

The Challenge to MOND from Ultra-faint Dwarf Galaxies

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Abstract

Modified Newtonian dynamics (MOND) at low acceleration has been astonishingly powerful at explaining the rotation curve of galaxies and the relation between the baryonic content of the galaxies and their observed circular velocity, known as the baryonic Tully–Fisher relationship. It is known that MOND fails at explaining the observed velocity dispersion of the ultra-faint dwarf galaxies (UFDs) with the justification that UFDs are more prone to tidal disruption in MOND compared to cold dark matter model. We show that (i) the ratio of tidal to internal acceleration in UFDs is extremely low, (ii) there is no correlation between the deviation of UFDs from MOND's prediction as a function of tidal susceptibility, and (iii) recent constraints from Gaia proper motion analysis on the orbital parameters of the UFDs exacerbate the challenge to MOND. In particular, Gaia data indicate that Ursa Major I is experiencing a recent infall into the Milky Way's halo, and its inconsistency with MOND at the 7σ level cannot be attributed to being an early infall satellite. Moreover, the new data from Gaia DR2 show Willman I to have the least eccentric orbit of all UFDs, and its deviation from MOND at the 4σ level cannot be attributed to a highly eccentric orbit as previously suggested. Finally, given that Tuc III is the only UFD observed to show tidal features, Reticulum II and Segue I are two other UFDs that potentially challenge MOND as they have comparable Galactocentric distances to Tuc III while showing no tidal features.

Unified Astronomy Thesaurus concepts: Cold dark matter (265); Modified Newtonian dynamics (1069); Dwarf galaxies (416)

1. Introduction

Despite all the efforts from multiple theoretical and experimental directions, the nature of dark matter remains unknown (see Profumo et al. 2019 for a recent review). One important clue to its nature could come from the fact that it should manifest itself on Galactic scales as is successfully described by modified Newtonian dynamics (MOND; Milgrom 1983). MOND has specific predictions for systems with acceleration below $a_0 = 1.2 \times 10^{-8} \, \mathrm{cm \, s^{-2}} = 3.8 \, \mathrm{pc \, Myr^{-2}}$ such that the acceleration becomes $g = \sqrt{g_N a_0}$, where g_N is the Newton's acceleration. Intriguingly, the value of a_0 is related to the Hubble constant by $a_0 \approx cH_0/6$ where c is the speed of light.

Two successes of MOND on galactic scales are (i) explaining the observed steep relation with low scatter between baryonic mass in galaxies and their circular velocity known as the baryonic Tully–Fisher relationship (BTFR; Tully & Fisher 1977), and (ii) explaining the rotation curve of spiral galaxies (Rubin et al. 1980). While in the standard ΛCDM cosmology fine-tuning of baryonic physics is required to recover the BTFR (McGaugh et al. 2000; Abadi et al. 2003; Governato et al. 2007; Bouché et al. 2010; Vogelsberger et al. 2013; Chan et al. 2015; Sales et al. 2017), MOND can reproduce the observed relationship with no fine-tuning as it requires no dark matter in its formulation (McGaugh 2005). Likewise, the modified dynamics have been astonishingly successful at reproducing the observed rotation curve of the galaxies (Sanders & McGaugh 2002).

In the limit of isolated systems, MOND's prediction for the 1D velocity dispersion of a dispersion-supported spherical galaxy for $g_{\rm in} \ll a_0$ is given by

$$\sigma_{\rm iso} \approx \left(\frac{4}{81}a_0GM_*\right)^{1/4},\tag{1}$$

where M_{*} is the stellar mass of the galaxy, and the internal acceleration, $g_{\rm in}$, is given by

$$g_{\rm in} \approx \frac{3\sigma_{\rm iso}^2}{r_{1/2}},\tag{2}$$

where $r_{1/2}$ is the half-light radius. However, if the galaxy is a satellite of the Milky Way (MW) in the regime of $g_{\rm in} < g_{\rm ex} < a_0$, where $g_{\rm ex}$ is the gravitational acceleration of the MW, given by

$$g_{\rm ex} \approx \frac{V_{\rm MW}^2}{D_{\rm GC}},$$
 (3)

MOND's predicted dispersion due to the external field effect (EFE; McGaugh & Milgrom 2013a) is given by

$$\sigma_{\text{efe}} \approx \left(\frac{a_0 G M_*}{3 g_{\text{ex}} r_{1/2}}\right)^{1/2},\tag{4}$$

where $V_{\rm MW}$ is the circular velocity of the MW, for which we adopt a constant value of 200 km s⁻¹ for all the calculations in this work (Reid et al. 2014). $D_{\rm GC}$ is the Galactocentric distance of the satellite from the MW. We note that the EFE of M31 on MW's satellites is irrelevant since the internal accelerations of the satellites are stronger than M31's external acceleration.

By definition, satellites with luminosity less than $10^5 L_{\odot}$ are considered to be ultra-faint dwarf galaxies (UFDs). In the standard Λ CDM cosmology, UFDs are dark matter-dominated systems with quenched star formation history (Simon & Geha 2007; Simon 2019). New UFDs were recently discovered in the Dark Energy Survey (Bechtol et al. 2015; Koposov et al. 2015a). As relics from the early universe, UFDs are testbeds for examining various theories from the standard collisionless dark matter (Nadler et al. 2021), to fuzzy dark matter (Burkert 2020;

Safarzadeh & Spergel 2020; Hayashi et al. 2021) and MOND (McGaugh & Wolf 2010).

A notable example of the success of MOND for the velocity dispersion of the Milky Way's satellites is MOND's prediction for the ultra-diffuse galaxy Crater 2 (McGaugh 2016), which has been proven by observation (Caldwell et al. 2017). This is an important prediction for MOND since Crater 2 is free of current tidal interference (though it may have suffered in the past). However, it has been shown that MOND's prediction for the velocity dispersion of the UFDs does not match the observed values, but the discrepancy has been attributed to tidal susceptibility (McGaugh & Wolf 2010). In this work, we revisit this claim in light of the recently discovered UFDs. We argue that tidal susceptibility does not seem to justify the deviation of MOND's predictions from the observed velocity dispersion of UFDs.

The structure of this Letter is as follows: In Section 2 we present our method of approximating tidal susceptibility. We investigate which UFDs are within the regime of MOND's EFE. We point out a few UFDs that challenge MOND and present the caveats around those systems. In Section 3, we summarize our results and discuss the most promising avenues toward understanding the nature of dark matter.

2. Data

The data on the velocity dispersion and half-light radius of the UFDs are primarily collected from the compilation of Wolf et al. (2010). We use the estimated Galactocentric distances of the UFDs from the recent proper motion analysis of the Gaia DR2 (Fritz et al. 2018; Simon 2018). The stellar mass of the UFDs is from the compilation of McConnachie (2012). For the UFDs that are not presented in Wolf et al. (2010) we have collected the data from the following references: Grus II and Tucana IV from Simon et al. (2020); Eridanus II from Li et al. (2017) and its stellar mass from Gallart et al. (2021) and Bechtol et al. (2015). We obtain the data for Horologium I (Hor I) from Koposov et al. (2015b) and its stellar mass from Bechtol et al. (2015), which are consistent with the data reported in Jerjen et al. (2018). Hor I is close to the Magellanic Clouds, and whether this is a satellite of the Large Magellanic Cloud (LMC) is discussed in Nagasawa et al. (2018), and we elaborate on in the next section. The data for Triangulum II are obtained from Kirby et al. (2017), and for Segue 2 from Kirby et al. (2013a). The data for Reticulum II is obtained from Simon et al. (2015). For Bootes I we adopt a velocity dispersion of 2.4 km s⁻¹, which comes from the majority of cold components of the stellar population (Koposov et al. 2011). As Leo T is a gas-rich satellite with a total H I mass of $4 \times 10^5 M_{\odot}$ (Adams & Oosterloo 2018) we consider a total baryonic mass of $\log(M_b/M_\odot) = 5.75$ for this system. In cases where there is no estimate of the stellar mass of the system, we use the relationship between the metallicity of the system and its stellar mass derived from dwarf satellites of the MW (Kirby et al. 2013b).

3. Method

To begin with, we display the predicted velocity dispersion of MOND for the UFDs as compared to the observed line-of-sight estimates in Figure 1. We have divided the satellites into two groups, one in which the $g_{\rm in} < g_{\rm ex}$, as shown in the red points, and those where internal acceleration is larger than $g_{\rm ex}$,

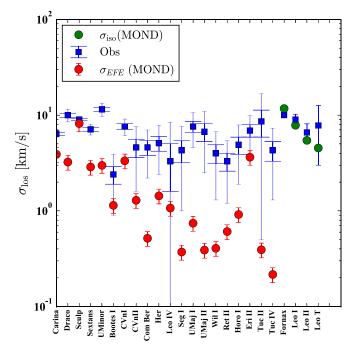


Figure 1. The predicted velocity dispersion of the UFDs and dwarf spheroidal galaxies from MOND are shown in red circles. The observed line-of-sight values (blue squares) of velocity dispersion of the UFDs and dwarf spheroidal galaxies are shown as blue squares with the longer cap error bars showing the 1σ (1 std) error, and the longer and thinner caps showing the 3σ error. We have divided the sample into two groups: those with $g_{\rm in}/g_{\rm ex}<1$ shown with red points, and those with $g_{\rm in}>g_{\rm ext}$ shown with green points. For the former category, we use the $\sigma_{\rm efe}$ of MOND, and for the latter systems, we use the isolated estimated prediction, $\sigma_{\rm iso}$, of MOND. The error bars for MOND predictions come from assuming the $V_{\rm MW}$ is a range between 170 and 230 km s⁻¹.

as shown in the green points. The squares show the observational estimates, the longer cap error bars show the 1σ (1 standard deviation) error, and the longer and thinner caps show the 3σ error bars.

To investigate whether tidal susceptibility can explain the deviation of MOND's prediction and the observed velocity dispersion of the UFDs, we calculate the ratio of the tidal acceleration and the internal acceleration of a satellite. The tidal accelerations are estimated by

$$g_t = g_{\rm ex} \frac{2r_{1/2}}{D_{\rm GC}}. (5)$$

The yellow hexagons in Figure 2 show the satellites where $0.15 \leqslant g_{\rm in}/g_{\rm ex} \leqslant 1$, and the red points show the system where $g_{\rm in}/g_{\rm ex} \leqslant 0.15$. The reason for this separation is that the EFE prediction is only valid when $g_{\rm in} \ll g_{\rm ex}$, and its validity when $g_{\rm in} \approx g_{\rm ex}$ is unknown (McGaugh & Milgrom 2013a). Therefore, we show the results once for all the sample with $g_{\rm in}/g_{\rm ex} < 1$, and another time for when $g_{\rm in}/g_{\rm ex} < 0.15$, a regime in which EFE is shown to give consistent results with the observations. The value of 0.15 is chosen based on the McGaugh (2016) EFE model's success in predicting the correct velocity dispersion for Crater II, which has a ratio of $g_{\rm in}/g_{\rm ex} \approx 0.12$.

The red points in Figure 2 imply that if tidal susceptibility is the reason for the discrepancy between the predictions of MOND and the observed σ_{los} of the UFDs, we have to see a positive correlation between the ratio of the tidal over internal acceleration and the deviation of MOND from the observed velocity dispersion. Does such a correlation exist? With low

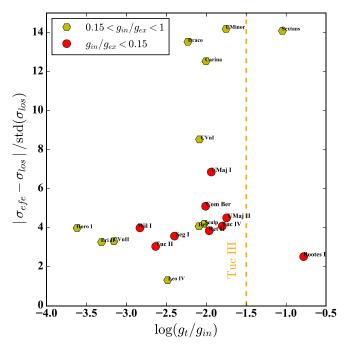


Figure 2. Deviation of the predicted MOND velocity dispersion of the UFDs and their observed value as a function of the $\log(g_r/g_{\rm in})$ for systems in which $g_{\rm in} < g_{\rm ex} < a_0$. Yellow hexagons: for systems with $g_{\rm in} < g_{\rm ex} < 1$. Red points: for systems with $g_{\rm in}/g_{\rm ex} < 0.15$. Tuc III is shown with a vertical line since, at the moment, only an upper limit exists for its velocity dispersion, $\sigma_{\rm los} < 1.5~{\rm km~s^{-1}}$ at 95% confidence. Therefore, the true ratio of tidal over internal acceleration of Tuc III could be any value smaller than $10^{-1.5}$.

number statistics, one can either argue for or against a positive correlation. However, the fact that Willman I still deviates from MOND's prediction at 4σ remains a challenge to MOND. Moreover, Hor I with $g_{\rm in}/g_{\rm ex}\approx 0.3$ stands at 4σ from the EFE prediction while having the lowest $g_t/g_{\rm in}$ in our sample.

But the most stringent constraint comes from Ret II due to its proximity to the Sun for the following reason: Tucana III (Tuc III) is the first UFD observed to show a velocity gradient of $8.0 \pm 0.4 \,\mathrm{km \, s^{-1} \, deg^{-1}}$, suggesting the galaxy has ongoing tidal disruption (Li et al. 2018). Simon et al. (2017) estimated the $\sigma_{\rm los}$ to be less than 1.5 km s⁻¹ at 95% C.I. With a half-light radius of about 34 pc and a distance of about 25 kpc, Tuc III is our nearest neighbor UFD with $g_{\rm in}/g_{\rm ex} \approx 0.04$. Given the old stellar population of the UFDs, one can adopt a value for the stellar mass-to-light ratio of $M_*/L_V \approx 2M_{\odot}/L_{\odot}$ assuming a standard initial mass function (Simon 2019). This leads to an estimate of the stellar mass of Tuc III to be $M_* \approx 1500 \ M_{\odot}$. The same estimate yields a stellar mass of about 5000 M_{\odot} for Ret II. With $\sigma_{los} = 1.5 \text{ km s}^{-1}$ for Tuc III (which is the estimated lower limit for this system), the ratio of the tidal and internal acceleration for Tuc III is $\log(g_t/g_{in}) = -1.5$. We show this with a vertical line in the bottom panel of Figure 2. This value is comparable to the ratio of tidal over internal acceleration for Ret II.

Ret II and Tuc III have similar Galactocentric distances, implying that if we see tidal features in Tuc III, we would also expect to see tidal features in Ret II as well, but no such features have been observed. However, one can argue that the orbital motion of the two satellites is different: based on Gaia DR2, Simon (2018) estimates a pericenter and apocenter distance for Tuc III to be 3^{+1}_{-1} kpc and 49^{+6}_{-5} kpc with an orbital period of 0.7 Gyr, and the corresponding values for Ret II are derived from being 29^{+14}_{-6} kpc for pericenter approach and

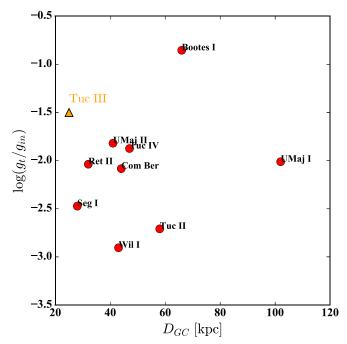


Figure 3. The distribution of MW satellites with $g_{\rm in}/g_{\rm ex} < 0.15$ in Galactocentric distance vs. the ratio of tidal to internal acceleration. Tuc III is shown with a triangle since we only know the lower limit on $g_{\rm r}/g_{\rm in}$ for Tuc III. The fact that tidal features are observed for Tuc III but not for Ret II or Seg I is concerning. However, this could be justified given the highly eccentric orbit of Tuc III with a pericenter approach of 3^{+1}_{-1} kpc.

 91^{+91}_{-39} kpc for apocenter approach with an orbital period of 1.6 Gyr. These numbers agree with the results reported in Fritz et al. (2018). The inferred orbital motion based on Gaia DR2 suggests that despite their current similar Galactocentric distance, Tuc III has been more prone to tidal disruption as opposed to Ret II. However, these orbital motion estimates are based on cold dark matter potential for the MW, and whether the same orbital dynamics would be inferred in MOND remains to be verified with detailed orbital motion simulation. Moreover, we note that although Ret II is an elongated UFD with an ellipticity of about $\epsilon = 0.6$, there have been no signs of tidal features within our field of view (Mutlu-Pakdil et al. 2018).

Figure 3 shows the distribution of the satellites in the Galactocentric distance and $\log(g_t/g_i)$ plane. Other than Ret II, Seg I also has a comparable Galactocentric distance to Tuc III. The pericenter and apocenter approaches for Segue I (Seg I) are estimated to be 20^{+4}_{-5} and 61^{+34}_{-18} kpc with an orbital period of about 1.1 Gyr. This makes Seg I more similar to Tuc III than Ret II. However, Simon et al. (2011) do not find evidence suggesting Seg I is being tidally disrupted.

Willman I stands 4σ away from MOND's prediction. This satellite has pericenter and apocenter approaches of 44^{+15}_{-19} and 53^{+57}_{-13} kpc, respectively, which yield this UFD at the lowest orbital eccentricity of $e \approx 0.2$. While the deviation of Willman I was identified in McGaugh & Wolf (2010), this was attributed to speculating a highly eccentric orbit for Willman I while the new data from Gaia DR2 actually suggest Willman I to be actually on the least eccentric orbit of all the UFDs (Simon 2018).

Similar to Willman I, Hor I also stands 4σ away from MOND's prediction; however, with the ratio of $g_{\rm in}/g_{\rm ex}\approx 0.3$, it is not clear whether the isolated or the EFE regime is adequate for predicting its internal dynamics. Furthermore, the close

proximity of Hor I to the LMC (Nagasawa et al. 2018) might further complicate the expected dynamics of this satellite, although $g_{\rm in}/g_{\rm ex}$ with respect to the Magellanic Clouds remains less than 1 for this satellite.

Perhaps the most deviant of all the satellites is U Maj I standing about 7σ from MOND's prediction. This deviation was attributed to the early infall time of this satellite in McGaugh & Wolf (2010). While early analysis of the infall times of the satellites categorized this satellite as one indeed with an early infall time of about 8 Gyr (Rocha et al. 2012), recent Gaia DR2 suggests that this satellite is actually entering the MW's halo recently in the past $1.5^{+5.1}_{-1.6}$ Gyr (Fillingham et al. 2019).

So far, all calculations have been done based on the current distance of the satellites from the Galactic center. However, we know that the satellites have experienced a pericenter approach. and if their orbit is highly eccentric, tidal acceleration at pericenter passage is important. This is shown in Figure 4. We obtain the estimated pericenter approach from Fritz et al. (2018) assuming two different halo masses for the MW's halo. The obtained estimates from the low virial mass calculation of Fritz et al. (2018) agree with the results in Simon (2018). We note that the agreement is not perfect, and some deviations in the estimated orbital parameters are present. If the existence of a correlation between MOND's deviation and tidal effects is the desired feature to look for, then the data show that such a correlation disappears if we focus on satellites with low ratios of tidal to internal acceleration even at pericenter passage. We note that the data point for U Maj I indicates its estimated pericenter approach while the satellite has not reached its pericenter distance. Tucana II and Seg I deviate at the 3σ and 4σ levels from MOND's prediction despite having a low tidal to internal acceleration at pericenter approach. Coma Berenices stands at 5σ deviation from MOND's prediction while having a rather large pericenter approach of 43 kpc. However, since this satellite is an early infall system, its deviation could be justified by being in the tidal field of its host for about a Hubble time. Moreover, despite its high ratio of tidal to internal acceleration, Hercules does not deviate from MOND's predictions as much as is expected from Draco, U Min, and Sextans.

4. Discussion

The latest Gaia DR2 data show a lack of strong correlation between tidal susceptibility and deviation from MOND's predictions and observed line-of-sight velocity dispersion of the UFDs, as presented in Figure 2. While the data challenge MOND, more data are needed. In this work, we solely focused on tidal features as a proxy for deviation from MOND. We did not investigate correlations with other estimators such as ellipticity of the UFD as projected on the sky, or adiabaticity (which is a measure of the number of internal orbits of stars over the number of orbits of the satellite around its host). Ellipticity, adiabaticity, and other estimators have been used in McGaugh & Wolf (2010) and are shown to be a proxy to tidal features and we do not investigate these in this work.

Could we be biased in inferring the intrinsic velocity dispersion of the UFDs? We know that binary stars can inflate the estimated 1D velocity dispersion specifically for systems in which the total number of the stars is ≤ 10 . For example, a binary star did inflate the velocity dispersion for Triangulum II (Kirby et al. 2017). However, for well-studied systems such as Seg I, binaries contribute to the velocity dispersion at the 10%

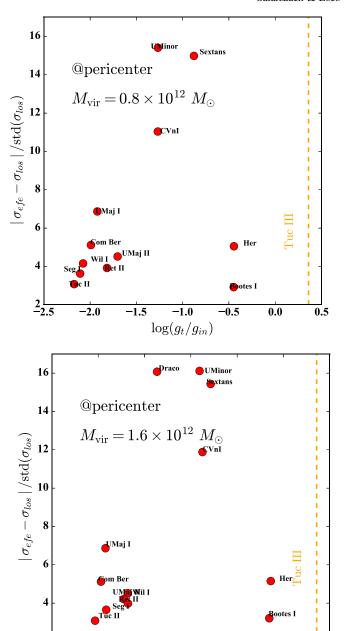


Figure 4. Same as in Figure 2, but calculated at the distance of pericenter approach. Top (bottom) panel shows the derived pericenter distance assuming a MW's potential with a virial mass of $0.8 \times 10^{12}~(1.6 \times 10^{12})~M_{\odot}$. The correlation between deviations from MOND's prediction disappears in the limit of low tidal to internal acceleration ratio, which can be a challenging result for MOND to account for. Hercules stands out as a satellite with rather high tidal to internal acceleration ratio at its pericenter passage but not deviating as much as Draco, U Mi, and Sextans would suggest.

-1.0

 $\log(g_t/g_{in})$

-0.5

0.0

0.5

-1.5

-2.0

level (Simon et al. 2011). In the meantime, a detailed analysis of Draco and U Min shows that the contribution of binaries to the observed velocity dispersion can be significant (Spencer et al. 2018). While single-epoch observations of stars are vulnerable to binaries, future multi-epoch campaigns will say the final word on the issue of binaries.

It is also worth mentioning the remarkable prediction of MOND for M31 dwarf satellites (McGaugh & Milgrom 2013a, 2013b). Moreover, the original prediction of

2<u>∟</u> -2.5 MOND for Cetus and Tucana of 8.2 and 5.3 ± 0.4 km s⁻¹, respectively, was not in agreement with the observations at the time (estimated values to be about 17.2 ± 2 and $15.8^{+4.1}_{-3.1}$ km s⁻¹ for Cetus and Tucana, respectively). However, recent observations point to a much better agreement with MOND's prediction with Tucana having a velocity dispersion of $6.2^{+1.6}_{-1.3}$ km s⁻¹ (Taibi et al. 2020). This can be taken as MOND will survive with time as better data are gathered for the UFDs.

The issue pertaining to MOND's predictions is not limited to the line-of-sight velocity dispersion. For example, it is known that dynamical friction is stronger in MOND (shorter timescale for a satellite) compared to dark matter models (Nipoti et al. 2008), which makes Fornax and its globular cluster a serious problem for MOND. However, Fornax is a problem for the standard model of Λ CDM too, and a core-like structure for its dark matter halo can allow for the dynamics of globular clusters observed in Fornax. We note that other circular orbits outside the tidal radius of Fornax can justify the dynamics of globular clusters in Fornax, even in MOND (Angus & Diaferio 2009).

What do the seemingly simple modification to Newtonian gravity and its success at the galactic scale teach us about the nature of dark matter? Why does MOND have shortcomings at larger scales, such as in clusters of galaxies? And based on the results we analyzed in this work, apparently shortcoming in the scale of UFDs? Is this shortcoming in satellite scale an issue with EFE modeling? Attempts to successfully merge these apparently different behaviors of dark matter under one umbrella have been limited (see for example the superfluid interpretation of the dark matter, Berezhiani & Khoury 2015; Khoury 2015; and the emergent gravity paradigm, Verlinde 2017), but perhaps such avenues hold the key to understanding the nature of dark matter.

We are hopeful that future data obtained by the Vera C. Rubin Observatory, which will find all satellites of the MW within its virial radius down to V-band absolute magnitude fainter than $M_V = -4$, discover enough UFDs to have a statistically significant sample for testing MOND (Tollerud et al. 2008).

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