New Equations for Energy Dissipation for Spillways with Slope of 30 Degrees and Height of 0.305m

Okechukwu Ozueigbo [a*] and J. C. Agunwamba [a]

[a] *Department of Civil Engineering, University of Nigeria, Nsukka, University Road, Nsukka, Nigeria.*

Authors' contributions

This work was carried out in collaboration between both authors. Author OO designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author JCA managed the analyses of the study. Both authors read and approved the final manuscript.

Original Research Article

ABSTRACT

Stepped spillways are current method of choice for safe discharge of flood water due to their inherent ability to employ their stepping nature to safely dissipate substantial energy. Hence, the authors in this study aim to provide to design engineers new models for estimating energy dissipation in a stepped spillway with an equal step heights and widths of 0.3 m. Many researchers have investigated both the hydraulic and the geometric relationships of stepped spillways of varying sizes that resulted in significant energy dissipation, but quite a few of them has conducted researches on stepped spillways with equal heights and widths of 0.3 m, thereby leading to limited information at the disposal of design engineers involved in the design of stepped spillway with equal heights and widths of 0.3 m. The authors obtained more than 500 measured data from the Engineering Research Center Colorado State University Fort Collins, Colorado work published in April, 2002 and about 300 of them that had complete data were re-analyzed and used to develop energy dissipation model that govern nappe and skimming flows over a wide range of operating conditions. This energy dissipation model was formulated in terms of the number of the steps of the stepped spillway and the flow critical depth using multiple regression analysis and matrix method. This developed model was later calibrated and yielded very high coefficients of correlation that ranged from 0.8808 to 0.9982, which upon verification gave good predictions between the measured and estimated data. Results showed that the rate of energy dissipation along a stepped spillway increases with increasing numbers of steps, but decreases with increasing rate of discharge.

_____

*Corresponding author: Email: okechukwuozueigbo@gmail.com, Ozueigbo.okechukwu.pg78995@unn.edu.ng;

1. INTRODUCTION

Stepped spillways are current method of choice for safe discharge of flood water due to their inherent ability to employ their stepping nature to safely dissipate substantial energy. Many researchers have investigated both the hydraulic and the geometric relationships of stepped spillways of varying sizes that resulted in significant energy dissipation. These researchers provided design guidelines for stepped spillway channels with various channel geometries and slopes [1-6]. Most recent research focused on the air-water flow properties and energy dissipation performances, including some more detailed air-water flow properties such as bubble count rate, turbulence, and microscopic air-water properties (Chanson & Toombes, 2002; Gonzalez, 2005; Chanson & Carosi, 2007; Toombes and Chanson, 2008a, b; Felder & Chanson, 2009b, 2011a); Meireles et al., 2009). However, despite the enormous amount of information and guidelines on stepped spillways provided by these scholars to design engineers, huge information gap and guidelines still existed in the area of stepped spillways with equal treads and heights of 0.305 m [7-17]. It is, therefore, the purpose of this present study to obtain near-prototype scale data on the hydraulic characteristics of stepped spillway flow including air concentration, bulked flow depth, clear water depth, and flow velocity from the Engineering Research Center Colorado State University Fort Collins, Colorado work published in April, 2002 [18-23]. These collected data were analyzed to quantify energy dissipation and to develop design guidelines and models that would aid design engineers involved in the design of stepped spillways with equal heights and widths of 0.3 m.
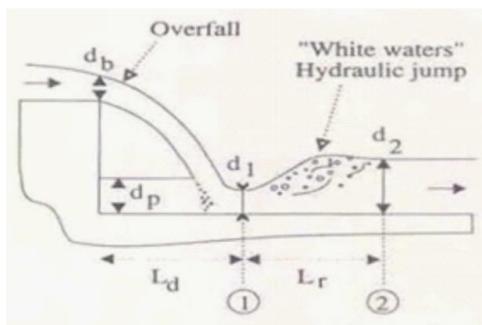
Fig. 1. Nappe flow regime (Flow at a drop structure)

These Researchers were able to identify three kinds of flows that take place over a stepped spillway such as a) nappe flow regime, b) transional flow regime, and c) skimming flow regime.

In nappe flow regime, sequence of drops from one step to the next step below it with the formation of hydraulic jump at every drop is observed. This type of flow can be likened to a sequence of separates drop structures [24, 25].

The water flows over one step of the spillway and lands on the next step with energy loss happening from a) the disintegration of the spout in the air, and b) the blending of flow on the steps, with or without the development of hydraulic jump on the step [26, 27] and these energy losses could be computed using equations 2.2a and 2.2b.

$$\frac{\Delta E}{E_o} = 1 - \frac{\frac{d_1}{y_c} + \frac{1}{2}\left(\frac{y_c}{d_1}\right)^2}{H_{max} + \frac{3}{2}} \quad (ungated\ spillway) \quad [1.1a]$$

$$\frac{\Delta E}{E_o} = 1 - \frac{\frac{d_1}{y_c} + \frac{1}{2}\left(\frac{y_c}{d_1}\right)^2}{\frac{(H_{max} + H_o)}{d_c}} \quad (gated\ spillways) \quad [1.1b]$$

And Chanson [26] later expressed these equations – [1a] and [1b] – in terms the spillway step height, the critical flow depth, and the dam height as:

$$\frac{\Delta E}{E_o} = 1 - \left[\frac{0.54\left(\frac{y_c}{h}\right)^{0.275} + \frac{3.43}{2}\left(\frac{y_c}{h}\right)^{-0.55}}{\frac{3}{2} + \frac{H_{dam}}{y_c}}\right] \quad ungated\ spillway \quad [1.2a]$$

$$\frac{\Delta E}{E_o} = 1 - \left[\frac{0.54\left(\frac{y_c}{h}\right)^{0.275} + \frac{3.43}{2}\left(\frac{y_c}{h}\right)^{-0.55}}{\frac{H_{dam} + H_o}{y_c}}\right] \quad gated\ spillway \quad [1.2b]$$

Where ΔE is the energy loss, E_o is the maximum available energy, y_c is critical flow depth, h is the height of the spillway step, H_{dam} is the dam height.

Nappe flow with completely established hydraulic jump (Fig. 1), usually arises from small discharges with shallow flow depths and flow over the step with formation of supercritical at the edge of the step and returns to subcritical flow downstream of the jump.

In skimming flow regime, the flow occurs with the submergence of the steps with the development of fully aerated uniform flow in the downstream

region in a long chute [28-34]. Along the upstream steps, a non-aerated flow region exists within which a turbulent boundary layer develops. Air entrainment in the flow begins where the boundary layer intersects the free surface, referred to as the point of inception [35-40].

Downstream from the point of inception, the flow continues to aerate and varies gradually in depth (Fig. 2). The flow eventually becomes fully aerated, uniform flow in which the water depth, velocity, and air concentration become constant [41] (Fig. 3).
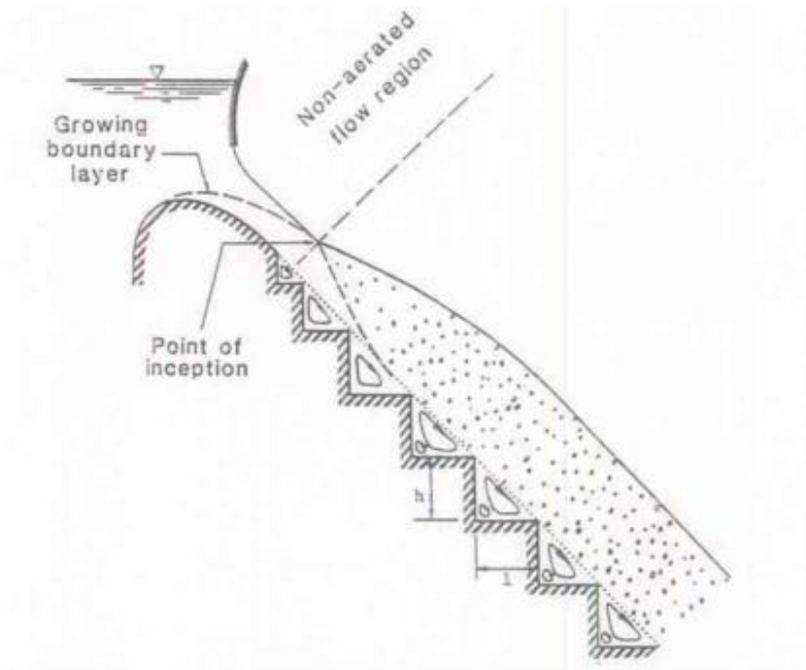
Fig. 2. Skimming flow regime - Sorensen [42]
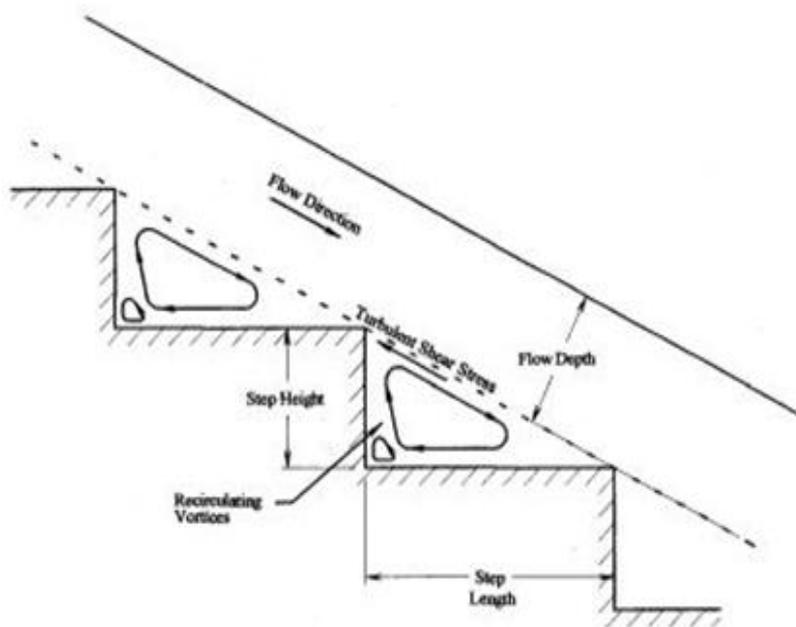
Fig. 3. Skimming flow regime with uniform flow conditions
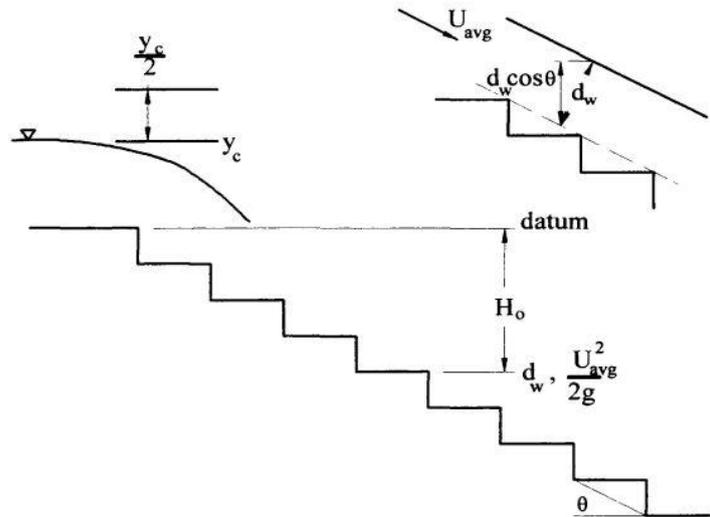
Fig. 4. Arrangement of the spillway with the definition of the variables

In skimming flow, most of the energy is dissipated in the maintenance of stable depression vortices. If uniform flow conditions are reached at the downstream end of the spillway, this energy loss could be computed as follows

$$\frac{\Delta E}{E_o} = \frac{d_w \cos\theta + \frac{U_{avg}^2}{2g}}{Nh + \frac{3}{2}y_c} \qquad [1.3\}$$

Where

ΔE is energy loss,
E_o is the maximum available energy,
d_w is the uniform aerated depth flow,
θ is the chute slope
U_{avg} is the average flow velocity

the total head loss may be rewritten in terms of the friction factor, the spillway slope, the critical depth and the dam height:

$$\frac{\Delta E}{E_o} = 1 - \frac{\left(\frac{f}{8\sin\theta}\right)^{1/3}\cos\theta + \frac{E}{2}\left(\frac{f}{8\sin\theta}\right)^{-2/3}}{\frac{H_{dam}}{y_c} + \frac{3}{2}} \qquad [1.4]$$

Eq [1.4] was computed for spillway slope with θ = 52 (degrees) and friction factor, f = 0.03 and f = 1.30, that represent average flow resistance on smooth spillways and stepped spillways. where E is the kinetic energy correction coefficient, θ is the dam slope in degrees,

2. MATERIALS AND METHODS

The authors obtained and re-analyzed the experimental data from the published works of the Final report of the Research Project 99FC800156 HYDRAULIC DESIGN OF STEPPED SPILLWAYS Prepared for: U.S. Bureau of Reclamation Denver, Colorado Prepared by: James F. Ruff and Jason P. Ward Engineering Research Center Colorado State University Fort Collins, Colorado published in April, 2002 and used them to develop an energy dissipation models that governed both nappe and skimming flows over a wide range of operating conditions. These improved models were verified with the measured data fro m these researchers and compared with the results from the existing models. These new models were later verified using part of the experimental data that were not used in the calibration. The extent of fit between measured to predicted ratio of energy dissipation to maximum elevation found by estimation of the Spearman – Pearson coefficient of correlation were compared with the existing model.

2.1 Formulation of the Model

In modeling, it is necessary to determine the values of the parameters that can fit the model to the system it shall describe (Agunwamba, 2007). By least square method, the best fit curve for this study was formulated as a function of the ratio of number of steps to the critical water depth as:

$$\frac{\Delta E}{E_o} = \left[\alpha_o \frac{Nh}{y_c} \right]^{\alpha_1} N^{\alpha_2} \qquad [2.1]$$

These improved energy dissipation models were formulated as a function of the ratio of number of steps to the critical water depth using multiple regression analysis.

where
ΔE is head loss,
E_o is maximum energy,
N = number of steps,
h = height of step (m),
y_c = critical water depth (m),
α_o, = constant,
α_1 and α_2 = coefficients.

Using this equation and the measured data, the authors used multiple regression analysis and matrix method to solve the resultant equations, which yielded the constant α_o along with the coefficients α_1, and α_2 for the developed models.

Prediction of new models verified with the experimental data is equated with the outcomes from the current models [43-47].

3. RESULTS AND DISCUSSION

The analyses yielded the values of constant α_o, the coefficients α_1, and α_2 for nappe, transition, skimming flow regimes, respectively, which were then substituted in Equation [2.1] to give the developed models in 3.1.

3.1 Developed Models for Nappe, Transition, and Skimming Flow Regimes

3.1.1 Developed model for nappe flow regime ($y_c/h \leq 0.46$ and h = 0.305 m)

$$\frac{\Delta E}{E_o} = \left[0.486 \frac{Nh}{y_c} \right]^{13.21} N^{-13.32} \qquad [3.1]$$

3.1.2 Developed model for transition flow regime ($0.46 < y_c/h \leq 0.95$ and h = 0.305 m)

$$\frac{\Delta E}{E_o} = \left[1.10 \frac{Nh}{y_c} \right]^{79.92} N^{-82.39} \qquad [3.2]$$

3.1.3 Developed model for skimming flow regime ($0.95 < y_c/h \leq 2.69$ and h = 0.305 m)

$$\frac{\Delta E}{E_o} = \left[\frac{Nh}{y_c} 1.23 e^{-10} \right]^{0.11} N^{0.55} \qquad [3.3]$$

3.2 Chart for Nappe Flow Regime for $y_c/h \leq 0.46$

Figure 5 depicts the energy losses rate and an expression of a dam height divided by the critical depth, and plotted with the measured data from Ruff and Ward [48], the developed analytical formulation (Eq [3.1]) as well as the existing model for the computation of energy dissipation (Eq [1.2a]). The figure shows some traditional concave shape distributions for all the three plotted data for energy dissipation for all the flow rates (Chanson, 2001). As indicated in the chart, the energy losses increase with decreasing discharges and again increase with increasing dam height for a particular discharge which is in accordance with the earlier investigations [49], (Chanson, 2001b; Felder & Chanson, 2009a). The data sets from the field work and developed model (Eq [3.1]) are in very close agreement with the coefficient of correlation computed as 0.99. However, the case is entirely different when the field data are compared with the data sets using the existing model (Eq [1.2a]): the existing model predicts unrealistically high energy dissipation values compared to the measured data for Nh/y_c within the values of 13 and 48. For instance, the figure predicts energy dissipation values of 79% and 94% as against the actual energy dissipation values of 22% and 81% when the values of Nh/y_c are 13 and 47 respectively. It is, therefore, not advisable to use the existing equation (Eq [1.2a]) for values of Nh/y_c between 13 and 47 when h is equal to 0.305 m.

3.3 Chart for Transition Flow Regime for $0.46 < y_c/h \leq 0.95$

Fig. 6 depicts the energy loss rate and an expression of a dam height divided by the critical depth, and plotted with the measured data from Ruff and Ward [48], the developed analytical formulation (Eq [3.2]) as well as the existing model for the computation of energy dissipation (Eq [1.2a]). Again, the figure shows same traditional concave shape distributions for all the three plotted data for energy dissipation for all the flow rates (Chanson, 2001). As shown in the chart, the energy losses increase with decreasing discharges and increase with increasing dam height for a particular discharge which is in accordance with the earlier investigations [49], (Chanson, 2001b; Felder & Chanson, 2009a). The data sets from the field work and the developed model (Eq [3.2]) are in very close agreement with the coefficient of
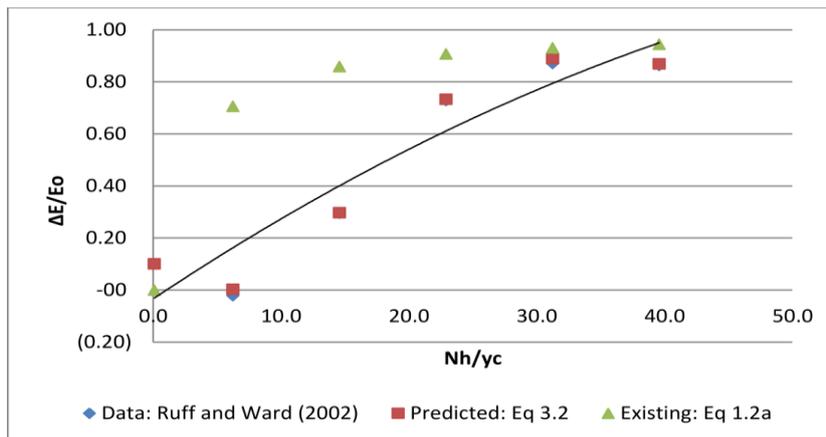
Fig. 5. Variation of relative head loss $\Delta E/E$ with Nh/y_c for y_c/h $\leq$ 0.46
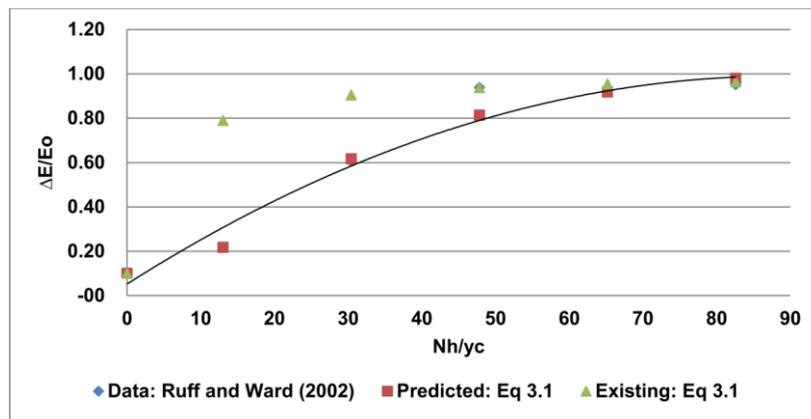
Fig. 6. Variation of relative head loss $\Delta E/E$ with Nh/y_c for 0.46 $<$ y_c/h $\leq$ 0.95

correlation computed as 0.98. However, the case is again entirely different when the field data are compared with the predicted data using the existing model (Eq [1.2a]): the figure shows again that the existing model predicts unrealistically high energy dissipation values compared to the measured data for Nh/y_c within the values of 6 and 23. For instance, the existing equation predicts energy dissipation values of 71% and 91% as against the actual energy dissipation values of 20% and 73% for the values of Nh/yc between 6 and 23 respectively. It is, therefore, not advisable to use the existing equation (Eq [1.2a]) for values of Nh/yc of between 6 and 23 when h is equal to 0.305 m.

3.4 Charts for Skimming Flow Regime for 0.95 $<$ y_c/h $\leq$ 1.48, 1.48 $<$ y_c/h $\leq$ 1.91, 1.91 $<$ y_c/h $\leq$ 2.32, and 2.32 $<$ y_c/h $\leq$ 2.69

Figs. 7, 8, 9 and 10 display patterns that are closely related and would, therefore, be discussed as a group here. These figures depict the energy loss rate and an expression of a dam height divided by the critical depth and plotted with the measured data from Carosi & Chanson (2006), the developed analytical formulation (Eq [3.3]) as well as the existing model for the computation of energy dissipation (Eq [1.3]). The figures show again the traditional concave shape distributions for all the three plotted data for energy dissipation for all the flow rates (Chanson, 2001). As shown in the charts, energy losses increase with decreasing discharges and increase with increasing dam height for a particular discharge which is in accordance with the earlier investigations [49], Chanson, 2001b; Felder & Chanson, 2009a). These data sets from the field work, the developed model (Eq [3.3]), and the existing model (Eq [1.3]) for computing energy dissipation are all in a very close agreement with the coefficients of correlation of 0.99 computed for the measured data and the developed model, and 0.98 computed for the measured data and the existing model for energy dissipation. These figures have, therefore, confirmed the appropriateness for use of the
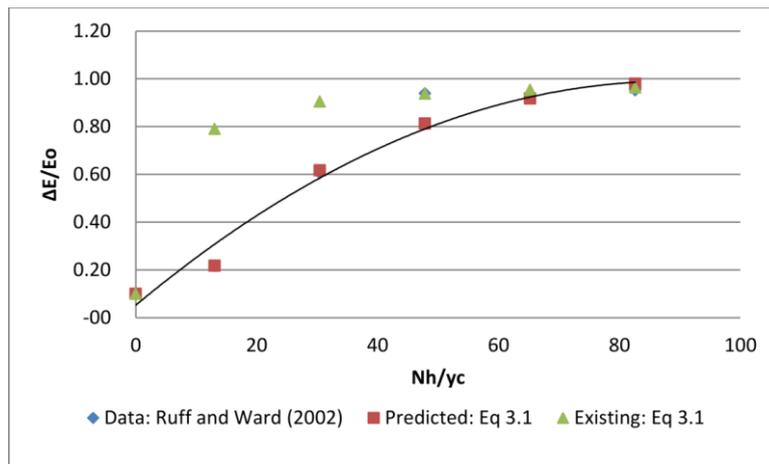
Fig. 7. Variation of relative head loss $\Delta E/E$ with Nh/y$_c$ for $0.95 < y_c/h \leq 1.48$
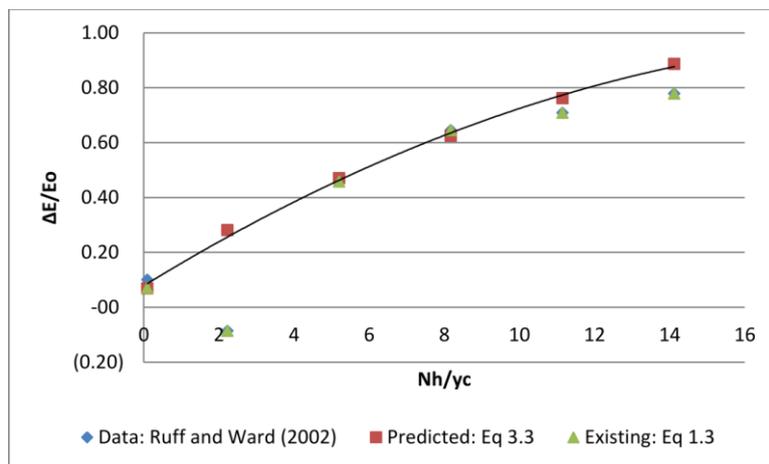
Fig. 8. Variation of relative head loss $\Delta E/E$ with Nh/y$_c$ for $1.48 < y_c/h \leq 1.91$
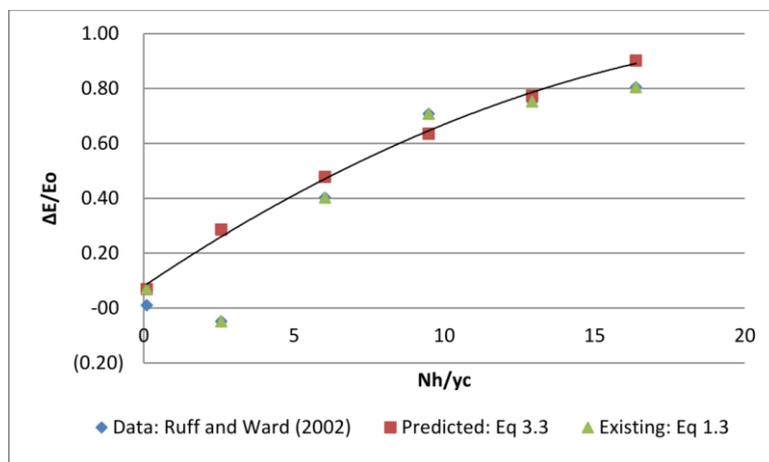
Fig. 9. Variation of relative head loss $\Delta E/E$ with Nh/y$_c$ for $1.91 < y_c/h \leq 2.32$

existing model (Eq [1.3]) when the spillway step height, h, is 0.305 m. The developed model (Eq [3.3]) is, however, simple, easy to use, and produces better accuracy than the existing model (Eq [1.3]).

16
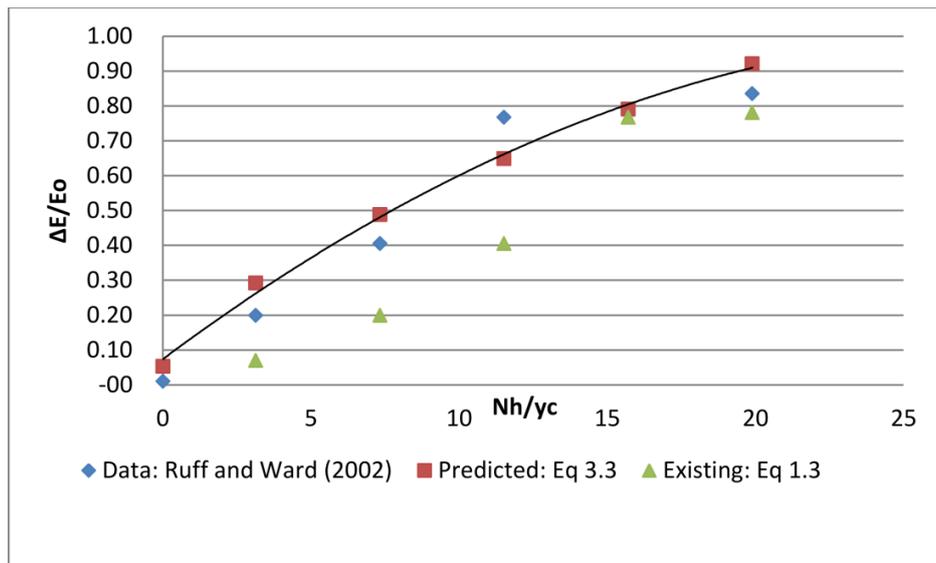
Fig. 10. Variation of relative head loss $\Delta E/E$ with Nh/y_c for 2.32 $<$ y_c/h $\leq$ 2.69

4. CONCLUSION

This present study presents thorough analytical investigations on a stepped spillway with a slope of 30 degrees and step height of 0.30 m to expand the knowledge of the features of the nappe flow, transition flow, and skimming flow regimes on a stepped waterfall, describing the most in depth representation of nappe flows, transition, and skimming flow, to date. The experiments are conducted for a wide range of nappe flow discharges emphasizing that nappe flows is typically related with supercritical flows without hydraulic jumps, while only for very small flow rates (dc/h ≤ 0.46) nappe flows with hydraulic jumps are found. The energy dissipation rate is recorded at the steps edges showing strong dissipation potential of stepped spillways in nappe flows. At the several steps, the measured data sets highlight an increase in aeration efficiency with increasing dam height. The measured data sets are overall in agreement with the data set from the developed model (Eq [3.1]) with h = 0.30 m. However, more study is advocated to document the growth of the air-water flow types along the stepped chute in more aspect, and to evaluate any effects of channel width on the nappe flow. The transition flow regime is a flow pattern happening on stepped chutes at intermediary discharges and is accompanied by robust free-surface aeration, some disordered movement and strong globule discharge. Two-phase flow properties are measured with an intrusive phase-detection probe and propose the occurrence of two transition flow sub-regimes. A simple model (Eq [3.2] is suggested to define the flow bulking in the rapidly-varied flow region and shows good agreement with the measured data set, but further work is needed to comprehend the difference between model and prototype data. Self-aeration occurs when the turbulent boundary layer outer edge intermingles with the free surface. The rates of energy dissipation increase with increasing discharge along the stepped chute without reaching uniform equilibrium conditions. In skimming flows down stepped chutes, the outward ends of the steps form a pseudo-bottom over which the flow passes [50-54]. Underneath this, recirculating whirlpools appear and are sustained through the spread of shear stress from the waters flowing past the step ends. Skimming flows are accompanied by huge flow obstruction, with resultant form losses A simple model (Eq [3.3] is suggested to define the uniform flow region and shows good agreement with the measured data set, but more work is recommended to find the difference between model and prototype data.The figures, 6 to 10, stress the robust energy dissipation performances for all stepped chutes linked with the air–water edges.

17

COMPETING INTERESTS

Authors have declared that no competing interests exist

REFERENCES

1. Matos J, Frizell K. Air concentration and velocity measurements on self-aerated flow down stepped chutes. Project Notes, Technical University of Lisbon, Lisbon, Portugal; 2000.
2. American Society of Civil Engineers, Acse. Roller-Compacted Concrete From The Us. Army Corps of Engineers, Asce Press, New York; 1994.
3. American Society of Mechanical Engineers, Asme. Fluid meters, their theory and application, report of the Asme Research Committee on Fluid Meters, 5th Edition, New York; 1959.
4. Boes RM, Hager WH. Fiber-optical experimentation in twophase cascade flow. Proceedings of the International Rcc Dams Seminar, K. Hansen (Ed), Denver; 1998.
5. Bradley JN, Peterka AJ. The hydraulic design of stilling basins: Short stilling basin for canal structures, small outlet works and small spillways (Basin Iii). Journal of Hydraulics Division. 1957;83(Hy5):Paper 1403, October.
6. Cain P. Measurements within self-aerated flow on a large spillway, dissertation. University of Canterbury, Christchurch New Zealand; 1978.
7. Houston KL. Stepped spillway design with a rcc dam. Asce Hydraulic Engineering Proceedings. 1987;163.
8. Jacobs ML. Air concentration meter electronics package manual. U.S. Bureau of Reclamation Technical Service Center, Project Notes. 1997;8450-98-01, October.
9. Johnson B, Cowan G. Hydraulic Model For A New Dam Significant Savings. Public Works. 1997;128(8):July.
10. Lamb OP, Killen JM. An electrical method for measuring air concentration in flowing air-water mixtures. Technical Paper Number 2, Series B, University of Minnesota, St. Anthony Falls Hydraulic Laboratory, Minneapolis, Minn; 1950.
11. Lejeune M, Lejeune A. About the energy dissipation of skimming flows over stepped spillways. Proceedings of the First Conference on Hydroinformatics, Deflt, Netherlands. 1994;2:September.
12. Matos J, Quintela A. Flow resistance and energy dissipation in skimming flow over stepped spillways. Proceedings of the First International Conference on Water Resources, Asce Water Resources Engineering Division, San Antonio, Tx. 1995;2, August.
13. Mckenna. Air-water gas transfer on stepped spillways. Thesis, Colorado State University, USA; 2001.
14. Montgomery DC, Runger GC. Applied statistics and probability for engineers. John Wiley & Sons, 200 Edition, New York; 1999.
15. Omega Instruments. The flow and level handbook. Omega Engineering, Inc. Stamford, Ct. 1995;29.
16. Perterka AJ. Hydraulic design of stilling basins and energy dissipaters. Engineering Monograph No. 25, U.S. Bureau of Reclamation, Denver, Colorado; 1983.
17. Peyras L, Royet P, Degoutte G. Flow and energy dissipation over stepped gabion weirs. Journal of Hydraulic Engineering. 1992;118(5), May.
18. Falvey HT. Personal Communications; 2001.
19. Falvey HT. Mean Air Concentration of Self-aerated flows. Journal of the Hydraulics Division. 1979;105(Hyl), January.
20. Frizell KH. Hydraulics of stepped spillways for rcc dams and dam rehabilitations. Proceedings; 1992. Asce Roller Compacted Concrete Iii Conference, San Diego, Ca, Februar; 1992.
21. Frizell KH, Ehler DG, Mefford BW. Developing air concentration and velocity probes for measuring highly-aerated, high-velocity flow. Proceedings Hydraulic Engineering Conference, Asce, Buffalo, Ny; 1994.
22. Gaston ML. Air entrainment and energy dissipation on a stepped block spillway. Thesis, Colorado State University, USA; 1995.
23. Hewlett et al. Design of stepped-block spillways. Construction Industry Research and Information Association, Ciria Special Punlication, London. 1997;142.
24. Chamani MR, Rajaratnam N. Jet flow on stepped spillways. Journal of Hydraulic Engineering. 1994;120(2):February.

25. Chamani MR, Rajaratnam N. Characteristics of skimming flow over stepped spillways. Journal of Hydraulic Engineering. 1999;125(4):April. 162.
26. Chanson H. Stepped spillway flows and air entrainment. Canadian Journal of Civil Engineering. 1993;20(3):June.
27. Chanson, Hubert. Hydraulic design of stepped cascades, channels, weirs and spillways. Pergamon, Oxford, England; 1994.
28. Cain P, Wood IR. Instrumentation for aerated flow on spillways. Journal Of The Hydraulics Division. 1981;107(Hyl L). November.
29. Chamani MR. Skimming flow in a large model of a stepped spillway, dissertation. University Of Alberta, Canada; 1997.
30. Christodoulou GC. Energy dissipation on stepped spillways. Journal of Hydraulic Engineering. 1993;119(5):May.
31. Chow VT. Open channel hydraulics. Mcgraw-Hill, New York; 1959.
32. Diez-Cascon J, Blanco JL, Revilla J, Garcia R. Studies of the hydraulic behaviour of stepped spillways. International Water Power and Dam Construction. 1991; 43:September.
33. Ehrenberger R. Flow of water in steep chutes with special reference to self-aeration. wasservewegung in steilen rinnen (Schusstennen) Mit Besonderer Berucksichtigung Der Selbstbelu:Ftung, Osterreichischen Ingenieurund Architektenvereines, Nos. 15/16 And 17/18, Translated By E.F. Wilsey, United States Bureau Of Reclamation; 1926.
34. Essery ITS, Homer MW. The hydraulic design of stepped spillways. Report 33, Construction Industry Research And Information Association, London, England, January; 1978.
35. Rand W. Flow geometry at straight drop spillways. Proceedings, ASCE. 1955; 108(791):September.
36. Rice CE, Kadavy KC. Model study of a roller compacted concrete stepped spillway. Journal of Hydraulic Engineering. 1996;122(6):June.
37. Rice CE, Kadavy KC. Physical Model study of the proposed spillway for cedar run site 6, Faquier County, Virginia. Asae Applied Engineering in Agriculture. 1997;13(6).
38. Robinson KM, Rice CE, Kadavy KC, Talbot JR. Energy losses on a roller compacted concrete stepped spillway.

asce water resources engineering division. Proceedings Of The 1998 International Water Resources Engineering Conference, Memphis, Tn. 1998;2:August.
39. Ruh JF, Frizell KH. Air concentration in highly turbulent flow on a steeply-sloping chute. Hydraulic Engineering '94: Proceedings of the 1994 Conference. 1994.
40. Schnitter NJ. A history of dams: the useful pyramids. Balkema Publications, Rotterdam, The Netherlands; 1994.
41. Rajaratnam N. Skimming flow m stepped spillways. Journal of Hydraulic Engineering. 1990; 116(4):April. 164
42. Bindo M, Gautier J, Lacroix F. The stepped spillway of m'bali dam. International Water Power and Dam Construction. 1993;45(1):January.
43. Schuyler JD. Reservoirs for irrigation, water power and domestic water supply. John Wiley & Sons, 2nd Edition, New York; 1912.
44. Smith N. A history of dams. The Citadel Press, Secaucus, New Jersey; 1972.
45. Stephenson D. Energy dissipation down stepped spillways. International Water Power and Dam Construction. 1991;43: September.
46. Straub LG, Lamb OP. Experimental studies of air entrainment in open channel flow. Proceedings, Minnesota International Hydraulics Convention, Minneapolis, Minnesota, September; 1956.
47. Straub LG, Anderson AG. Experiments on self-aerated flow in open channels. Journal of the Hydraulics Division. 1958; 84(Hy7):December.
48. Sorensen RM. Stepped Spillway Hydraulic Model Investigations. Journal of Hydraulic Engineering. 1985;111(12). December.
49. Ruff JF, Ward JP. 'Final report research project 99fc800156 hydraulic design of stepped spillways prepared for: U.S. Bureau of Reclamation Denver, Colorado'' Prepared By: Jam Es F. Ruff and Jason P. Ward Engineering Research Center Colorado State University Fort Collins, Colorado; April 2002.
50. Tozzi MJ. Residual energy in stepped spillways. International Water Power and Dam Construction. 1994;46, May.
51. White MP. energy loss at the base of a free overfall - discussion. Transactions, Asce. 1943;108.165.
52. Wilhelms SC. Self-aerated flow on corps engineers spillways, technical report no.

W-94-2, U.S. Army Engineers Waterways Experiment Station, Vicksburg, Miss; 1994.

53. Wood IR. Uniform flow region of self-aerated flow. Journal of Hydraulic Engineering. 1983;109(3).

54. Wood IR. Air entrainment in free-surface flows. Jahr Hydraulic Strucutres Design Manual No. 4, Hydraulic Design Considerations, Balkema Publications, Rotterdam, The Netherlands; 1991.

Peer-review history:
The peer review history for this paper can be accessed here:
https://www.sdiarticle5.com/review-history/89345