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Dominator and total dominator colorings in vague graphs

Lian Chen[1], Huiqin Jiang[2], Zehui Shao[1,*] and Marija Ivanović[3]

1 Institute of Computing Science and Technology, Guangzhou University, Guangzhou 510006, China.; chenlian@gzhu.edu.cn(L.C); zshao@gzhu.edu.cn(Z.S)

2 School of Information Science and Engineering, Chengdu University, Chengdu 610106, China.; hq.jiang@hotmail.com(H.J)

3 Faculty of Mathematics, University of Belgrade, Studentski trg 16/IV, 11 000 Belgrade, Serbia.; maria.ivanovic@gmail.com(M.I)

* Correspondence: zshao@gzhu.edu.cn

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Abstract: The concept of vague graph was introduced early by Ramakrishna and substantial graph parameters on vague graphs were proposed such graph coloring, connectivity, dominating set, independent set, total dominating number and independent dominating number. In this paper, we introduce the concept of the dominator coloring and total dominator coloring of a vague graph and establish mathematical modelling for these problems.

Keywords: Dominator coloring, fuzzy graph theory, total dominator coloring, vague graph.

1. Introduction

Fuzzy set generalize classical sets by use of a membership function such that each element is assigned a number in the real unit interval [0,1], which measures its grade of membership in the set. The theory of fuzzy sets was proposed by Zadeh in 1965 [1]. Since then, the theory was used in a wide range of domains in which information is incomplete or imprecise, such as such as management science, medical science, social science, financial science, environment science and bioinformatics [2]. In 1993, Gau *et al.* [3] presented the concept of vague set theory as a generalization of fuzzy set theory, which allow a separation of evidence for membership (grade of membership) and evidence against membership (negation of membership). They used a subinterval of [0,1] to replace the value of an element in a set. That is, a vague set is characterized by two functions. Namely, a truth-membership function $t_v(x)$ and false-membership function $f_v(x)$ are used to describe the boundaries of the membership degree.

Graph theory is a very useful and well developed branch of discrete mathematics, and it also is an important tool for modeling many types of relations and processes in biological, physical, social and information systems. Realizing the importance of graph theory and inspiring of Zadeh's fuzzy relations [4], Kauffman [5] proposed the definition of fuzzy graph in 1973. Then Rosenfeld [6] proposed another elaborated definition of fuzzy graph in 1975. Since then, there was a vast research on fuzzy graph [7–19]. Inspired by fuzzy graph, in 2009, Ramakrishna [20] introduced the concept of vague graphs and studied some important properties. After that, Samata *et al.* [21] analysed the concepts of vague graphs and its strength. Rashmanlou *et al.* [22] introduced the notion of vague h-morphism on vague graphs and regular vague graphs, and they investigated some properties of an edge regular vague graph [23]. At the same time, they introduced some connectivity concepts in the vague graphs [24].

The Dominator coloring of a graph was proposed by Gera et al [25] in 2006. In the same paper, they showed that dominator chromatic number is NP-complete. After that, they studied the bounds and realization of the dominator chromatic number in terms of chromatic number and domination number [26] and the dominator colorings in bipartite graphs [27]. Recently, several researchers have theoretically investigated the dominator coloring number of Claw-free graph [28], Certain Cartesian Products [29], trees [30] and more [31,32]. Motivated by dominator chromatic number, Kazemi [33] studied the new concept of a total dominator chromatic number of a graph. And they showed that total dominator chromatic number is NP-complete. A survey of total dominator chromatic number in graphs can also be found in [34,35].

Borzooei *et al.* [36] in their work introduced the concepts of special kinds of dominating sets in vague graph. Kumar *et al.* [37] discuss the new concepts of coloring in vague graphs with application. In this paper, we introduce the concept of the dominator coloring and total dominator coloring of a vague graph and establish mathematical modelling for these problems.

2. Preliminaries

A vague set A in an ordinary finite non-empty set X is a pair (t_A, f_A), where $t_A : X \rightarrowtail [0,1]$, $f_A : X \rightarrowtail [0,1]$, and $0 \leq t_A(x) + f_A(x) \leq 1$ for each element $x \in X$. Note that the truth-membership $t_A(x)$ is considered as the lower bound on grade of membership of x derived from the evidence for $x \in X$ and the false-membership $f_A(x)$ is the lower bound on negation of membership of x derived from the evidence against $x \in X$. The grade of membership for x is characterized by the interval $[t_A(x), 1 - f_A(x)]$ not a crisp value. And if $t_A(x) = 1 - f_A(x)$ for all $x \in X$, the vague set degrades to a fuzzy set.

In this paper, we denote by P_n, C_n, K_n the path, cycle and complete graph on n vertices, respectively. The complete bipartite graph with part size m, n is denoted by $K_{m,n}$ and the ladder graph is the Cartesion product of P_2 and C_n, denoted by $P_2 \square C_n$.

Definition 1. Let $G = (V, E)$ be a graph. A pair $G' = (A, B)$ is called a vague graph on G where $A = (t_A, f_A)$ is a vague set on V and $B = (t_B, f_B)$ is a vague set on E such that $t_B(uv) \leq min\{t_A(u), t_A(v)\}$, $f_B(uv) \geq max\{f_A(u), f_A(v)\}$ for each $uv \in E$.

Definition 2. For a vague graph $G = (A, B)$, an edge uv is called a strong edge if $t_B(uv) = min\{t_A(u), t_A(v)\}$, $f_B(uv) = max\{f_A(u), f_A(v)\}$. Let $N(u) = \{v | uv \text{ is a strong edge in } G\}$ and $N[u] = N(u) \cup \{u\}$.

We say u dominates all vertices in $N(u)$ and totally dominates all vertices in $N[u]$.

Definition 3. Dominator coloring of a vague graph G is a coloring of the vertices of G such that every vertex dominates all vertices of at least one other class. The dominator chromatic number $\chi^d(G)$ of G is the minimum number of colors among all dominator colorings of G.

Definition 4. Total dominator coloring of a vague graph G is a coloring of the vertices of G such that every vertex totally dominates all vertices of at least one other class. The total dominator chromatic number $\chi_t^d(G)$ of G is the minimum number of colors among all total dominator colorings of G.

3. Dominator coloring problems

Let $[k] = \{1, 2, \ldots, k\}$. Let $V_c \subseteq V$ denotes set of vertices with assigned color c. Further, let decision variables $x_{i,c}$ be defined as

$$x_{i,c} = \begin{cases} 1, & i \in V_c \\ 0, & i \in V_c \end{cases}$$

For a vague graph G and an integer k, let E^s be the set of all strong edges of G. We propose integer linear programming (ILP) formulations (called Dominator Coloring ILP and Total Dominator Coloring ILP, respectively), for the dominator coloring problem and total dominator coloring problem as follows:

Dominator coloring ILP

$$\sum_{c=1}^{k} x_{i,c} = 1, \quad i \in V \tag{1}$$

$$\sum_{i \in V}^{k} x_{i,c} \geq 1, \quad c \in [k] \tag{2}$$

$$x_{i,c_1} + x_{j,c_2} + M_{i,c_1,c_2} \leq 2, \quad c_1, c_2 \in [k], i \in V(G), j \in V \backslash N[i] \tag{3}$$

$$\sum_{c_2=1}^{k} M_{i,c_1,c_2} \geq 1, \quad c_1 \in [k], i \in V \tag{4}$$

$$x_{i,c} + x_{j,c} \leq 1, \quad c \in [k], (i,j) \in E^s \tag{5}$$

$$x_{i,c} \in \{0,1\}, \quad c \in [k], i \in V \tag{6}$$

$$M_{i,c_1,c_2} \in \{0,1\}, \quad \{c_1,c_2\} \subseteq [k], i \in V \tag{7}$$

Theorem 5. *Conditions* $(1) - (7)$ *defined for the graph G are satisfied if and only if G admits a dominator coloring with k colors.*

Proof. Condition (1) ensures that each vertex is assigned with exactly one color. Condition (2) ensures that each color should be used. Conditions (3) and (4) ensure that every vertex dominates all vertices of at least one other class. Condition (5) ensures that the assignment is a proper coloring. Conditions (6) and (7) ensure that each variable is boolean. Therefore, if each condition is satisfied, then G admits a dominator coloring with k colors.

$\Leftarrow$ By the definition of the dominator coloring, it is clear that Conditions $(1) - (7)$ defined for the graph G are satisfied. □

Total dominator coloring ILP

$$\sum_{c=1}^{k} x_{i,c} = 1, \quad i \in V \tag{8}$$

$$\sum_{i \in V}^{k} x_{i,c} \geq 1, \quad c \in [k] \tag{9}$$

$$x_{i,c_1} + x_{j,c_2} + M_{i,c_1,c_2} \leq 2, \quad \{c_1,c_2\} \in [k], i \in V(G), j \in V \backslash N[i] \tag{10}$$

$$\sum_{c_2 \neq c_1, c_2=1}^{k} M_{i,c_1,c_2} \geq 1, \quad c_1 \in [k], i \in V \tag{11}$$

$$x_{i,c} + x_{j,c} \leq 1, \quad c \in [k], (i,j) \in E^s \tag{12}$$

$$x_{i,c} \in \{0,1\}, \quad c \in [k], i \in V \tag{13}$$

$$M_{i,c_1,c_2} \in \{0,1\}, \quad \{c_1,c_2\} \subseteq [k], i \in V \tag{14}$$

Theorem 6. *Conditions* $(8) - (14)$ *defined for the graph G are satisfied if and only if G admits a total dominator coloring with k colors.*

Proof. Condition (8) ensures that each vertex is assigned with exactly one color. Condition (9) ensures that each color should be used. Conditions (10) and (11) ensure that every vertex totally dominates all vertices of at least one other class. Condition (12) ensures that the assignment is a proper coloring. Conditions (13) and (14) ensure that each variable is boolean. Therefore, if each condition is satisfied, then G admits a total dominator coloring with k colors.

$\Leftarrow$ By the definition of the total dominator coloring, it is clear that Conditions $(1) - (7)$ defined for the graph G are satisfied. □

Example 1. Let G be a vague graph depicted in Figure 1. Then the set of strong edges is $\{(u_1u_2), (u_2v_2), (u_2u_4), (u_2u_3), (u_3u_4), (u_4v_3), (u_4u_5), (u_5v_4), (u_5u_6), (u_6u_7), (v_1v_2), (v_2v_3), (v_3v_4), (v_4v_5)\}$.

Example 2. Let G be a vague graph depicted in Figure 2. Then by solving the instance from Dominator Coloring ILP, we obtain $\gamma^d(G) = 6$. A dominator coloring f with 6 colors is $f(u_1) = 1,\ f(u_2) = 2,\ f(u_3) = 4,\ f(u_4) = 1, f(u_5) = 4, f(u_6) = 6,\ f(u_7) = 1, f(v_1) = 3,\ f(v_2) = 1,\ f(v_3) = 4,\ f(v_4) = 5,\ f(v_5) = 1$ which is presented in Figure 2.

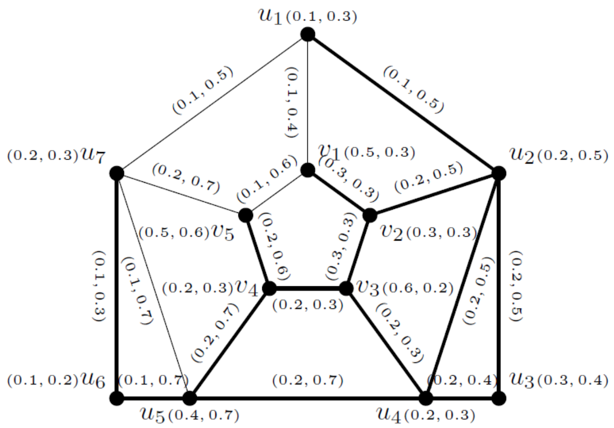

Figure 1. An example of a vague graph G

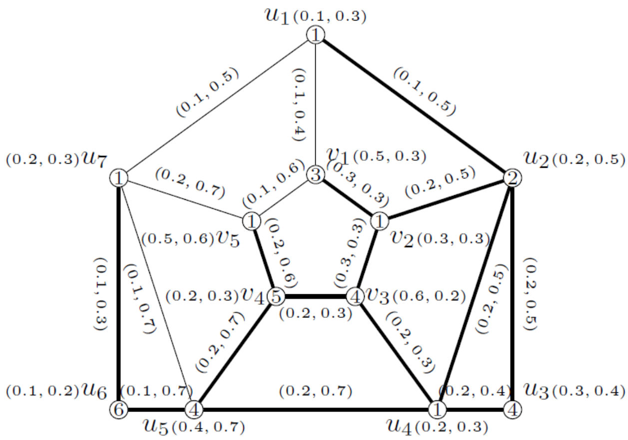

Figure 2. A dominator coloring of G with 6 colors

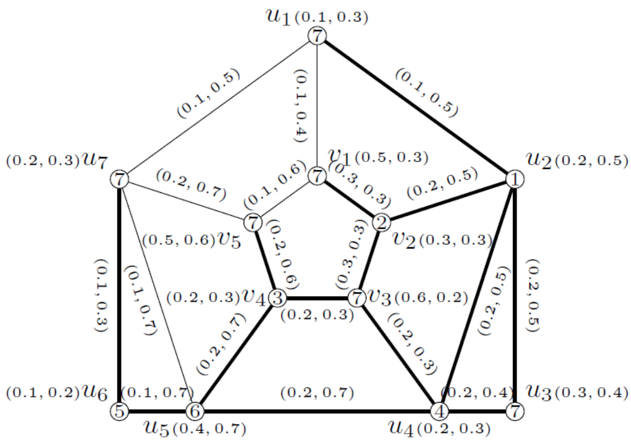

Figure 3. A total dominator coloring of G with 7 colors

Example 3. Let G be a vague graph depicted in Figure 3. Then by solving the instance from Total Dominator Coloring ILP, we obtain $\gamma_t^d(G) = 7$. A dominator coloring f with 7 colors is $f(u_1) = 7,\ f(u_2) = 1, f(u_3) = 7, f(u_4) = 4,\ f(u_5) = 6, f(u_6) = 5, f(u_7) = 7,\ f(v_1) = 7,\ f(v_2) = 2, f(v_3) = 7,\ f(v_4) = 3, f(v_5) = 7$ which is presented in Figure 3.

4. Dominator coloring number of some classes of vague graphs

Definition 7. For a vague graph G, we define an underlying graph of G, denoted by $\widetilde{G}$, with $V(\widetilde{G}) = V(G)$, and $xy \in E(\widetilde{G})$ if and only if $x \in N(y)$ in G.

By the definition of Dominator coloring, we have

Proposition 8. *For any vague graph G, $\chi^d(G) = \chi^d(\widetilde{G})$ and $\chi_d^t(G) = \chi_d^t(\widetilde{G})$.*

Proposition 9. *(see [28]) For any vague graph G with $\widetilde{G} \cong K_n$, we have $\chi_d(G) = \chi_d^t(G) = n$.*

Proof. By the definition, we have $\chi_d^t(G) \geq \chi_d(G) \geq \chi(G) = n$. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$. We consider a function $f : V(G) \rightarrowtail \{1, 2, \ldots, n\}$ with $f(v_i) = i$ for any i, then we have f is a total dominator coloring of G with n colors. Therefore, we have $\chi_d^t(G) \leq n$ and so the desired result holds. □

The following results are straightforward:

Proposition 10. *(see [28]) For any vague graph G with $\widetilde{G} \cong K_{m,n}$, we have $\chi^d(G) = \chi_d^t(G) = 2$.*

Proposition 11. *(see [28]) For any vague graph G with $\widetilde{G} \cong C_n(n \geq 3)$, we have $\chi^d(G) = 3$ for $n \equiv 3$ (mod 6) and $\chi_d(G) = 2$ otherwise.*

Proposition 12. *(see [35]) For any vague graph G with $\widetilde{G} \cong P_n$ or $C_n(n \geq 3)$, we have $\chi_d^t(G) = \left[\frac{n}{2}\right] + \left[\frac{n}{4}\right] - \left[\frac{n}{4}\right]$.*

The Cartesian product $G\square H$ of two graphs G and H is a graph with $V(G) \times V(H)$ and two vertices (g_1, h_1) and (g_2, h_2) are adjacent if and only if either $g_1 = g_2$ and $(h_1, h_2) \in E(H)$, or $h_1 = h_2$ and $(g_1, g_2) \in E(G)$. Let $V(C_n) = \{1, 2, 3, \ldots n\}$, $E(C_n) = \{i(i+1)\}$, $1n|i = 1, 2, \ldots n-1\}$ and $V(P_2) = \{1, 2\}$, $E(P_2) = \{(12)\}$. Let $u_{i,j}$ be a vertex of $P_2\square C_n$ where $i = 1, 2,\ j = 1, 2, \ldots n$. We have the following result:

Proposition 13. *For any vague graph G with $\widetilde{G} \cong P_2\square C_n$ with $n \geq 6$,*

$$\gamma_t^d(G) \leq \begin{cases} \frac{2n}{3} + 2, & n \equiv 0 \quad (mod\ 6) \\ \frac{2n}{3} + 4, & n \equiv 1, 2\ (mod\ 6) \\ \frac{2n}{3} + 3, & n \equiv 3 \quad (mod\ 6) \\ \frac{2n}{3} + 2, & n \equiv 4, 5\ (mod\ 6) \end{cases}$$

Proof. We use two lines of numbers to denote a total dominator coloring of $P_2\square C_n$. The total dominator coloring can be represented as a $2 \times n$ array as follows:

$$f(P_2\square C_n) = \begin{cases} f(u_{1,1})\ f(u_{1,2}) \ldots f(u_{1,n-1})\ f(u_{1,n}) \\ f(u_{2,1})\ f(u_{2,2}) \ldots f(u_{2,n-1})\ f(u_{2,n}) \end{cases}$$

If $i = 1$ and $j \equiv 1, 3$ (mod 6), let $f(u_{i,j}) = 1$.
If $i = 1$ and $j \equiv 2, 4$ (mod 6), let $f(u_{i,j}) = 2$.
If $i = 1$ and $j \equiv 5$ (mod 6), let $f(u_{i,j}) = 4\frac{j}{6} + 5$.
If $i = 1$ and $j \equiv 0$ (mod 6), let $f(u_{i,j}) = 4\frac{j}{6} + 6$.
If $i = 2$ and $j \equiv 4, 5$ (mod 6), let $f(u_{i,j}) = 1$.
If $i = 2$ and $j \equiv 0, 5$ (mod 6), let $f(u_{i,j}) = 2$.
If $i = 2$ and $j \equiv 2$ (mod 6), let $f(u_{i,j}) = 4\frac{j}{6} + 3$.
If $i = 2$ and $j \equiv 3$ (mod 6), let $f(u_{i,j}) = 4\frac{j}{6} + 4$.
We will consider the following cases:

Case 1. $n \equiv 0 \pmod 6$.
Obviously, f is total dominator coloring with desired number of colors. For example, let $n = 12$, we have

$$f(P_2 \square C_{12}) = \begin{cases} 1\ 2\ 1\ 2\ 5\ 6\ 1\ 2\ 1\ 2\ 9\ 10 \\ 2\ 3\ 4\ 1\ 2\ 1\ 2\ 7\ 8\ 1\ 2\ 1 \end{cases}$$

Case 2. $n \equiv 1, 2, 4, 5 \pmod 6$.
Let $h(x) = f(x)$ for any $x \in V(P_2 \square C_n) \backslash \{u_{1,n}, u_{2,n}\}$, $h(u_{1,n}) = 2 \times \frac{n}{3}$, $h(u_{2,n}) = 2 \times \frac{n}{3} + 4$. Obviously, h is total dominator coloring with desired number of colors. For example, let $n = 13$, we have

$$f(P_2 \square C_{13}) = \begin{cases} 1\ 2\ 1\ 2\ 5\ 6\ 1\ 2\ 1\ 2\ 9\ 10\ 11 \\ 2\ 3\ 4\ 1\ 2\ 1\ 2\ 7\ 8\ 1\ 2\ 1\ 12 \end{cases}$$

Let $n = 14$, we have

$$f(P_2 \square C_{14}) = \begin{cases} 1\ 2\ 1\ 2\ 5\ 6\ 1\ 2\ 1\ 2\ 9\ 10\ 1\ 11 \\ 2\ 3\ 4\ 1\ 2\ 1\ 2\ 7\ 8\ 1\ 2\ 1\ 2\ 12 \end{cases}$$

Let $n = 16$, we have

$$f(P_2 \square C_{16}) = \begin{cases} 1\ 2\ 1\ 2\ 5\ 6\ 1\ 2\ 1\ 2\ 9\ 10\ 1\ 2\ 1\ 13 \\ 2\ 3\ 4\ 1\ 2\ 1\ 2\ 7\ 8\ 1\ 2\ 1\ 2\ 11\ 12\ 14 \end{cases}$$

Let $n = 17$, we have

$$f(P_2 \square C_{17}) = \begin{cases} 1\ 2\ 1\ 2\ 5\ 6\ 1\ 2\ 1\ \ 2\ 9\ 10\ 1\ 2\ 1\ 2\ 13 \\ 2\ 3\ 4\ 1\ 2\ 1\ 2\ 7\ 8\ 1\ 2\ 1\ 2\ 11\ 12\ 1\ 14 \end{cases}$$

Case 3. $n \equiv 3 \pmod 6$.
Let $h(x) = f(x)$ for any $x \in V(P_2 \square C_n) \backslash \{u_{1,n}\}$, $h(u_{1,n}) = \frac{2n}{3} + 3$. Obviously, h is total dominator coloring with desired number of colors. As an example, let $n = 15$, we have

$$f(P_2 \square C_{15}) = \begin{cases} 1\ 2\ 1\ 2\ 5\ 6\ 1\ 2\ 1\ 2\ 9\ 10\ 1\ 2\ 13 \\ 2\ 3\ 4\ 1\ 2\ 1\ 2\ 7\ 8\ 1\ 2\ 1\ 2\ 11\ 12 \end{cases}$$

Now the proof is complete. □

5. Conclusion

Fuzzy graph theory has substantial applications for real-world life in different domains, such as in the fields of biological science, neural networks, decision making, physics and chemistry. At present, the graph coloring problem can be applied in sequencing, timetabling, scheduling, electronic bandwidth allocation, computer register allocation and printed circuit board testing. Also the domination is also one of the fundamental concepts in graph theory and it has been wide used to distributed computing, biological networks, resource allocation and social networks. In this paper, motivated with the combination of fuzzy graph theory, graph coloring and graph domination, we introduce the concept of the dominator coloring and total dominator coloring of a vague graph and establish mathematical modelling for these problems.

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References

[1] Zadeh, L. A. (1965). Fuzzy sets. *Information and control, 8*(3), 338-353.
[2] Pappis, C. P., Siettos, C. I., & Dasaklis, T. K. (2013). Fuzzy sets, systems, and applications. *Encyclopedia of Operations Research and Management Science*, 609-620.
[3] Gau, W. L., & Buehrer, D. J. (1993). Vague sets. *IEEE transactions on systems, man, and cybernetics, 23*(2), 610-614.
[4] Zadeh, L. A. (1971). Similarity relations and fuzzy orderings. *Information sciences, 3*(2), 177-200.

[5] Kaufmann, A. (1975). *Introduction to the theory of fuzzy subsets* (Vol. 2). Academic Pr.

[6] Zadeh, L. A. (1977). Fuzzy sets and their application to pattern classification and clustering analysis. In *Classification and clustering* (pp. 251-299). Academic press.

[7] Samanta, S., Pal, M., Rashmanlou, H., & Borzooei, R. A. (2016). Vague graphs and strengths. *Journal of Intelligent & Fuzzy Systems, 30*(6), 3675-3680.

[8] Borzooei, R. A., & Rashmanlou, H. (2015). Ring sum in product intuitionistic fuzzy graphs. *Journal of advanced research in pure mathematics, 7*(1), 16-31.

[9] Jun, Y. B. (2006). Intuitionistic fuzzy subsemigroups and subgroups associated by intuitionistic fuzzy graphs. *Communications of the Korean Mathematical Society, 21*(3), 587-593.

[10] Rashmanlou, H., Samanta, S., Pal, M., & Borzooei, R. A. (2015). A study on bipolar fuzzy graphs. *Journal of Intelligent & Fuzzy Systems, 28*(2), 571-580.

[11] Rashmanlou, H., Samanta, S., Pal, M., & Borzooei, R. A. (2015). Bipolar fuzzy graphs with categorical properties. *International Journal of Computational Intelligence Systems, 8*(5), 808-818.

[12] Rashmanlou, H., & Jun, Y. B. (2013). Complete interval-valued fuzzy graphs. *Annals of Fuzzy Mathematics and Informatics, 6*(3), 677-687.

[13] Samanta, S., & Pal, M. (2011). Fuzzy tolerance graphs. *International Journal of Latest Trends in Mathematics, 1*(2), 57-67.

[14] Samanta, S., & Pal, M. (2011). Fuzzy threshold graphs. *CIIT International Journal of Fuzzy Systems, 3*(12), 360-364.

[15] Sunitha, M. S., & Vijayakumar, A. (2002). *Complement of a fuzzy graph. Indian Journal of pure and applied Mathematics, 33*(9), 1451-1464.

[16] Samanta, S., & Pal, M. (2015). Fuzzy planar graphs. *IEEE Transactions on Fuzzy Systems, 23*(6), 1936-1942.

[17] Samanta, S., Akram, M., & Pal, M. (2015). M-Step fuzzy copetition graphs. *Journal of Applied Mathematics and Computing, 47*(1-2), 461-472.

[18] Samanta, S., & Pal, M. (2013). Fuzzy k-competition graphs and p-competition fuzzy graphs. *Fuzzy Information and Engineering, 5*(2), 191-204.

[19] Samanta, S., Pal, M., & Pal, A. (2014). New concepts of fuzzy planar graph. *International Journal of Advanced Research in Artificial Intelligence, 3*(1), 52-59.

[20] Ramakrishna, N. (2009). Vague graphs. *International Journal of Computational Cognition, 7*(51-58), 19.

[21] Samanta, S., Pal, M., Rashmanlou, H., & Borzooei, R. A. (2016). Vague graphs and strengths. *Journal of Intelligent & Fuzzy Systems, 30*(6), 3675-3680.

[22] Rashmanlou, H., Samanta, S., Pal, M., & Borzooei, R. A. (2016). A study on vague graphs. *SpringerPlus, 5*(1), 1234.

[23] Borzooei, R. A., Rashmanlou, H., Samanta, S., & Pal, M. (2016). Regularity of vague graphs. *Journal of Intelligent & Fuzzy Systems, 30*(6), 3681-3689.

[24] Rashmanlou, H., & Borzooei, R. A. (2016). Vague graphs with application. *Journal of Intelligent & Fuzzy Systems, 30*(6), 3291-3299.

[25] Gera, R., Rasmussen, C. W., & Horton, S. (2006). Dominator colorings and safe clique partitions. *Faculty Publications 181*(7) (2006), 19-32. **Need to cross check this reference**

[26] Gera, R. (2007). On dominator colorings in graphs. *Graph Theory Notes of New York, 52*, 25-30.

[27] Gera, R. (2007, April). On the dominator colorings in bipartite graphs. In *Fourth International Conference on Information Technology (ITNG'07)* (pp. 947-952). IEEE.

[28] Abdolghafurian, A., Akbari, S., Ghorban, S. H., & Qajar, S. (2014). Dominating Coloring Number of Claw-free Graphs. *Electronic Notes in Discrete Mathematics, 45*, 91-97.

[29] Chen, Q., Zhao, C., & Zhao, M. (2017). Dominator colorings of certain cartesian products of paths and cycles. *Graphs and Combinatorics, 33*(1), 73-83.

[30] Merouane, H., & Chellali, M. (2012). On the dominator colorings in trees. *Discussiones Mathematicae Graph Theory, 32*(4), 677-683.

[31] Chellali, M., & Maffray, F. (2012). Dominator colorings in some classes of graphs. *Graphs and Combinatorics, 28*(1), 97-107.

[32] Bagan, G., Boumediene-Merouane, H., Haddad, M., & Kheddouci, H. (2017). On some domination colorings of graphs. *Discrete Applied Mathematics, 230*, 34-50.

[33] Kazemi, A. P. (2015). Totat Dominator Chromatic number of a Graph. *Transactions on Combinatorics* 4(2), 57-68.

[34] Kazemi, A. P. (2014). Total dominator coloring in product graphs. *Util. Math, 94*, 329-345.

[35] Henning, M. A. (2015). Total dominator colorings and total domination in graphs. *Graphs and Combinatorics, 31*(4), 953-974.

[36] Borzooei, R. A., & Rashmanlou, H. (2015). Domination in vague graphs and its applications. *Journal of Intelligent & Fuzzy Systems 29*(5), 1933-1940.

[37] Kishore Kumar, P. K., Lavanya, S., Broumi, S., & Rashmanlou, H. (2017). New concepts of coloring in vague graphs with application. *Journal of Intelligent & Fuzzy Systems*, *33*(3), 1715-1721.