
\end{aligned}$$

This shows that all conditions of Theorem 3.2 are satisfied and so T *has a unique fixed point in* X *which is* $x^* = 0$.

Now, we give a new definition of (F, L)-weak contraction that will be used later.

Definition 3.3. Let (X, p) be a weak partial metric space and $T, S : X \to X$ be two mappings. Then the pair (T, S) is said to be the (F, L)-weak contraction if $F \in \mathcal{F}$ and there exist $\tau > 0$ and $L \geq 0$ such that, for all $x, y \in X$ satisfying $p(Tx, Sy) > 0$, the following holds:

$$\begin{aligned}
\tau + F(p(Tx, Sy)) \leq\ & F\left(\max\left\{p(x,y), p(x,Tx), p(y,Sy), \frac{p(x,Sy)+p(y,Tx)}{2}\right\}\right) \\
& + L\min\{d_w(x,Sy), d_w(y,Tx)\}. \qquad (3.14)
\end{aligned}$$

Theorem 3.6. *Let* (X, p) *be a complete weak partial metric space and* $T, S : X \to X$ *be two mappings such that the pair* (T, S) *satisfy the* (F, L)*-weak contraction condition. If* F *is continuous, then* T *and* S *have a common fixed point in* X.

Proof. Take $x_0 \in X$ as an arbitrary point and we construct a sequence $\{x_n\}$ in X by $x_{2n+1} = Tx_{2n}$ and $x_{2n+2} = Sx_{2n+1}$ for all $n \in \mathbb{N}$. Suppose $p(x_n, x_{n+1}) = 0$ for some $n \in \mathbb{N}$. Without loss of generality, we assume $n = 2k$ for some $k \in \mathbb{N}$. Thus, we get $p(x_{2k}, x_{2k+1}) = 0$. Now, suppose $p(x_{2k+1}, x_{2k+2}) > 0$, then by (3.14), we obtain

$$\begin{aligned}
F(p(x_{2k+1}, x_{2k+2})) &= F(p(Tx_{2k}, Sx_{2k+1})) \\
&\leq F(\max\{p(x_{2k}, x_{2k+1}), p(x_{2k}, Tx_{2k}), p(x_{2k+1}, Sx_{2k+1}), \\
&\quad [p(x_{2k}, Sx_{2k+1}) + p(x_{2k+1}, Tx_{2k})]/2\}) \\
&\quad + L\min\{d_w(x_{2k}, Sx_{2k+1}), d_w(x_{2k+1}, Tx_{2k})\} - \tau \\
&= F\left(\max\left\{p(x_{2k+1}, x_{2k+2}), \frac{p(x_{2k}, x_{2k+2}) + p(x_{2k+1}, x_{2k+1})}{2}\right\}\right) \\
&\quad + L\min\{d_w(x_{2k}, x_{2k+2}), d_w(x_{2k+1}, x_{2k+1})\} - \tau \\
&\leq F\left(\max\left\{p(x_{2k+1}, x_{2k+2}), \frac{p(x_{2k}, x_{2k+1}) + p(x_{2k+1}, x_{2k+2})}{2}\right\}\right) - \tau \\
&= F(p(x_{2k+1}, x_{2k+2})) - \tau.
\end{aligned}$$

It is a contradiction. Then, we conclude that $p(x_{2k+1}, x_{2k+2}) = 0$ for all $k \in \mathbb{N}$, that is, $x_{2k+1} = x_{2k+2}$. Therefore $x_{2k} = Tx_{2k} = Sx_{2k}$ and hence x_n is a common fixed point of T and S. Thus, we may assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Given $n \in \mathbb{N}$. If n is even, then $n = 2t$ for some $t \in \mathbb{N}$.

By (3.14), we obtain

$$\begin{aligned}
F(p(x_{2t},x_{2t+1})) &= F(p(x_{2t+1},x_{2t})) \\
&= F(p(Tx_{2t},Sx_{2t-1})) \\
&\leq F(\max\{p(x_{2t},x_{2t-1}),p(x_{2t},Tx_{2t}),p(x_{2t-1},Sx_{2t-1}), \\
&\quad [p(x_{2t},Sx_{2t-1})+p(x_{2t-1},Tx_{2t})]/2\}) \\
&\quad +L\min\{d_w(x_{2t},Sx_{2t-1}),d_w(x_{2t-1},Tx_{2t})\}-\tau \\
&= F\left(\max\left\{p(x_{2t},x_{2t-1}),p(x_{2t},x_{2t+1}),\frac{p(x_{2t},x_{2t})+p(x_{2t-1},x_{2t+1})}{2}\right\}\right) \\
&\quad +L\min\{d_w(x_{2t},x_{2t}),d_w(x_{2t-1},x_{2t+1})\}-\tau \\
&\leq F\left(\max\left\{p(x_{2t},x_{2t-1}),p(x_{2t},x_{2t+1}),\frac{p(x_{2t-1},x_{2t})+p(x_{2t},x_{2t+1})}{2}\right\}\right)-\tau \\
&= F(\max\{p(x_{2t},x_{2t-1}),p(x_{2t},x_{2t+1})\})-\tau. \qquad (3.15)
\end{aligned}$$

If $\max\{p(x_{2t},x_{2t-1}),p(x_{2t},x_{2t+1})\}=p(x_{2t},x_{2t+1})$, then (3.15) yields a contradiction. Thus, $\max\{p(x_{2t},x_{2t-1}),p(x_{2t},x_{2t+1})\}=p(x_{2t},x_{2t-1})$ and hence

$$F(p(x_{2t},x_{2t+1}))\leq F(p(x_{2t-1},x_{2t}))-\tau. \qquad (3.16)$$

If n is odd, then $n=2t+1$ for some $t\in\mathbb{N}$. By similar arguments as above we can show that

$$F(p(x_{2t+1},x_{2t+2}))\leq F(p(x_{2t},x_{2t+1}))-\tau. \qquad (3.17)$$

By (3.16) and (3.17), we have

$$F(p(x_n,x_{n+1}))\leq F(p(x_{n-1},x_n))-\tau. \qquad (3.18)$$

By repeating (3.18) n-times, we get

$$F(p(x_n,x_{n+1}))\leq F(p(x_0,x_1))-n\tau,$$

for all $n\in\mathbb{N}$. Therefore, $\{x_n\}$ is a Cauchy sequence in X as in the proof of Theorem 3.1. Since X is complete, there exists $x^*\in X$ such that $\lim_{n\to\infty}d_w(x_n,x^*)=0$ and so

$$p(x^*,x^*)=\lim_{n\to\infty}p(x_n,x^*)=\lim_{n,m\to\infty}p(x_n,x_m)=0.$$

Now we will prove that $Tx^*=Sx^*=x^*$. For this, suppose that F is continuous. Then we consider two cases.

Case 1. For each $n\in\mathbb{N}$, there exists $i_n\in\mathbb{N}$ such that $p(x_{i_n+1},Tx^*)=0$ and $i_n>i_{n-1}$ where $i_0=1$. Then we have

$$p(x^*,Tx^*)=\lim_{n\to\infty}p(x_{i_n+1},Tx^*)=0.$$

Therefore, $x^*=Tx^*$, that is, x^* is a fixed point of T. Suppose $p(Tx^*,Sx^*)>0$; then by (3.14) we obtain

$$\begin{aligned}
\tau+F(p(Tx^*,Sx^*)) &\leq F\left(\max\left\{p(x^*,x^*),p(x^*,Tx^*),p(x^*,Sx^*),\frac{p(x^*,Sx^*)+p(x^*,Tx^*)}{2}\right\}\right) \\
&\quad +L\min\{d_w(x^*,Sx^*),d_w(x^*,Tx^*)\} \\
&= F\left(\max\left\{p(x^*,x^*),p(x^*,Sx^*),\frac{p(x^*,Sx^*)+p(x^*,x^*)}{2}\right\}\right) \\
&\quad +L\min\{d_w(x^*,Sx^*),d_w(x^*,x^*)\} \\
&= F(p(x^*,Sx^*)) \\
&= F(p(Tx^*,Sx^*))
\end{aligned}$$

a contradiction as $\tau > 0$. Therefore, we have $p(Tx^*, Sx^*) = 0$, that is, $Tx^* = Sx^* = x^*$. Thus, x^* is a common fixed point of T and S.

Case 2. There exists $n_2 \in \mathbb{N}$ such that $p(x_n, Tx^*) > 0$ for all $n \geq n_2$. For any $n \geq n_2$, we get from (3.14) that

$$\begin{aligned}
\tau + F(p(Tx^*, x_{2n+2})) &= \tau + F(p(Tx^*, Sx_{2n+1})) \\
&\leq F(\max\{p(x^*, x_{2n+1}), p(x^*, Tx^*), p(x_{2n+1}, Sx_{2n+1}), \\
&\quad [p(x^*, Sx_{2n+1}) + p(x_{2n+1}, Tx^*)]/2\}) \\
&\quad + L \min\{d_w(x^*, Sx_{2n+1}), d_w(x_{2n+1}, Tx^*)\} \\
&= F(\max\{p(x^*, x_{2n+1}), p(x^*, Tx^*), p(x_{2n+1}, x_{2n+2}), \\
&\quad [p(x^*, x_{2n+2}) + p(x_{2n+1}, Tx^*)]/2\}) \\
&\quad + L \min\{d_w(x^*, x_{2n+2}), d_w(x_{2n+1}, Tx^*)\}.
\end{aligned}$$

Using continuity of F and letting $n \to \infty$ in the above inequality we obtain

$$\tau + F(p(x^*, Tx^*)) \leq F(p(x^*, Tx^*)),$$

which is a contradiction. Therefore, we get $p(x^*, Tx^*) = 0$, that is, x^* is a fixed point of T.Then, by similar arguments as in Case 1, we may show that $Sx^* = x^*$. So, x^* is a common fixed point of T and S. □

The common fixed point of T and S in Theorem 3.6 is unique if we replace $\min\{d_w(x, Sy), d_w(y, Tx)\}$ by $\min\{d_w(x, Tx), d_w(x, Sy), d_w(y, Tx)\}$ in (3.14). So, we have the following theorem.

Theorem 3.7. *Let (X, p) be a complete weak partial metric space and $T, S : X \to X$ be two mappings. Suppose that $F \in \mathcal{F}$ and there exist $\tau > 0$ and $L \geq 0$ such that, for all $x, y \in X$ satisfying $p(Tx, Sy) > 0$, the following holds:*

$$\begin{aligned}
\tau + F(p(Tx, Sy)) &\leq F\left(\max\left\{p(x, y), p(x, Tx), p(y, Sy), \frac{p(x, Sy) + p(y, Tx)}{2}\right\}\right) \\
&\quad + L \min\{d_w(x, Tx), d_w(x, Sy), d_w(y, Tx)\}. \qquad (3.19)
\end{aligned}$$

If F is continuous, then the common fixed point of T and S is unique.

Proof. The existence of the common fixed point of T and S follows from Theorem 3.6. To prove the uniqueness of the common fixed point of T and S, assume that x^* and y^* are two common fixed points of T and S and $x^* \neq y^*$. Then, we have $Tx^* = Sx^* = x^*$ and $Ty^* = Sy^* = y^*$. First, we show that $p(y^*, y^*) = 0$. Suppose that $p(y^*, y^*) > 0$. Then, from (3.19), we get

$$\begin{aligned}
\tau + F(p(y^*, y^*)) &= \tau + F(p(Ty^*, Sy^*)) \\
&\leq F\left(\max\left\{p(y^*, y^*), p(y^*, Ty^*), p(y^*, Sy^*), \frac{p(y^*, Sy^*) + p(y^*, Ty^*)}{2}\right\}\right) \\
&\quad + L \min\{d_w(y^*, Ty^*), d_w(y^*, Sy^*), d_w(y^*, Ty^*)\} \\
&= F(p(y^*, y^*)),
\end{aligned}$$

which is a contradiction. Thus $p(y^*, y^*) = 0$. Now, since $p(Tx^*, Sy^*) > 0$, from (3.19), we have

$$\begin{aligned}
\tau + F(p(x^*, y^*)) &= \tau + F(p(Tx^*, Sy^*)) \\
&\leq F\left(\max\left\{p(x^*, y^*), p(x^*, Tx^*), p(y^*, Sy^*), \frac{p(x^*, Sy^*) + p(y^*, Tx^*)}{2}\right\}\right) \\
&\quad + L\min\{d_w(x^*, Tx^*), d_w(x^*, Sy^*), d_w(y^*, Tx^*)\} \\
&= F\left(\max\left\{p(x^*, y^*), p(x^*, x^*), p(y^*, y^*), \frac{p(x^*, y^*) + p(y^*, x^*)}{2}\right\}\right) \\
&\quad + L\min\{d_w(x^*, x^*), d_w(x^*, y^*), d_w(y^*, x^*)\} \\
&= F(p(x^*, y^*)).
\end{aligned}$$

It is a contradiction.Then, we conclude that $p(x^*, y^*) = 0$, that is $x^* = y^*$. This proves that the common fixed point of T and S is unique. □

Taking $T = S$ in Theorems 3.6 and 3.7, we have the following results.

Corollary 3.8. *Let (X, p) be a complete weak partial metric space and $T : X \to X$ be a mapping. Suppose that $F \in \mathcal{F}$ and there exist $\tau > 0$ and $L \geq 0$ such that, for all $x, y \in X$ satisfying $p(Tx, Ty) > 0$, the following holds:*

$$\begin{aligned}
\tau + F(p(Tx, Ty)) &\leq F\left(\max\left\{p(x, y), p(x, Tx), p(y, Ty), \frac{p(x, Ty) + p(y, Tx)}{2}\right\}\right) \\
&\quad + L\min\{d_w(x, Ty), d_w(y, Tx)\}.
\end{aligned}$$

If F is continuous, then T has a fixed point in X.

Corollary 3.9. *Let (X, p) be a complete weak partial metric space and $T : X \to X$ be a mapping. Suppose that $F \in \mathcal{F}$ and there exist $\tau > 0$ and $L \geq 0$ such that, for all $x, y \in X$ satisfying $p(Tx, Ty) > 0$, the following holds:*

$$\begin{aligned}
\tau + F(p(Tx, Ty)) &\leq F\left(\max\left\{p(x, y), p(x, Tx), p(y, Ty), \frac{p(x, Ty) + p(y, Tx)}{2}\right\}\right) \\
&\quad + L\min\{d_w(x, Tx), d_w(x, Ty), d_w(y, Tx)\}.
\end{aligned}$$

If F is continuous, then T has a unique fixed point in X.

We see that the following corollary becomes a direct result, if we take $L = 0$ and $T = S$ in Theorem 3.6.

Corollary 3.10. *Let (X, p) be a complete weak partial metric space and $T : X \to X$ be an F-weak contraction. If T or F is continuous, then T has a unique fixed point in X.*

From Corollary 3.10, we get the following result:

Corollary 3.11. *Let (X, p) be a complete weak partial metric space and $T : X \to X$ be a mapping. Suppose that there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x, y \in X$ satisfying $p(Tx, Ty) > 0$, the following holds:*

$$\tau + F(p(Tx, Ty)) \leq F(ap(x, y) + bp(x, Tx) + cp(y, Ty) + e[p(x, Ty) + p(y, Tx)]), \tag{3.20}$$

where $a, b, c, e \geq 0$ and $a + b + c + 2e < 1$. If T or F is continuous, then T has unique a fixed point in X.

Proof. The proof will follow from Corollary 3.10, if we can show that condition (3.1) holds for all $x, y \in X$ with $p(Tx, Ty) > 0$. Then, from (3.20), we have

$$\begin{aligned} & ap(x,y) + bp(x,Tx) + cp(y,Ty) + e[p(x,Ty) + p(y,Tx)] \\ \leq \quad & (a+b+c+2e)\max\left\{p(x,y), p(x,Tx), p(y,Ty), \frac{p(x,Ty)+p(y,Tx)}{2}\right\} \\ \leq \quad & \max\left\{p(x,y), p(x,Tx), p(y,Ty), \frac{p(x,Ty)+p(y,Tx)}{2}\right\}, \end{aligned}$$

for all $x, y \in X$ with $p(Tx, Ty) > 0$. Then, by using (F1), we obtain

$$\begin{aligned} \tau + F(p(Tx,Ty)) \quad & \leq \quad F(ap(x,y) + bp(x,Tx) + cp(y,Ty) + e[p(x,Ty) + p(y,Tx)]) \\ & \leq \quad F\left(\max\left\{p(x,y), p(x,Tx), p(y,Ty), \frac{p(x,Ty)+p(y,Tx)}{2}\right\}\right), \end{aligned}$$

for all $x, y \in X$ with $p(Tx, Ty) > 0$. Then the proof is completed. □

Now, we give an illustrative example.

Example 3.12. *Let $X = [0,1]$ and $p(x,y) = |x-y| = d_w(x,y)$. Therefore, since (X, d_w) is complete, then by Lemma 2.2 (X,p) is a complete weak partial metric space. Let $T, S : [0,1] \to [0,1]$,*

$$Tx = Sx = \begin{cases} \frac{1}{2}, & x \in [0,1), \\ \frac{1}{4}, & x = 1. \end{cases}$$

Therefore, by choosing $F\alpha = \ln \alpha$, $\alpha \in (0, +\infty)$ and $\tau = \ln 3$, for $x \in [0,1)$ and $y = 1$, we obtain

$$p(Tx, Sy) = p\left(\frac{1}{2}, \frac{1}{4}\right) = \left|\frac{1}{2} - \frac{1}{4}\right| = \frac{1}{4} > 0$$

and

$$\max\left\{p(x,1), p(x,Tx), p(1,S1), \frac{p(x,S1)+p(1,Tx)}{2}\right\} \geq p(1,S1) = \frac{3}{4}.$$

Then, for any $L \geq 0$, we obtain

$$\begin{aligned} \tau + F(p(Tx,Sy)) \quad & = \quad \ln 3 + \ln \frac{1}{4} \\ & = \quad F(p(1,S1)) \\ & \leq \quad F\left(\max\left\{p(x,1), p(x,Tx), p(1,S1), \frac{p(x,S1)+p(1,Tx)}{2}\right\}\right) \\ & \qquad + L\min\{d_w(x,Tx), d_w(x,S1), d_w(1,Tx)\}. \end{aligned}$$

This shows that all conditions of Theorem 3.7 are satisfied and so $\frac{1}{2}$ is the unique common fixed point of T and S.

Theorem 3.13. *Let (X,p) be a complete weak partial metric space and $T : X \to X$ be a mapping. Suppose that $F \in \mathcal{F}$ and there exist $\tau > 0$ and $L \geq 0$ such that, for all $x, y \in X$ satisfying $p(Tx, Ty) > 0$, the following holds:*

$$\begin{aligned} \tau + F(p(Tx,Ty)) \quad & \leq \quad F(\max\{p(x,y), p(x,Tx), p(y,Ty)\}) \\ & \qquad + L\min\{d_w(x,Tx), d_w(x,Ty), d_w(y,Tx)\}. \end{aligned} \tag{3.21}$$

If F is continuous, then T has a unique fixed point in X.

Proof. Let $x_0 \in X$ be an arbitrary point. Let us define the sequence $\{x_n\} \subset X$ as $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$. Assume $p(x_n, x_{n+1}) > 0$ for all $n \in \mathbb{N}$. By (3.21), we have

$$\begin{aligned} F(p(x_n, x_{n+1})) &= F(p(Tx_{n-1}, Tx_n)) \\ &\leq F(\max\{p(x_{n-1}, x_n), p(x_{n-1}, Tx_{n-1}), p(x_n, Tx_n)\}) \\ &\quad +L\min\{d_w(x_{n-1}, Tx_{n-1}), d_w(x_{n-1}, Tx_n), d_w(x_n, Tx_{n-1})\} - \tau \\ &= F(\max\{p(x_{n-1}, x_n), p(x_{n-1}, x_n), p(x_n, x_{n+1})\}) \\ &\quad +L\min\{d_w(x_{n-1}, x_n), d_w(x_{n-1}, x_{n+1}), d_w(x_n, x_n)\} - \tau \\ &= F(\max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}) - \tau. \end{aligned}$$

If $\max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} = p(x_n, x_{n+1})$, then

$$F(p(x_n, x_{n+1})) \leq F(p(x_n, x_{n+1})) - \tau,$$

a contradiction. Thus, $\max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} = p(x_{n-1}, x_n)$ and hence

$$F(p(x_n, x_{n+1})) \leq F(p(x_{n-1}, x_n)) - \tau, \tag{3.22}$$

for all $n \in \mathbb{N}$. Repeating (3.22) n-times, we get that

$$F(p(x_n, x_{n+1})) \leq F(p(x_0, x_1)) - n\tau.$$

Rest of the proof can be obtained as in the proof of Theorem 3.1. Also, it can be shown that the fixed point of T is unique as in the proof of Theorem 3.7. □

By the aid of Lemma 2.4, we have the following result of Theorem 3.13.

Corollary 3.14. *Let (X, p) be a weak partial metric space and $T, S : X \to X$ be two mappings. Suppose that $F \in \mathcal{F}$ and there exist $\tau > 0$ and $L \geq 0$ such that, for all $x, y \in X$ satisfying $p(Tx, Ty) > 0$, the following holds:*

$$\begin{aligned} \tau + F(p(Tx, Ty)) &\leq F(\max\{p(Sx, Sy), p(Sx, Tx), p(Sy, Ty)\}) \\ &\quad +L\min\{d_w(Sx, Tx), d_w(Sx, Ty), d_w(Sy, Tx)\}. \end{aligned}$$

Also, assume that

i. $TX \subseteq SX$.

ii. SX is a complete subspace of the weak partial metric space X.

If F is continuous, then T and S have a unique point of coincidence in X. Moreover, if T and S are weakly compatible, then T and S have a unique common fixed point.

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Competing Interests

Authors have declared that no competing interests exist.

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