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A Comparison of Univariate and Multivariate Time Series Approaches to Modeling Currency Exchange Rate

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Authors' contributions

This work was carried out in collaboration between both authors. Author OAK designed the study, performed the statistical analysis. Author IMA wrote the protocol and the first draft of the manuscript and also managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

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Abstract

This paper describes a study using Average Monthly Exchange Rates (AMER) of Naira (Nigerian currency) to six other currencies of the World to evaluate and compare the performance of univariate and multivariate based time series models. The data from 2002 -2014 was used for modeling and forecasting the actual values of the AMER for 2014 of the six currencies. The Mean Absolute Percentage Error (MAPE) forecast accuracy measure was also used in determining if Univariate Times Series Model or Multivariate Time Series Models is best for forecasting the future AMER value of a given currency. The result of data showed that the Univariate time series fits better for Dollar, Pounds Sterling, Yen, WAUA and CFA, while only Euro fits well for the Multivariate time series.

Keywords: Autoregressive integrated moving average; vector autoregressive and mean absolute percentage error.

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1 Introduction

Time series modeling is a dynamic research area which has attracted attentions of researchers' community over last few decades. The main aim of time series modeling is to carefully collect and rigorously study the past observations of a time series to develop an appropriate model which describes the inherent structure of the series and also the determination of the temporal ordering among some variables through Granger causality tests. This model is then used to generate future values for the series, i.e. to make forecasts. Time series forecasting thus can be termed as the act of predicting the future by understanding the past [1]. Due to the indispensable importance of time series forecasting in numerous practical fields such as business, economics, finance, science and engineering, etc. ([2,3-5]), proper care should be taken to fit an adequate model to the underlying time series. It is obvious that a successful time series forecasting depends on an appropriate model fitting. A lot of efforts have been done by researchers over many years for the development of efficient models to improve the forecasting accuracy. As a result, various important time series forecasting models have been evolved in literature (see [6-11,1,12-17,4]). Furthermore, in many forecasting problems, it may be the case that there are more than just one variable to consider. Attempting to model each variable individually may at times work. But in such situations, it is often the case that these variables are sometimes cross-correlated, and that structure can be effectively taken advantage of in forecasting. It has been argued that the nature of the problem allows fairly strong restrictions to be imposed in a univariate model. These restrictions are not normally enforced with the traditional ARIMA framework. For a multivariate set-up, the number of parameters to be estimated increases rapidly as more series are included and a vector ARMA model the issues concerned with identifiability becomes quite complicated [10]. Hence it is even more important to formulate models which take account of the nature of the problem. Nowadays, more and more investors are interested in investing in the foreign exchange market. However, the international financial market is changing over time due to exchange rate volatility. It causes an inevitable risk in the investment since we don't even know if the exchange rate would increase or decrease tomorrow. Thus, how to avoid or reduce this risk requires a model to forecast the result accurately by eliminating the fitting errors involved with classical data forecasting. Indistinguishable linked-predictive let-downs suggest the inadequacy of a model. Therefore, a good forecasting model should result in the fewest fitting errors while maximizing accuracy.

In this study, we compared two types of time-series-based forecasting models: univariate and multivariate to see which of these models best fits the exchange rate data using MAPE. The remainder of the paper is structured as follows: Section 2 provides a unified framework for univariate and multivariate time series models. In Section 3 we describe our procedure for finding in a robust way the transformation parameters. In Section 4 we apply the suggested procedure to real time series Section 5 contains comparison from the two models and concluding remarks.

1.1 Univariate time series (ARMA Model)

The process $\{Y_t, t = 0, \pm 1, \pm 2, ...\}$ is said to be an ARMA(*p*, *q*) process if $\{Y_t\}$ is stationary and if for every *t*, Box and Jenkins [13],

$$Y_t - \phi_1 Y_{t-1} - \cdots - \phi_p Y_{t-p} = e_t - \theta_1 e_{t-1} - \cdots - \theta_q e_{t-q} \tag{1}$$

where $\{e_t\} \sim N\left(0, \sigma^2\right)$. The Equation (1) can be written symbolically in a more compact form

$$\phi(B) Y_t = \theta(B) e_t, \qquad t = 0, \pm 1, \pm 2, ..., \tag{2}$$

Where ϕ and θ are the p^{th} and q^{th} degree polynomials.

$$\phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p \tag{3}$$

And

$$\theta(B) = 1 + \theta_1 Z + \cdots + \theta_q Z^q \tag{4}$$

B and Z are the backward shift operators defined by

$$B^j Y_t = Y_{t-j}, \qquad j = 0, \pm 1, \pm 2, ... \tag{5}$$

$$Z^j e_t = e_{t-j}, \qquad j = 0, \pm 1, \pm 2, ... \tag{6}$$

The polynomials ϕ and θ will be referred to as the autoregressive and moving average polynomials respectively of the difference Equation (3).

If $\phi(B) \equiv 1$ then

$$Y_t = \theta(B) e_t \tag{7}$$

The process is said to be a moving-average process of order q (or MA(q)). Similarly, If $\theta(B) \equiv 1$ then

$$\phi(B) Y_t = e_t \tag{8}$$

The process is said to be an autoregressive process of order p (or AR(p)).

A series which becomes stationary after first differencing is said to be integrated of order one, denoted $I(1)$. If ΔY_t is described by a stationary ARMA(p, q) model, we say that Y_t is described by an autoregressive integrated moving average (ARIMA) model of order p,1,q, or mathematically ARIMA(p,1,q) is written as

$$\phi\ (B)(1-B)^d Y_t = \theta(B) e_t$$

Let

$$X_t = Y_t^{(1-B)} \tag{9}$$

$$W_t = (1-B)^d X_t = \nabla^d X_t \tag{10}$$

Then, (10) admits an ARMA (p, q) Model if ;

$$\begin{aligned} W_t = {} & \alpha_1 + \alpha_2 W_{t-1} + \alpha_3 W_{t-2} + \alpha_4 W_{t-3} + ... + \alpha_{p+1} W_{t-p} + e_t - \alpha_{p+2} e_{t-1} - \alpha_{p+3} e_{t-2} \\ & - \alpha_{p+4} e_{t-3} - ... - \alpha_{p+q+1} e_{t-q} \end{aligned} \tag{11}$$

$$\begin{aligned} e_t = {} & -\alpha_1 + W_t - \alpha_2 W_{t-1} - \alpha_3 W_{t-2} - \alpha_4 W_{t-3} - ... - \alpha_{p+1} W_{t-p} + \alpha_{p+2} e_{t-1} + \alpha_{p+3} e_{t-2} \\ & + \alpha_{p+4} e_{t-3} + ... + \alpha_{p+q+1} e_{t-q} \end{aligned} \tag{12}$$

$$S(\boldsymbol{\alpha}) = \sum_{t=1}^{n} e_t^2 \tag{13}$$

where Y_t is the original series, X_t is the transformed series, $\langle \alpha_i, i = 1,2,..., p+1, p+2,..., p+q+1 \rangle$ are the sequence of the parameters of ARMA(*p*, *q*) process, e_t is the white noise process, $e_t \sim N(0, \sigma^2)$ and $S(\boldsymbol{\alpha})$ is the model residual sum of squares.

1.2 Multivariate time series

Multivariate time series is an extension of the Univariate time series. Multivariate time series in practice are best considered as components of some vector valued time series $\{Y_t\}$ having not only serial dependence within each component $\{Y_{it}\}$ and $\{Y_{jt}\}$, $i \neq j$. Much of the theory of univariate time series extends in a natural way to the multivariate case. In multivariate time series, attention is confined to vector autoregressive (or VAR) models. The univariate autoregressive moving average models can be readily extended to the multivariate case, in which the stochastic process that generates the time series of a vector of variables is modeled. The most common approach is to consider a vector autoregressive (VAR) model. A VAR describes the dynamic evolution of a number of variables from their common history [3]. If we consider two variables, Y_{1t} and Y_{2t}, a first order VAR(1) would be given by

$$Y_{1t} = \mu_1 + \phi_{11} Y_{1(t-1)} + \phi_{12} Y_{2(t-1)} + z_{1t} \tag{14}$$

$$Y_{2t} = \mu_2 + \phi_{21} Y_{1(t-1)} + \phi_{22} Y_{2(t-1)} + z_{2t} \tag{15}$$

Where z_{1t} and z_{2t} are two white noise processes (independent of the history of Y_{1t} and Y_{2t}) that may be correlated. Hence $\phi_{12} \neq 0$ it means that the history of Y_{1t} helps explaining Y_{2t}. Equations (17 and 18) can be written as:

$$\begin{pmatrix} Y_{1t} \\ Y_{2t} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} + \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} \tag{16}$$

Or with appropriate definitions, as

$$\vec{Y}_t = \mu + \Theta_1 \vec{Y}_{t-1} + \vec{z}_t \tag{17}$$

where $\vec{Y}_t = (Y_{1t}, Y_{2t})'$ and $\vec{z}_t = (z_{1t}, z_{2t})'$.

This can be extended to a general vector autoregressive model of order *p* (VAR(*p*)), hence for a *k*-dimensional vector $\vec{Y}_t$ given by

$$\vec{Y}_t = \mu + \Theta_1 \vec{Y}_{t-1} + ... + \Theta_p \vec{Y}_{t-p} + \vec{z}_t \tag{18}$$

where each Θ_i is a $k \times k$ matrix and $\vec{z}_t$ is a *k*-dimensional vector of white noise terms with covariance matrix Σ. As in the univariate case, we can use the lag operator to define a matrix lag polynomial

$$\Theta(L) = I_k - \Theta_1 L - \ldots - \Theta_p L^p \tag{19}$$

Where I_k is the *k*-dimensional identity matrix, hence VAR can be written as

$$\Theta(L)\vec{Y}_t = \mu + \vec{z}_t \tag{20}$$

The matrix lag polynomial is a $k \times k$ matrix where each element corresponds to *p*-th order polynomial.

2 Materials and Methods

The data used in this research is the daily exchange rate of the Nigerian currency naira to other currencies for the period of January, 2002 – December 2014 which was obtained from the Central Bank of Nigeria. The Average Monthly Exchange Rate (AMER) was computed for six selected currencies namely, Dollar, Pounds Sterling, Euro, Yen, West Africa Unit of Account (WAUA) and Commuaute Financiere Africaine (CFA). The AMER became necessary in order to obtain a more central measure of the daily exchange rates due to calendar problem often encountered in time series analysis. Thus, the AMER was computed for the period of January 2002 – December 2014.

2.1 Data transformation

In most time series data, transformation is required in order to stabilize the variance of the data in other words the series do not depend on the mean of the data. Transformation is a preliminary analysis often associated with non-stationary time series data, hence the reason for transformation could be that the amount of variability in a time series is not constant across time or to study what is left in a data set after having removed the trends (see [18,14,19]). A simple but often effective way to stabilize the variance across time is to apply the common transformations $\left[\log Y, \sqrt{Y}, \frac{1}{Y}, Y^2, \frac{1}{Y^2}, \frac{1}{\sqrt{Y}}\right]$ to the time series. In this study we would apply the power transformation to time series data this is necessary in order to have a uniform transformation for all the variables under consideration. In applying the power transformation, we split the observed time series $\{Y_t, t = 1, 2, \ldots, n\}$ chronologically into m fairly equal different groups and compute their means $\{\overline{Y}_i, i = 1, 2, \ldots, m\}$ and standard deviations $\{\hat{\sigma}_j,\ j = 1, 2, \ldots, m\}$ for the groups. We then regress the natural logarithms of the group standard deviations $\{\hat{\sigma}_j,\ j = 1, 2, \ldots, m\}$ against the natural logarithms of the group means $\{\overline{Y}_i, i = 1, 2, \ldots, m\}$ and then determine the slope, β, of the relationship. [20]:

$$\log_e \hat{\sigma}_i = \alpha + \beta \log_e \overline{Y}_i + \varepsilon_i \tag{21}$$

The power transformation is given as

$$X_t = \begin{cases} \log_e Y_t, & \beta = 1 \\ Y_t^{(1-\beta)} & \beta \neq 1 \end{cases} \tag{22}$$

3 Data Analysis and Applications

Based on (2), the required transformation for each of the variables were obtained, resulting from estimates of the fitted line slopes ($\beta's$). It was observed that all the variables required transformation of their data except for CFA. Table 1 illustrates the transformation required for each of the variables.

Table 1. Transformation required for the variables

Variable (Currency)	β -Value	Transformation
Dollar	-9.253	$X_t = Y_t^{10.253}$
Pounds	-4.583	$X_t = Y_t^{5.583}$
Euro	-0.7024	$X_t = Y_t^{1.7024}$
Yen	0.7073	$X_t = Y_t^{0.2927}$
WAUA	-1.813	$X_t = Y_t^{2.813}$
CFA	0.1336	No transformation is required.

Having obtained the transformed series for each of the six variables and ARIMA models fitted to the variables, the AIC selection criteria was used to select which models gives a better fit based on a list of candidate models computed for each of variables. Table 2 summaries the fitted models for these variables

Table 2. Summary of fitted models

Variable (Currency)	Fitted model	Model equation
Dollar	ARIMA(1,1,1)	$\hat{Y}_t = -0.4589Y_{t-1} - 0.7143e_{t-1} + e_t$
Pounds	Random walk	$Y_t = Y_{t-1} + e_t$
Euro	ARIMA(0,1,1)	$Y_t = -0.2974e_{t-1} + e_t$
Yen	ARIMA(1,1,0)	$\hat{Y}_t = 0.3586Y_{t-1} + e_t$
WAUA	ARIMA(0,1,1)	$\hat{Y}_t = -0.3653e_{t-1} + e_t$
CFA	ARIMA(1,1,0)	$\hat{Y}_t = -0.2389Y_{t-1} + e_t$

The time series plots of the **AMER** for the six variables (currencies) are illustrated in Figs. 1-6 while the SACF and SPACF of the residual plots of the fitted models for five of the variables are illustrated in Figs. 7-16.

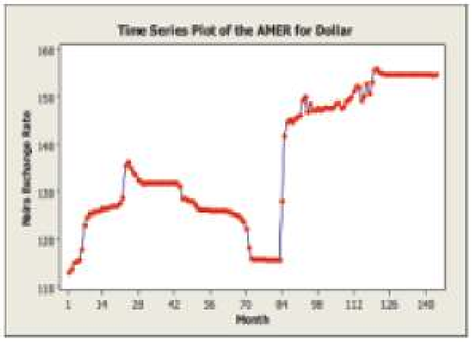

Fig. 1. Time series plot of the AMER for Dollar

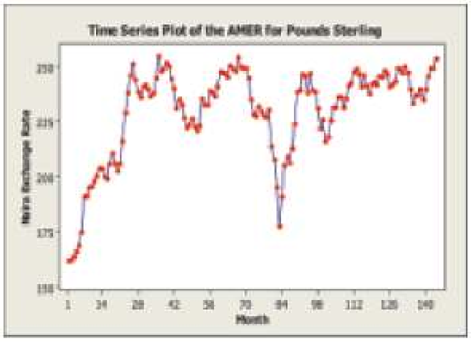

Fig. 2. Time series plot of the AMER for Pounds Sterling

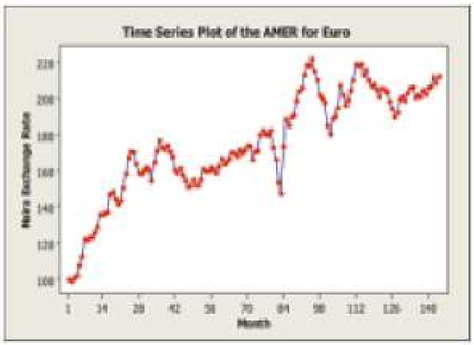

Fig. 3. Time series plot of the AMER for Euro

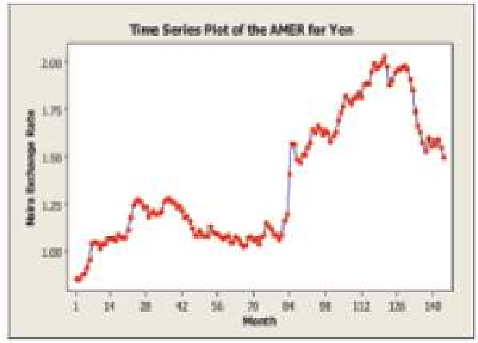

Fig. 4. Time series plot of the AMER for Yen

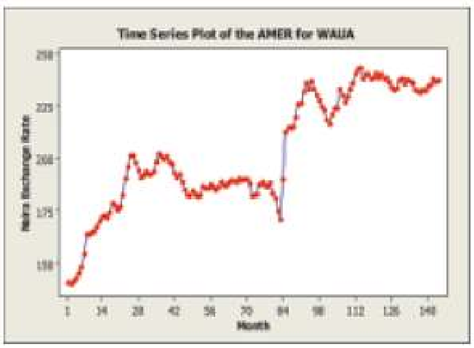

Fig. 5. Time series plot of the AMER for WAUA

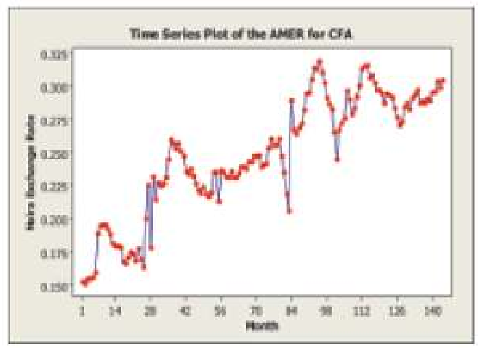

Fig. 6. Time series plot of the AMER for CFA

SACF and SPACF of the residual plots of the fitted models for five of the variables are shown below

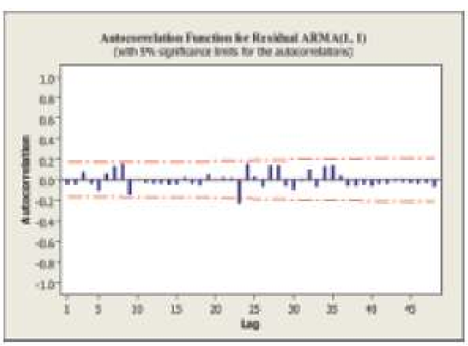

Fig. 7. Autocorrelation plot of residuals of ARIMA(1,1) model for Dollar

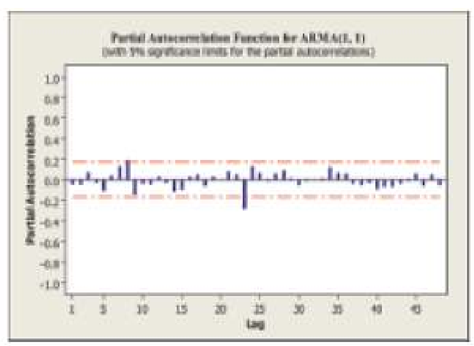

Fig. 8. Partial autocorrelation plot of residuals of ARIMA(1,1) model for Dollar

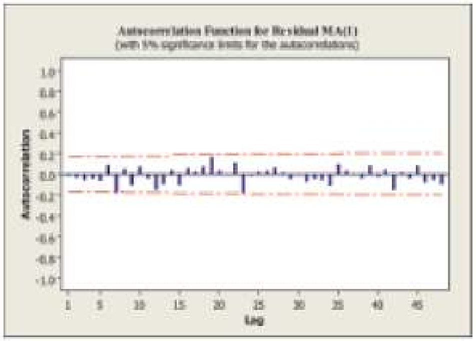

Fig. 9. Autocorrelation plot of residuals of MA(1) model for Euro

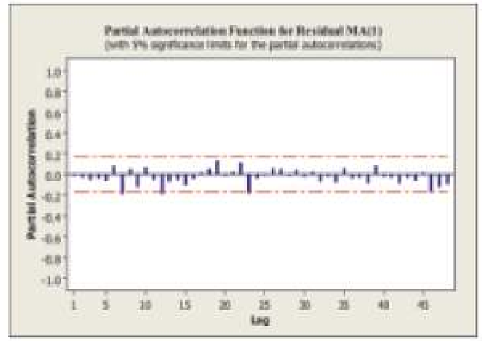

Fig. 10. Partial autocorrelation plot of residuals of MA(1) model for Euro

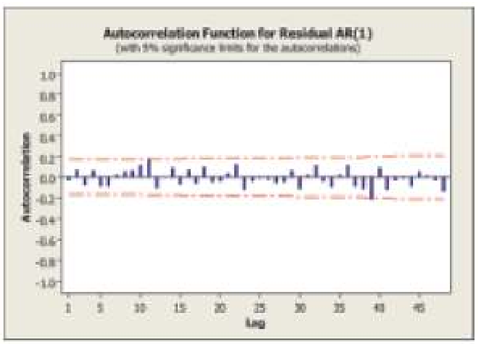

Fig. 11. Autocorrelation plot of residuals of AR(1) model for Yen

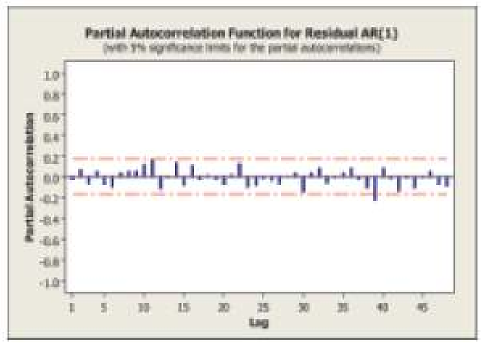

Fig. 12. Partial autocorrelation plot of residuals of AR(1) model for Yen

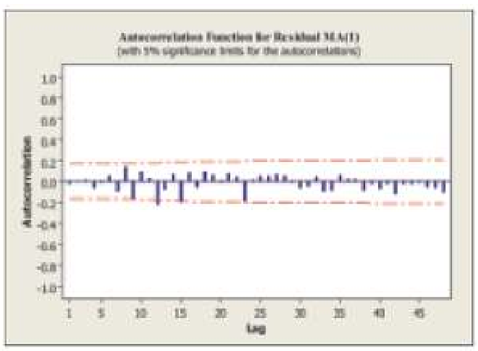

Fig. 13. Autocorrelation plot of residuals of MA(1) model for WAUA

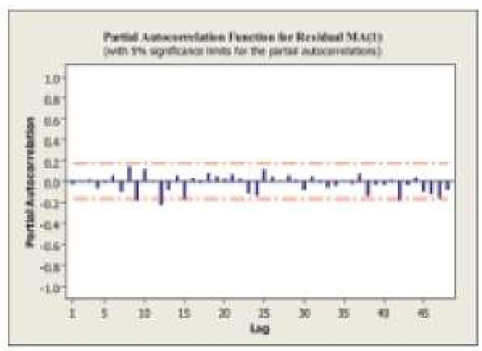

Fig. 14. Partial autocorrelation plot of residuals of MA(1) model for WAUA

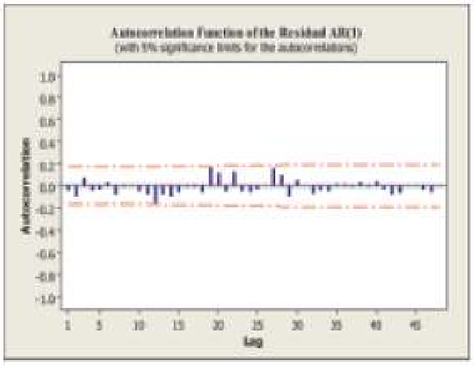

Fig. 15. Autocorrelation plot of residuals of AR(1) model for CFA

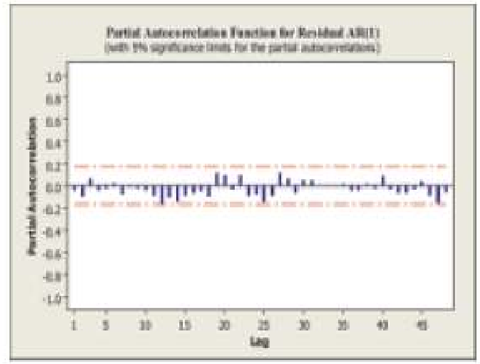

Fig. 16. Partial autocorrelation plot of residuals of AR(1) model for CFA

In the case of the Pounds Sterling, a preliminary analysis of the time series using the SACF and SPACF shows that the series looks like a white noise process. Figs. 17-18 depicts the SACF and SPACF of the Pounds Sterling series.

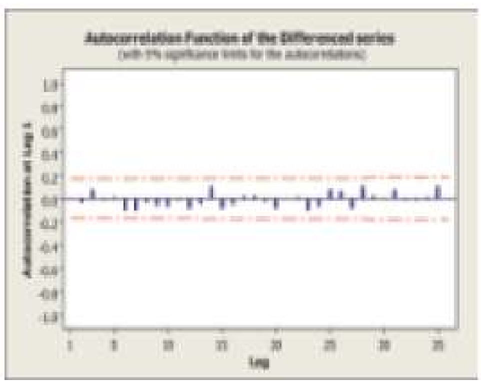

Fig. 17. Autocorrelation plot of the differenced series for Pound Sterling

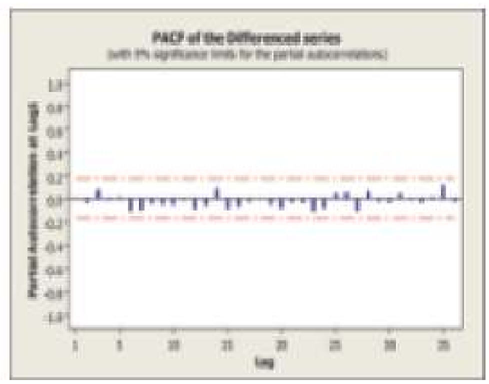

Fig. 18. Partial autocorrelation plot of the differenced series for Pound Sterling

3.1 Multivariate analysis

This section we consider fitting a multivariate time series model to the Average Monthly Exchange Rates (AMER) of the six variables. A Vector Autoregressive Model of order twelve VAR(12) is applied using GreI software to select the most appropriate *p* order of the VAR model using the HIC, BIC and AIC criterion.

3.2 Selection of the VAR order *p*

The choice of an appropriate order *p* for the estimates of the VAR model was determined using the AIC, BIC and HQC criterion using the Gretl software. Table 3 outlines the log-likelihood ratio for lags 1 to 12 and its corresponding AIC, BIC and HQC values. The asterisks below indicate the best (that is, minimized) values of the respective information criteria, Akaike criterion (AIC), Schwarz Bayesian criterion (BIC) and Hannan-Quinn criterion (HQC).

Table 3. VAR system, maximum lag order 12

Lags	Loglik	p(LR)	AIC	BIC	HQC
1	5458.24543		83.337052	84.254307*	83.709782
2	5394.72088	0.00000	82.920013*	84.623487	83.612227*
3	5360.51133	0.00089	82.947141	85.436834	83.958838
4	5342.23723	0.44320	83.215716	86.491627	84.546896
5	5318.08289	0.08247	83.395195	87.457325	85.045859
6	5292.14166	0.04207	83.547601	88.395950	85.517747
7	5263.37049	0.01275	83.657129	89.291696	85.946758
8	5230.99159	0.00229	83.711994	90.132780	86.321107
9	5201.90821	0.01108	83.816791	91.023796	86.745387
10	5157.46184	0.00000	83.688816	91.682039	86.936895
11	5105.59107	0.00000	83.448350	92.227792	87.015912
12	5058.80664	0.00000	83.284949	92.850610	87.171995

Based on the result of Table 3, a VAR model of order p=2 is the most appropriate model to use. We therefore proceed by fitting Vector Autoregressive model of order two VAR(2) to the six variables.

3.3 Vector autoregressive model for Dollar

The estimates of the VAR(2) of the AMER for Dollar are shown in Table 4.

Table 4. VAR(2) parameter estimates of the AMER for dollar

Variable	Co-efficient	Std error	t-ratio	Remarks
Constant	1205.65	657.092	-1.8348	Not Significant
Y_{1t-1}	0.994158	0.11275	8.8174	Significant
Y_{1t-2}	-0.0529731	0.113358	-0.4673	Not Significant
Y_{2t-1}	-1.371108e-012	6.28668e-12	-0.2181	Not Significant
Y_{2t-2}	3.37565e-012	6.30687e-012	-0.5352	Not Significant
Y_{3t-1}	0.103098	0.0700886	1.4710	Not Significant
Y_{3t-2}	-0.103098	0.0706959	-1.5380	Not Significant
Y_{4t-1}	3236.27	1318.4	2.4547	Significant
Y_{4t-2}	-2036.48	1447.33	-1.4071	Not Significant
Y_{5t-1}	-0.272922	0.194013	-1.4067	Not Significant
Y_{5t-2}	0.241743	0.18688	1.2936	Not Significant
Y_{6t-1}	1458.87	959.563	1.5203	Not Significant
Y_{6t-2}	-664.545	958.08	-0.6936	Not Significant

From Table 4 it shows that only one variable Yen (Y_4) contributes to the value of Dollar. It also reveals that the value of Dollar at any given time using the VAR (2) is determined by the value of Dollars at time t-1 and Yen at t-1.

The model equation is $\hat{Y}_{1t} = 0.9942 Y_{1t-1} + 3236.27 Y_{4t-1} + e_t$

3.4 Vector autoregressive model for pounds sterling

The estimates of the VAR(2) of the AMER for Pounds Sterling are shown in Table 5.

From Table 5 it shows that only one variable Euro (Y_3) contributes to the value of Pounds Sterling. It also reveals that the value of Pounds Sterling at any given time using the VAR (2) is determined by the value of Pounds Sterling at time t-1 and Euro at times t-1 and t-2.

Table 5. VAR(2) parameter estimates of the AMER for pounds sterling

Variable	Co-efficient	Std error	t-ratio	Remarks
Constant	9.57721e+011	1.29535e+012	0.0739	Not Significant
Y_{1t-1}	-2.35039e+09	2.22268e+09	-1.0575	Not Significant
Y_{1t-2}	2.31799e+09	2.23466e+09	1.0373	Not Significant
Y_{2t-1}	0.941758	0.123932	7.5900	Significant
Y_{2t-2}	-0.0269465	0.12433	-0.2167	Not Significant
Y_{3t-1}	3.68249e+09	1.38168e+09	2.6652	Significant
Y_{3t-2}	-3.75579e+09	1.39366e+09	-2.6949	Significant
Y_{4t-1}	2.08224e+013	2.59901e+013	0.8012	Not Significant
Y_{4t-2}	-1.85546e+013	2.85317e+013	-0.6530	Not Significant
Y_{5t-1}	-3.28038e+09	3.82465e+09	-0.8577	Not Significant
Y_{5t-2}	4.20743e+09	3.68404e+09	1.1421	Not Significant
Y_{6t-1}	-3.49117e+013	1.89163e+013	-1.8456	Not Significant
Y_{6t-2}	1.87796e+01	1.8887e+013	0.9943	Not Significant

The model equation is $\hat{Y}_{2t} = 0.9418Y_{2t-1} + 3.68249e+09Y_{3t-1} - 3.75579e+09Y_{3t-2} + e_t$.

3.5 Vector autoregressive model for Euro

The estimates of the VAR(2) of the AMER for Euro are shown in Table 6.

Table 6. VAR(2) parameter estimates of the AMER for Euro

Variable	Co-efficient	Std error	t-ratio	Remarks
Constant	-2137.62	1852.68	-1.1538	Not Significant
Y_{1t-1}	0.157544	0.317899	0.4956	Not Significant
Y_{1t-2}	-0.0123727	0.3196613	-0.0387	Not Significant
Y_{2t-1}	-6.29127e-012	1.77254e-011	-0.3549	Not Significant
Y_{2t-2}	2.58059e-012	1.77823e-011	0.1451	Not Significant
Y_{3t-1}	1.59251	0.197616	8.0586	Significant
Y_{3t-2}	-0.552697	0.199328	-2.7728	Significant
Y_{4t-1}	5932.98	3717.24	1.5961	Not Significant
Y_{4t-2}	-3137.53	4080.76	-0.7689	Not Significant
Y_{5t-1}	-1.23995	0.547021	-2.2667	Significant
Y_{5t-2}	0.847933	0.52691	1.6093	Not Significant
Y_{6t-1}	-871.734	2705.5	-0.3222	Not Significant
Y_{6t-2}	677.328	2701.32	0.2507	Not Significant

From Table 6 it shows that only one variable WAUA (Y_5) contributes to the value of Euro. It also reveals that the value of Euro at any given time using the VAR (2) is determined by the value of Euro at times *t*-1 and *t*-2 as well as the value WAUA at *t*-1.

The model equation is $\hat{Y}_{3t} = 1.59251Y_{3t-1} - 0.552697Y_{3t-2} - 1.23995Y_{5t-1} + e_t$.

3.6 Vector autoregressive model for Yen

The estimates of the VAR(2) of the AMER for Yen are shown in Table 7.

Table 7. VAR(2) parameter estimates of the AMER for Yen

Variable	Co-efficient	Std error	t-ratio	Remarks
Constant	-0.0215931	0.0506119	-0.4266	Not Significant
Y_{1t-1}	4.24777e-06	8.68443e-06	0.4891	Not Significant
Y_{1t-2}	-5.89148e-06	8.73127e-06	-0.6748	Not Significant
Y_{2t-1}	0	0	0.0159	Not Significant
Y_{2t-2}	0	0	-1.0994	Not Significant
Y_{3t-1}	3.99801e-06	5.39851e-06	0.7406	Not Significant
Y_{3t-2}	-1.38554e-06	5.44529e-06	-0.2544	Not Significant
Y_{4t-1}	1.36515	0.101549	13.4433	Significant
Y_{4t-2}	-0.33492	0.111479	-3.0043	Significant
Y_{5t-1}	-3.086326e-05	1.49437e-05	-2.0633	Significant
Y_{5t-2}	2.72085e-05	1.43943e-05	1.8902	Not Significant
Y_{6t-1}	0.0935117	0.0739096	1.2652	Not Significant
Y_{6t-2}	-0.114607	0.0737953	-1.5530	Not Significant

From Table 7 it shows that only one variable WAUA (Y_5) contributes to the value of Yen. It also reveals that the value of Yen at any given time using the VAR (2) is determined by the value of Yen at time *t*-1 and *t*-2 as well as the value WAUA at time *t*-1.

The model equation is $\hat{Y}_{4t} = 1.36515 Y_{4t-1} - 0.33492 Y_{4t-2} - 3.08326e - 05 Y_{5t-1} + e_t$

3.7 Vector autoregressive model for WAUA

The estimates of the VAR(2) of the AMER for WAUA are shown in Table 9.

Table 8. VAR(2) parameter estimates of the AMER For WAUA

Variable	Co-efficient	Std error	t-ratio	Remarks
Constant	-1597.03	711.832	-2.2436	Significant
Y_{1t-1}	0.197692	0.122142	1.6185	Not Significant
Y_{1t-2}	-0.138393	0.122801	-1.1270	Not Significant
Y_{2t-1}	-1.66543e-012	6.8104e-012	-0.2445	Not Significant
Y_{2t-2}	-1.68479e-012	6.83227e-012	-0.2466	Not Significant
Y_{3t-1}	0.386876	0.0759274	5.0953	Significant
Y_{3t-2}	-0.325448	0.0765853	-4.2495	Significant
Y_{4t-1}	5368.09	1428.23	3.7586	Significant
Y_{4t-2}	-3476.62	1567.9	-2.2174	Significant
Y_{5t-1}	0.120208	0.210175	0.5719	Not Significant
Y_{5t-2}	0.613945	0.202448	3.0326	Significant
Y_{6t-1}	586.987	1039.5	0.5647	Not Significant
Y_{6t-2}	-826.596	1037.89	-0.7964	Not Significant

From Table 8 it shows that the constant term, variables Y_3 (Euro) and Y_4 (Yen), contributes to the value of WAUA. It also reveals that the value of WAUA at any given time using the VAR (2) is determined by the value of WAUA at time *t*-2 and values of Euro and Yen at *t*-1 and *t*-2. It also confirms the reason for the presence of WAUA in the earlier models for Y_3 (Euro) and Y_4 (Yen). The model equation is

$$\hat{Y}_{5t} = -2.2436 + 0.3869 Y_{3t-1} - 0.3254 Y_{3t-2} + 5368.09 Y_{4t-1} - 3476.62 Y_{4t-2} + 0.6139 Y_{5t-2} + e_t$$

3.8 Vector autoregressive model for CFA

The estimates of the VAR(2) of the AMER for CFA are shown in Table 9.

Table 9. VAR(2) parameter estimates of the AMER For CFA

Variable	Co-efficient	Std error	t-ratio	Remarks
Constant	-0.0961157	0.0711225	-1.3514	Not Significant
Y_{1t-1}	-5.80468e-06	1.22038e-05	-0.4756	Not Significant
Y_{1t-2}	1.05132e-05	1.22696e-05	0.8569	Not Significant
Y_{2t-1}	0	0	1.4227	Not Significant
Y_{2t-2}	0	0	-1.1337	Not Significant
Y_{3t-1}	2.07073e-05	7.58627e-06	2.7296	Significant
Y_{3t-2}	-1.37394e-05	7.652e-06	-1.7955	Not Significant
Y_{4t-1}	0.183023	0.142701	1.2826	Not Significant
Y_{4t-2}	-0.0542741	0.156656	-0.3465	Not Significant
Y_{5t-1}	-3.91948e-05	2.0999e-05	-1.8665	Not Significant
Y_{5t-2}	1.99803e-05	2.02275e-05	0.9878	Not Significant
Y_{6t-1}	0.472836	0.103861	4.5526	Significant
Y_{6t-2}	0.365834	0.103701	3.5278	Significant

From Table 9 it shows that only one variable Y_3 (Euro) contributes to the value of CFA. It also reveals that the value of CFA at any given time using the VAR (2) is determined by the value of CFA at time *t*-1 and *t*-2 as well as the value of Euro time *t*-1.

The model equation is $\hat{Y}_{6t} = 2.071e-05Y_{3t-1} + 0.472836Y_{6t-1} + 0.365834Y_{6t-2} + e_t$.

3.9 Plots of the Univariate forecast and the multivariate forecast with the actual values of the AMER for 2014

The plots of the univariate and multivariate forecast of the 2014 AMER of the six currencies are illustrated in Figs. 19-24.

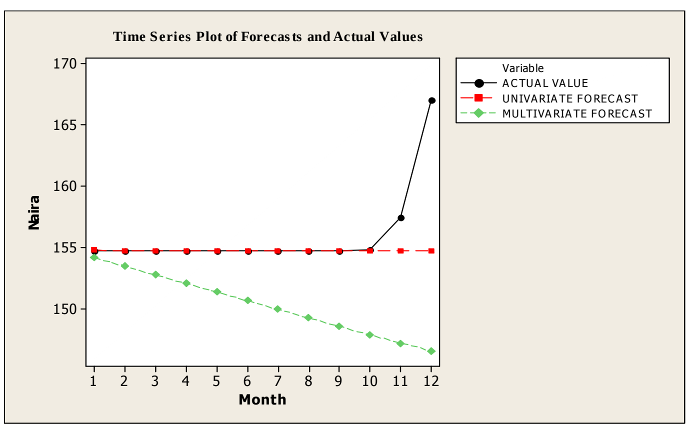

Fig. 19. Plot of the AMER univariate and multivariate forecast for Dollar with the actual values for 2014

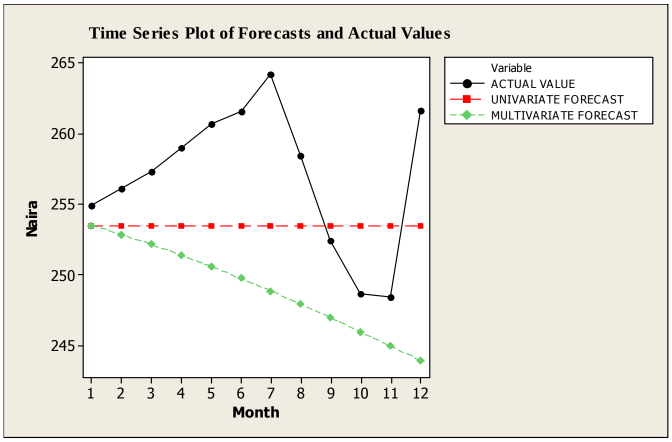

Fig. 20. Plot of the AMER univariate and multivariate forecast for Pounds Sterling with the actual values for 2014

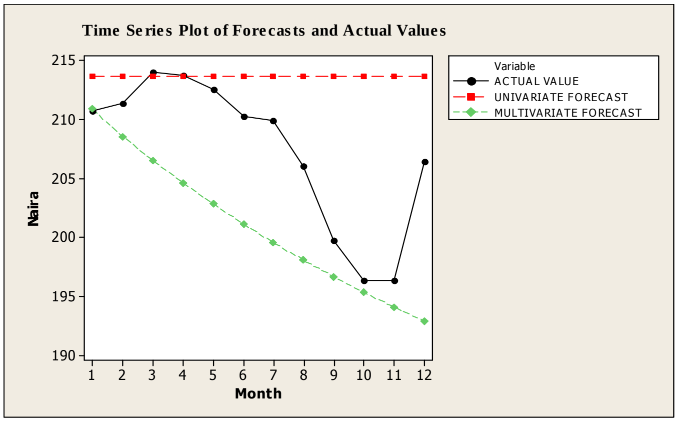

Fig. 21. Plot of the AMER univariate and multivariate forecast for Euro with the actual values for 2014

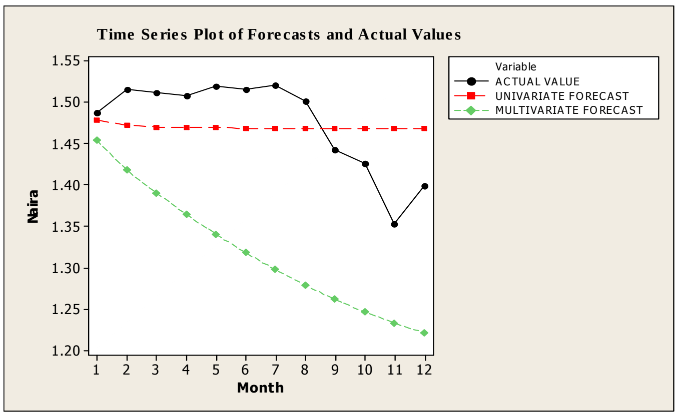

Fig. 22. Plot of the AMER univariate and multivariate forecast for Yen with the actual values for 2014

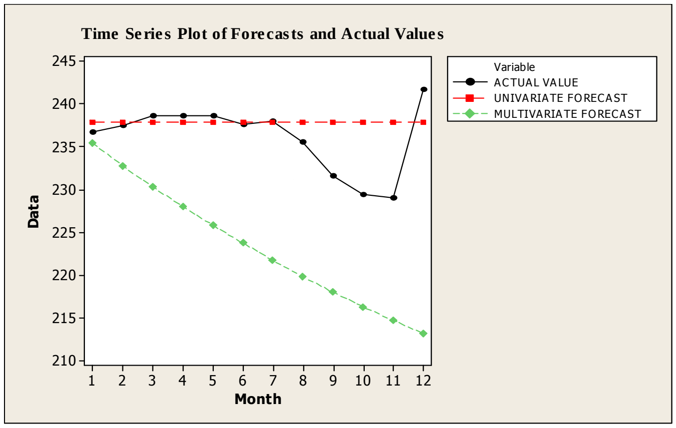

Fig. 23. Plot of the AMER univariate and multivariate forecast for WAUA with the actual values for 2014

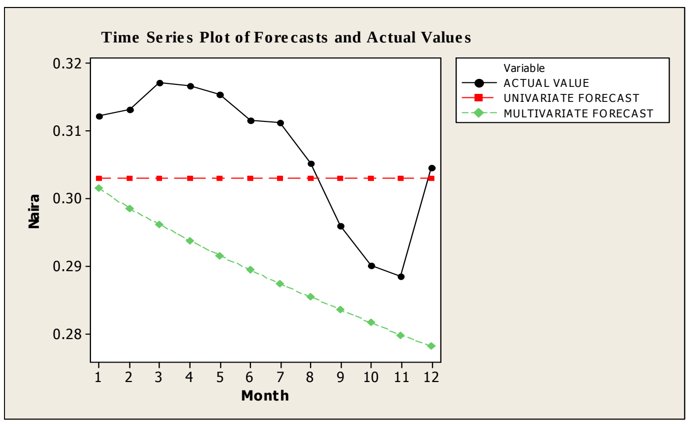

Fig. 24. Plot of the AMER univariate and multivariate forecast for CFA with the actual values for 2014

The plots of the univariate forecast and multivariate forecast with the actual values of AMER of 2014 showed general a downward slope of the multivariate forecast which suggests that most of the variables contributes negatively to the multivariate forecast values. On the other hand, the univariate forecast plots move almost in the direction of the actual values except for AMER of Euro.

4 Results and Discussion

4.1 Comparison of the univariate models and multivariate models

This section outlines a comparison of the univariate model and the multivariate model. Table 10 outlines the mean absolute percentage error (MAPE) of each of these variables by comparing the univariate method MAPE and the multivariate method MAPE. The MAPE of the univariate and the multivariate methods are compared, the method with a lesser MAPE is chosen as the preferable method for the variable been considered.

Table 10. Comparison of the univariate and multivariate models using mean absolute percentage error (MAPE)

Variable	Univariate model mape %	Multivariate model mape %	Remarks
Dollar	0.76	3.57	Univariate Model is Preferable
Pounds Sterling	2.05	3.04	Univariate Model is Preferable
Euro	3.20	3.06	Multivariate Model is Preferable
Yen	3.27	10.64	Univariate Model is Preferable
WAUA	1.22	5.41	Univariate Model is Preferable
CFA	3.12	5.79	Univariate Model is Preferable

4.2 Discussion of results

In line with the aim of this research study, a univariate and multivariate time series models for each of the six variables were fitted the results based on their mean absolute percentage error (MAPE) shows that univariate method is preferable for Dollar, Pounds Sterling, Yen, WAUA and CFA. On the other hand, only one of these variables Euro shows that the multivariate method is preferable. Furthermore, the justification of these results explains the reason why Euro contributes significantly to three other variables namely; Pounds, WAUA and CFA) which suggests a multivariate model fits better.

5 Conclusion

Based on the results obtained, the univariate time series gives a better model for Dollar, Pounds Sterling, Yen, West African Unit of Account and CFA. Similarly, the forecast plots compared to the actual values for these variables also illustrates how close these values are with the actual values. The Multivariate time series model is preferable for only Euro, forecast plots also illustrates how close these values are to the actual values. The mean absolute percentage error (MAPE) was used as a forecast accuracy measure in drawing the conclusion as to which of the two methods gives a much closer approximate values to the actual values.

Competing Interests

Authors have declared that no competing interests exist.

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