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Determination of ρR Parameter and Calculation Energy Gain for P-^{11}B Fusion Reaction

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Authors' contributions

This work was carried out in collaboration between all authors. Author SNH designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Authors MTY, SJJ and MJ managed the analyses of the study and literature searches. All authors read and approved the final manuscript.

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ABSTRACT

One of the important problems in the human life is obtaining clean energy and a good candidate for this demand is proton-Boron (P-^{11}B) fusion reaction. Hydrogen-boron fuel generates nearly all its energy in the form of charged particles, not neutrons, thus minimizing or eliminating induced radioactivity. Our main goal in this paper, is studying on the behavior of proton-Boron plasma nuclear fusion reaction in terms of variations of time and temperature, in the presence of Proton-Boron sources. Therefore by solving the time and temperature dependent balance equations on the system of P-^{11}B fusion we

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determine the optimum physical conditions with low boron consumption rate to obtain maximum energy gain.

Keywords: Plasma; proton; boron; time; gain; temperature.

1. INTRODUCTION

Fusion reactions can produce no neutrons (no radiation) are called aneutronic. One of the most aneutronic interesting fusion reaction [1] is a proton colliding with anion of Boron 11. (see Fig. 1) For a very brief period of time, an ion of Carbon 12 is formed which then immediately decays in three α particles [8]. Controlled fusion with advanced fuels, especially hydrogen-boron-11, is an extremely attractive potential energy source. Hydrogen-boron fuel generates nearly all its energy in the form of charged particles, not neutrons, thus minimizing or eliminating induced radioactivity.

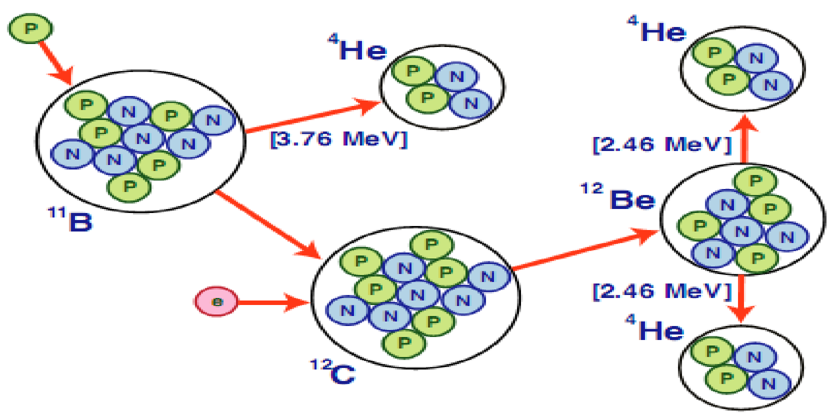

Fig. 1. P-^{11}B aneutronic fusion

Hydrogen-boron fuel also allows direct conversion of charged-particle energy to electric power, without the expensive intermediate step of generating steam for turbines [2-4]. While this fuel requires extremely high ion energies, above 200 keV, there is evidence that such energies can be achieved in the dense plasma focus [2] as well as in the z-pinch [5].

However, because of the z^2 dependence and boron's z of 5, bremsstrahlung x-ray radiation is enhanced for P-^{11}B fuel. Many analyses have indicated that fusion power can barely if at all exceed plasma cooling by bremsstrahlung [5]. If unavoidable, this situation would eliminate the heating of the plasma by the fusion-produced alpha particles and would require that all the energy be recovered from the x-ray radiation.

In 1975 A. W. Maschke suggested the use of relativistic heavy ion beams to ignite an inertially confined mass of thermonuclear fuel [1]. As in conventional inertial confinement fusion, the fuel was assumed to be pre compressed by a factor of the order of 100 in order to minimize the energy needed for ignition. Maschke suggested that lasers might compress the fuel, by high velocity impact, or by ion beams other than the ignition beam. Maschke's "fast ignition" scheme was finally abandoned in favor of the more conventional approach to inertial fusion where the implosion supplies the energy for both compression and ignition. In 1994 an important paper by Tabak et al. [2] rekindled interest in fast ignition, this time using short-pulse lasers to provide the ignition temperature. Later, at the 1997 HIF Symposium, Tabak estimated requirements for heavy-ion-driven fast ignition [3]. This approach is now the subject of intense investigation in a number of countries. If ion beams could be made to deliver the energy density needed for ignition, they would have a number of distinct advantages. The reliability, durability, high repetition rate, and high driver efficiency are expected to be advantages of any accelerator driven inertial fusion system. In the case of fast ignition, there are some additional advantages. Moreover, for ions of the appropriate range, the beam energy can be deposited directly in the fuel, eliminating the inefficiency of converting laser light to electrons or ions that then deposit their energy in the fuel. Finally, because of the reduced requirements on illumination symmetry and stability, it may be possible to devise simple illumination schemes using direct drive or tightly coupled indirect drive, or use of single sided ion illumination for indirect drive fuel compression. This could simplify chamber design and, since direct drive and or tightly coupled indirect drive are efficient implosion methods, it could lead to lower driver energy and, as in the laser case, higher energy gain. We consider ion beam requirements for fast ignition in general, where a single short pulse of ions comes in from one direction onto one side of a pre-compressed D+T fuel mass, heating a portion of that fuel mass to conditions of ignition and propagating burn in a pulse shorter than the time for the heated region to expand significantly. The igniter beam (or beams) would be arranged to penetrate the target in such a way that the Bragg peak occurs at the usual target hot spot. Or one might increase the expansion time by tamping the ignition region. In fact Magelssen published a paper in 1984 [4] in which he presented calculations of a target driven by ion beams having two very different energies. The lower energy ions arrived first and imploded the target to a spherical configuration with a rather dense pusher or tamper surrounding the fuel. The higher energy beams were then focused onto the entire assembly, heating both the fuel and the pusher. The combination of the exploding pusher and the direct ion energy deposition heated the fuel to ignition.

In summary, the fast ignition (FI) mechanism, in which a pellet containing the thermonuclear fuel is first compressed by a nanosecond laser pulse, and then irradiated by an intense "ignition" beam, initiated by a high power picosecond laser pulse, is one of the promising approaches to the realization of the inertial confinement fusion (ICF). The ignition beam could consist of laser-accelerated electrons, protons, heavier ions, or could consist of the laser beam itself. It had been predicted that the FI mechanism would require much smaller overall laser energies to achieve ignition than the more conventional central hot spot approach, and that it could deliver a much higher fusion gain, due to peculiarities of the pressure and density distributions during the ignition. It is clear, however, that interactions of electrons and ions with plasma, and most importantly the energy deposition mechanisms are essentially different. Moreover, if the ignition beam is composed of deuterons, an additional energy is delivered to the target, coming from fusion reactions of the beam-target type, directly initiated by particles from the ignition beam [5]. These and other effects had been of course taken into account in later works on this topic [6,7]. In this work, we choose the D+T fuel and at first step we compute the average reactivity in terms of temperature for first time at second step we use the obtained results of step one and calculate the total deposited energy of deuteron beam inside the target fuel at available physical condition then in third of step we introduced the dynamical balance equation of D+T mixture and solve these nonlinear differential coupled equations by programming maple-15 versus time .in forth step we compute the power density and energy gain under physical optimum conditions and at final step we analyzed our obtained results.

2. BALANCE EQUATIONS AND PHYSICAL PARAMETERS IN THE P-^{11}B SYSTEM

In the P-^{11}B reaction a proton fuses with boron 11 and becomes unstable Carbon 12 that quickly decays to three Helium 4 atoms and three photons [9]. Note that we consider the steady injection of Proton and Boron into the core with rate of s_P, and s_B, also, we consider τ_P, τ_B , τ_α, as half-life of proton, boron and alpha particles, respectively, such that τ_P=τ_B = $\tau_\alpha = \tau$ thus the balance equation of particle density of proton n_P(x), boron n_B(x) and alpha n_α(x), respectively, are given by [10]:

$$\frac{dn_p}{dt} = S_p - n_p n_B \langle \sigma v \rangle_{P-^{11}B} - \frac{n_p}{\tau} \tag{1}$$

$$\frac{dn_B}{dt} = S_B - n_B n_P \langle \sigma v \rangle_{P-^{11}B} - \frac{n_B}{\tau} \tag{2}$$

$$\frac{dn_\alpha}{dt} = n_B n_P \langle \sigma v \rangle_{P-^{11}B} - \frac{n_\alpha}{\tau} \tag{3}$$

By combining equations (1) and (2):

$$\frac{d}{dt}(n_p + n_B) = (S_P + S_B) - 2n_p n_B \langle \sigma v \rangle_{P-^{11}B} - (\frac{n_p}{\tau} - \frac{n_B}{\tau}) \tag{4}$$

By defining the following relative quantities, we try to simplify the above equation.

$$f_p = \frac{n_p}{n_i} \text{ , } f_B = \frac{n_B}{n_i} \text{ , } f_\alpha = \frac{n_\alpha}{n_i} \tag{5}$$

Where n_i is plasma density which is equal to the sum of density of proton, boron and alpha particles:

$$n_i = n_p + n_B + 3n_\alpha \tag{6}$$

namely, according to equations (5) and (6) following statement is easily verified:

$$f_p + f_B + 3f_\alpha = 1 \tag{7}$$

Considering equations (4) and (5) and by being time independence f_B and f_p , we have:

$$\frac{dn_i}{dt} = \frac{S_p + S_B}{f_p + f_B} - \frac{f_p f_B}{f_p + f_B} n_i^2 \langle \sigma v \rangle_{P-^{11}B} - \frac{n_i}{\tau} \tag{8}$$

So equation (8) is a differential equation as follows:

$$\frac{dz}{dt} = a + bz + cz^2 \tag{9}$$

By solving above differential equation to the finite element method, n_i(t) can be obtained by finding plasma density and considering equation (7) we have:

$$f_\alpha = \frac{1 - f_p - f_B}{3} \tag{10}$$

Now n_p, n_B, n_α can be obtained from equation (5), of course the constant values for various parameters are given in Table 1.

Table 1. Numerical values of different parameters

Parameter	$P-{}^{11}B$
S_P	$1.6056\times10^{28}\ \mathrm{cm}^3$
S_B	$5.0844\times10^{28}\ cm^3$
τ	$2\ s$
T	$200\,\mathrm{keV}$

As mentioned earlier, due to ever-increasing energy consumers, and need of using pure resources, $P-{}^{11}B$ fusion reactor [11-12] can be used as one of the proposed choices for producing energy, because the fuel for this reactor (proton, boron) can be found on the earth surface and neutron does not produce in this reactor [13].

Now back to the Table 1 data and explain more about them.

Since, the highest gain was obtained in terms of ion temperature $200\,\mathrm{keV}$, we will study $P-{}^{11}B$ fusion reaction in ion temperature of $200\,\mathrm{keV}$ and we will note that confinement time should be longer than the fusion time. Here we consider this time equal to 2s because the fusion time is of order but there is more discussion about s_B and s_P.

When there is no steady injection source of proton and boron in fusion reaction i.e s_P=0, s_B=0, this means that over the time, the number of proton and boron particles are decreased but the number of alpha particles during the fusion is increased. If s_P ≠0 and s_B ≠0 means that fusion reaction is done in the presence of proton and boron sources, we will encounter with 2 theories. Using equations (1) or (2), we will express these theories with considering equation (1) and we have:

$$\begin{aligned} &a)\ \text{if}\quad S_P \rangle\ n_p n_B \langle\sigma v\rangle_{P-{}^{11}B} - \frac{n_p}{\tau} \Rightarrow \frac{dn_p}{dt} \rangle\ 0 \\ &b)\ \text{if}\quad S_p \langle\ n_p n_B \langle\sigma v\rangle_{P-{}^{11}B} - \frac{n_p}{\tau} \Rightarrow \frac{dn_p}{dt} \langle\ 0 \end{aligned} \tag{11}$$

In fact in equation (11), the factor which determines whether n_P is ascending or descending, is fusion cross-section, that relates to average reactivity $\langle \sigma v \rangle_{P-^{11}B}$. When fusion cross section is small (case a)) the number of fusion reactions per unit time is low, this means that proton leakage or consumption is less than proton injection into the system. So we can say that during the fusion reaction $P-^{11}B$, density of proton particle increases in system and n_P is anupward function relative to the time.

The case that we here investigated is the case b, in which leakage rate or proton consumption is more than injection rate.

Indeed, in this case, $\langle \sigma v \rangle_{P-^{11}B}$ is big and we have more reactions per unit time which causes $\frac{dn_p}{dt} < 0$.

It is worth nothing that here, we use "Nevines" & "Swian" equation for $P-^{11}B$ average reactivity as follows:

$$\langle \sigma v \rangle_{P-^{11}B} = C_1 \varsigma^{\frac{-5}{6}} \xi^2 \exp(-3\varsigma^{\frac{1}{3}}\xi) + 5.41 \times 10^{-15} T^{\frac{-3}{2}} \times \exp(-148/T) \quad \frac{cm^3}{s} \tag{12}$$

Where ς and ξ are defined as $\varsigma = 1 - \frac{C_2 + C_4 T^2 + C_6 T^3}{1 + C_3 T + C_5 T^2 + C_7 T^3}$ and $\xi = \frac{C_0}{T^{\frac{1}{3}}}$. Also the numerical values of C_0 to C_7 are given in the Table 2 for different fusion reactions.

Fig. 2, shows the variations of average reactivity for $P-^{11}B$ reaction in terms of temperature.

As can be seen, the average reactivity is increased with increasing temperature until a maximum value and since that, the cross section decreases at $97.4785\ (\mathrm{keV})$ temperature, the maximum $\langle \sigma v \rangle_{P-^{11}B}$ occurs at a temperature of $97.4785\ (\mathrm{keV})$, at this temperature, $\langle \sigma v \rangle_{P-^{11}B}$ is equal to $2.4638 \times 10^{-18}\ (\frac{cm^3}{s})$ and this means that the probability of $P-^{11}B$ fusion reaction at temperature $97.4785\ (\mathrm{keV})$ is maximized.

The three dimensional variations of plasma density in terms of time and f_B is shown in the Fig. 3.

As we can see the plasma density decreases in every f_B over time, and also at every specific time, withrising f_B, plasma density first increases and then decreases.

For a more careful investigation, how plasma changes at different times, see Fig. 4. It is clear from the diagram that, at a specific time. The plasma density increases with f_B rise, but process of increasing for larger f_B has more intensity.

Table 2. Numerical values of C_0 to C_7 for different fusion reactions

Reaction	$T(d,n)\alpha$	$D(d,p)T$	$D(d,n)^3He$	$^3He(d,p)\alpha$	$^{11}B(p,\alpha)2\alpha$
C_0 $(keV^{1/3})$	6.6610	6.2696	6.2696	10.572	17.708
$C_1 \times 10^{16}$ (cm^3/s)	643.41	3.7212	3.5741	151.16	6382
$C_2 \times 10^3$ (keV^{-1})	15.136	3.4127	5.8577	6.4192	-59.357
$C_3 \times 10^3$ (keV^{-1})	75.189	1.9917	7.6822	$-2..290$	201.65
$C_4 \times 10^3$ (keV^{-2})	4.6064	0	0	-0.019108	1.0404
$C_5 \times 10^3$ (keV^{-2})	13.500	0.010506	-0.002964	0.13578	2.7621
$C_6 \times 10^3$ (keV^{-3})	-0.10675	0	0	0	-0.0091653
$C_7 \times 10^3$ (keV^{-3})	0.01366	0	0	0	0.00098305
T (keV)	$0.2-100$	$0.2-100$	$0.2-100$	$0.5-190$	$50-500$

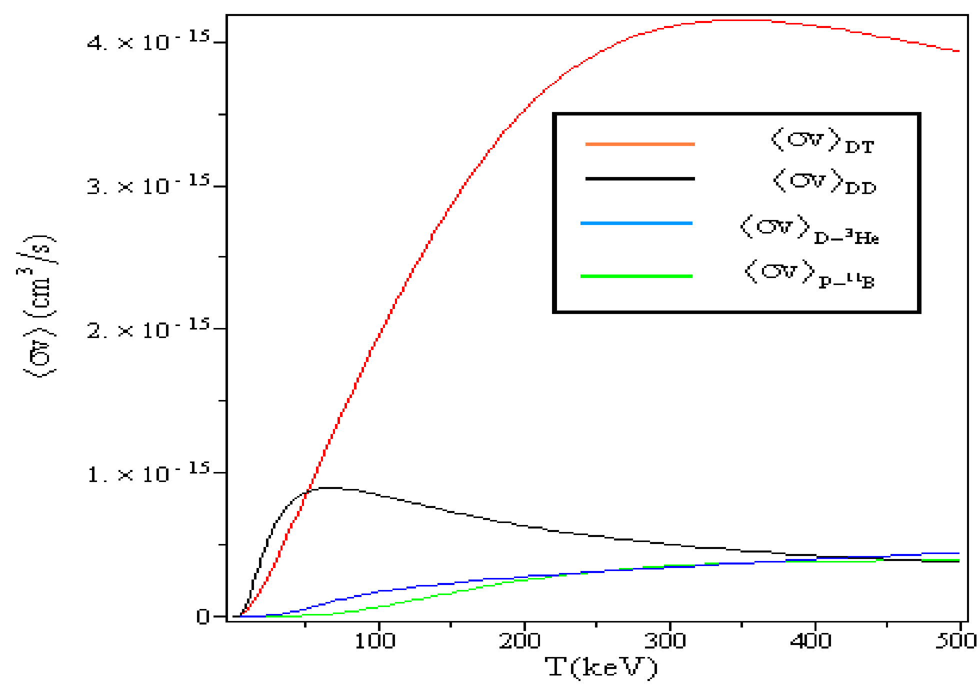

Fig. 2. Variations of average reactivity for fusion reactions in terms of temperature

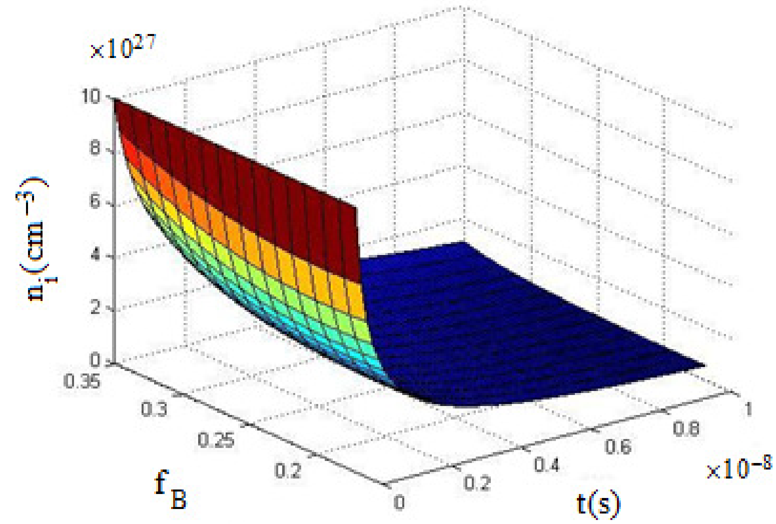

Fig. 3. Three dimensional variations of plasma density in terms of time and f_B

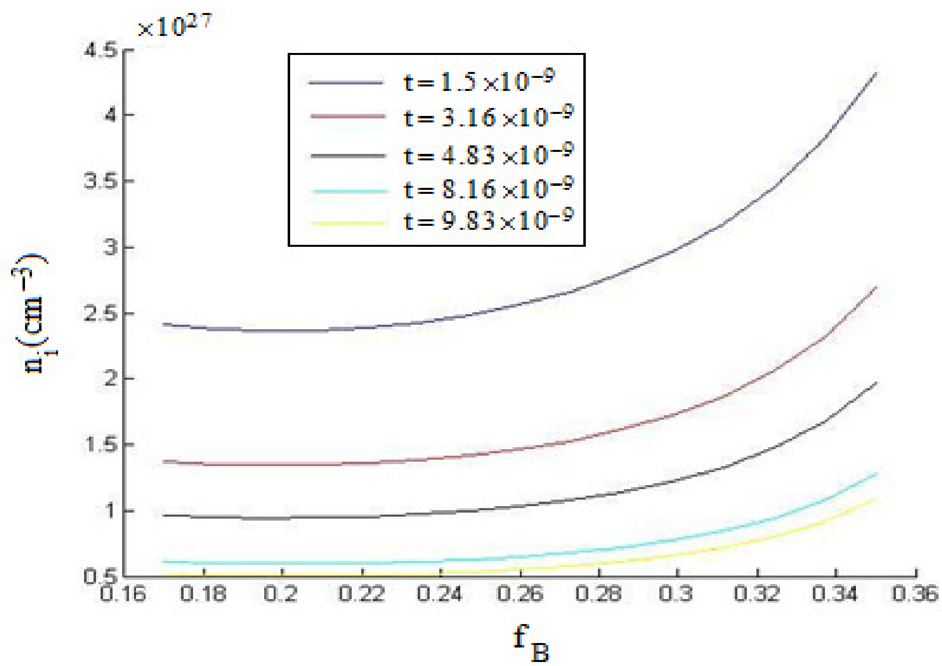

Fig. 4. Two dimensional variations of plasma density in terms of f_B at different times

We can conclude from Figs. 3 and 4 that in a specific case, with time increasing, the plasma density decreases, thus enhancement of f_B causes a rise in plasma density in a constant time.

Also by solving the point kinetic coupled differential equations with time, we will have the density of n_α, n_B, n_P changes in terms different values of f_B. Our computational results are given in the Figs. 5, 6 and 7.

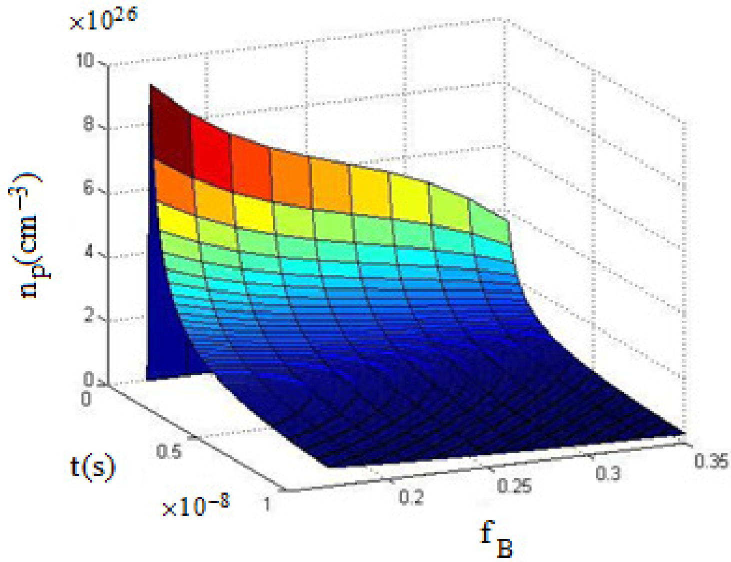

Fig. 5. Three dimensional variations of proton density in terms of f_B and time at temperature 200Kev

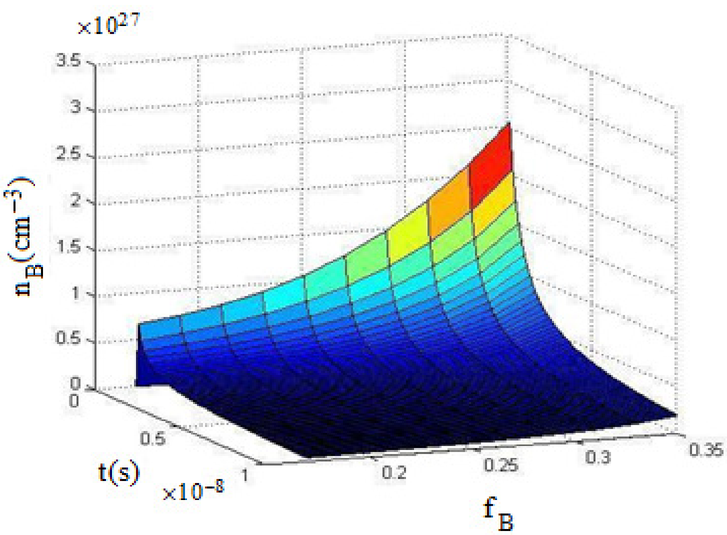

Fig. 6. Three dimensional variations of boron density in terms of f_B and time at temperature 200Kev

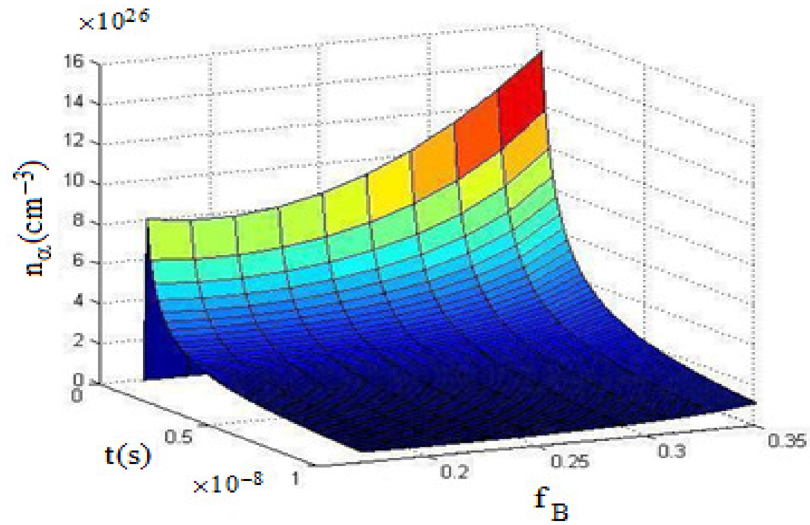

Fig. 7. Three dimensional variations of alpha particle density in terms of f_B and time at temperature 200Kev

We see clearly that from Figs. 5 to 7, n_P, n_B and n_α are reduced over time, and also in a certain time, n_P reduces by increasing f_B, but both $n_B \, and \, n_\alpha$ are increased, because by rising f_B and with considering the relation $f_B = 1 - f_B - 3f_\alpha$, $f_P$will be reduced and therefore n_P reduces. We can even show, how plasma density changes in terms of f_B and T see Fig. 8. As can be seen, by increasing temperature, plasma density for constantf_B first decreases and then increases and increases for equal temperature.

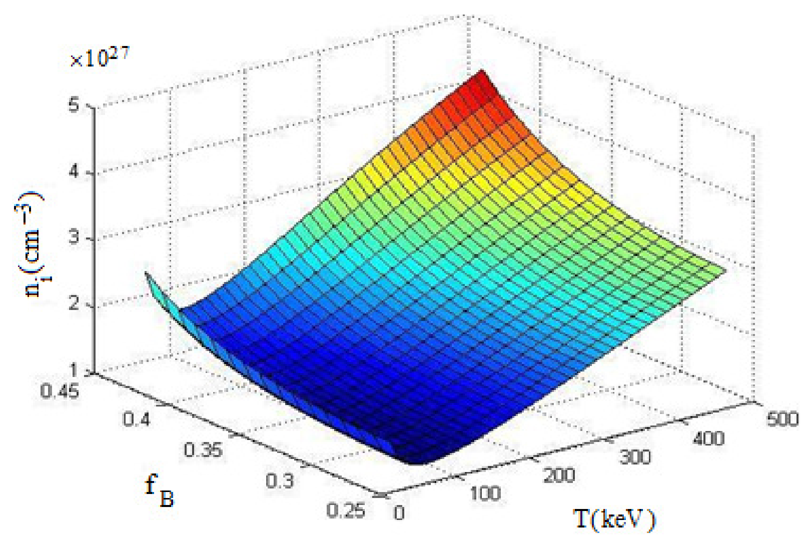

Fig. 8. Three dimensional variations of plasma density in terms of f_B and temperature

For more careful review, Fig. 9 shows plasma density curve for different f_B, in terms of temperature.

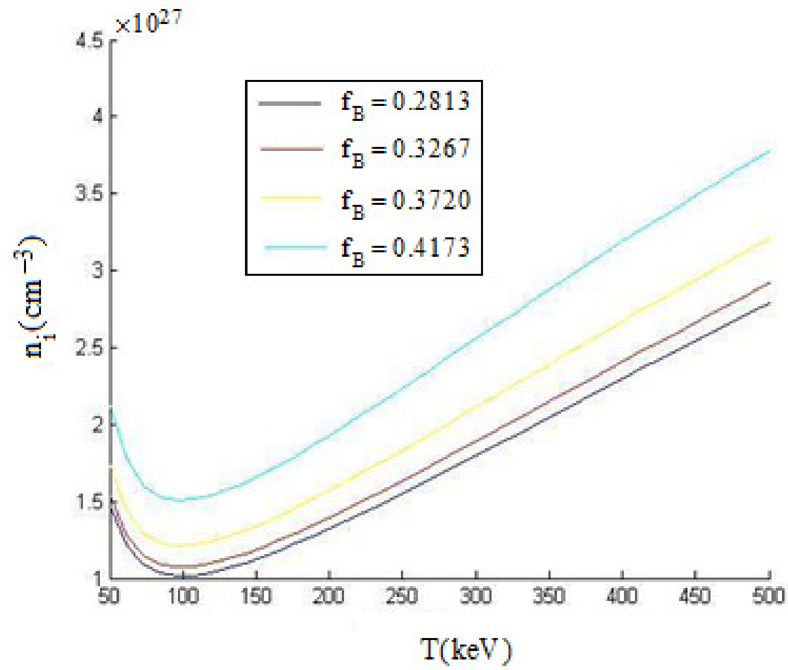

Fig. 9. Plasma density variations in terms of temperature at different f_B

As is evident from this figure, in a certain f_B, with increasing temperature, the plasma density first decreases and after reaching a minimum value increases rapidly. The reason is that, the P-"B fusion reaction cross section, according to Fig. 2, increases with increasing temperature up to a maximum value and then decreases with increasing temperature. We have the highest cross section in nearly 97Kev temperature, so we can say here, the lowest density is at 97kev, in other word, with increasing temperature from 0 to 97Kev, fusion cross section increases and thus, the number of reactions per unit time increases therefore, the rate of proton and boron consumption (leakage) compared with its injection in the system increases and eventually plasma density decreases. But the reverse mechanism is from 97 to 500Kev temperature, in the case that the fusion cross section decreases with increasing temperature and with fusion cross section decreasing, the number of reactions per unit time decreases and this leads to lower fuel consumption (leakage) in core which over time leads accumulating in fuel and thus increasing the plasma density. With regarding to the presence of proton and boron source, the fraction of fuel consumption is defined as follows that represents the rate of reaction:

$$f = \frac{(S_p + S_B)\tau_c + n_i(0) - n_i(\tau_c)}{(S_p + S_B)\tau_c + n_i(0)} \tag{13}$$

Assuming that the sum of proton and boron sources are multiples of initial density of plasma, i.e:

$$S_p + S_B = \gamma n_i(0) \tag{14}$$

then the following equation can be achieved from relations (13) and (14):

$$f = 1 - \frac{n_i(\tau_c)}{\gamma n_i(0)\tau_c + n_i(0)} \tag{15}$$

In relation (15), instead of $n_i(0)\tau_c$ we use its equal phrase namely $\frac{\rho R}{m_i C_s}$. Then equation (15) is changed to equation (16):

$$f = 1 - \frac{n_i(\tau_c)}{\frac{\gamma\rho R}{m_i C_s} + n_i(0)} = 1 - \frac{A}{\rho R + B} \tag{16}$$

Where A and B are given by:

$$A = \frac{m_i C_s n_i(\tau_c)}{\gamma}$$
$$B = \frac{m_i C_s n_i(0)}{\gamma} \tag{17}$$

As previously mentioned, the desired fusion condition is the confinement time be greater than the fusion time in other word, $\tau_C > \tau_{fusion}$. The fusion time is calculated from equation $\tau_{fusion} = \frac{1}{n_i<\sigma v>}$, so we can estimate τ_c with relation: $\tau_c = \frac{1}{n_i(0)\langle\sigma v\rangle}$. τ_{fusion} is minimized when n_i and $<\sigma v>$ are maximized and the maximum value of product of $n_i<\sigma v>$ is order of 10^{10}. Therefore, we can say minimum value of τ_{fusion} is the order of 10^{-10} or 10^{-11}. So τ_C should be greater than 10^{-11} . Since τ_C is very small, we can say nearly: $n_i(0) \cong n_i(\tau_c)$ Therefore we have: A=B. Thus, the fuel consumption fraction is converted to:

$$f = 1 - \frac{A}{\rho R + A} = \frac{\rho R}{\rho R + A} \tag{18}$$

Equation (18) shows that "f" is a function of ρR and A. For representing the dependence of "f" to ρR see Fig. 10.

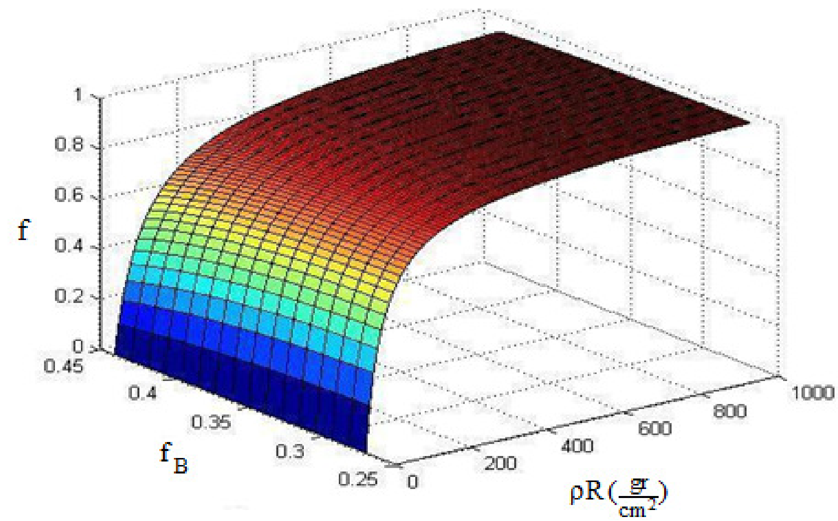

Fig. 10. Three dimensional variations of "f" in terms of f_B and ρRat T=200Kev

It is evident from this figure that in a certain value of f_B, the fuel consumption fraction will increase if ρR increases and in a constant ρR ,"f" decreases with increasing f_B. For more careful review, see Fig. 10.

It is clear from Fig. 11 that in a certain f_B, f increases if ρR increases and it happens in low ρR. As we know, ρR depends on the temperature so we can draw how changes of fuel consumption fraction in terms of f_B and T see Fig. 12.

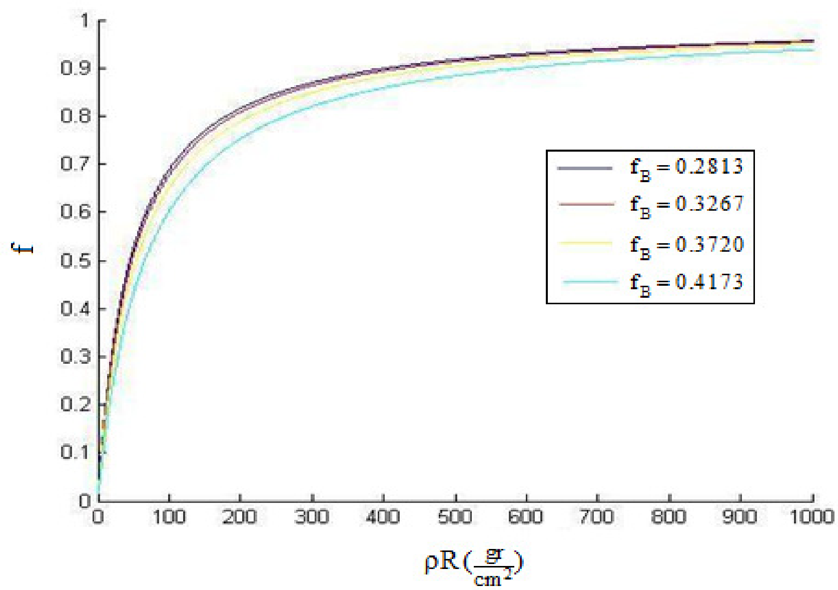

Fig. 11. Two dimensional variations of fuel consumption fraction in terms of ρR for different values of f_B in T=200Kev

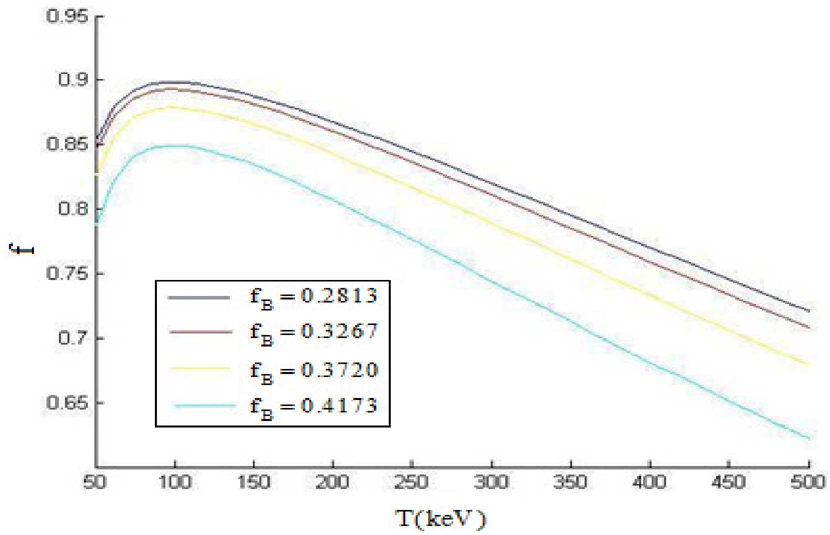

Fig. 12. Variations of f versus T for several values of f_B

3. DETERMINATION OF ρR PARAMETER FOR P-^{11}B FUSION REACTION IN THE PRESENCE Of BORON AND PROTON SOURCES

We know that the output energy from the system is given by:

$$E_{out} = \frac{4}{3}\pi R^3 \left[(S_p + S_B)\tau_c + n_i(0) - n_i(\tau_c)\right]\Delta E_{P-{}^{11}B} \tag{19}$$

First with multiplying and dividing the above equation at expression $\left[(S_p + S_B)\tau_c + n_i(0)\right]$ and then using equations (13) and (18) we have:

$$E_{out} = \frac{4}{3}\pi R^3 \left[(S_p + S_B)\tau_c + n_i(0)\right]\frac{\rho R}{\rho R + A}\Delta E_{P-{}^{11}B} \tag{20}$$

Then for having economical system we must have been $E_{out} \rangle E_{inp}$. Also the total input energy of system due to laser irradiation is given by:

$$E_{in} = \frac{2\pi(n_i kT_i + n_e kT_e)}{\eta_H} R^3 \tag{21}$$

Where, η_H,n_i,n_e,T_i,T_e and R are the system hydrodynamics efficiency, ion and electron density,ion and electron temperature and final radius of fuel pellet, respectively.

Replacing equations (20) and (21) inside $E_{out} \rangle E_{inp}$ we obtain a relation for ρR parameter. In the Fig. 13, we plotted the minimum of ρR parameter in terms of f_B and T at $\eta_H = 0.3$.

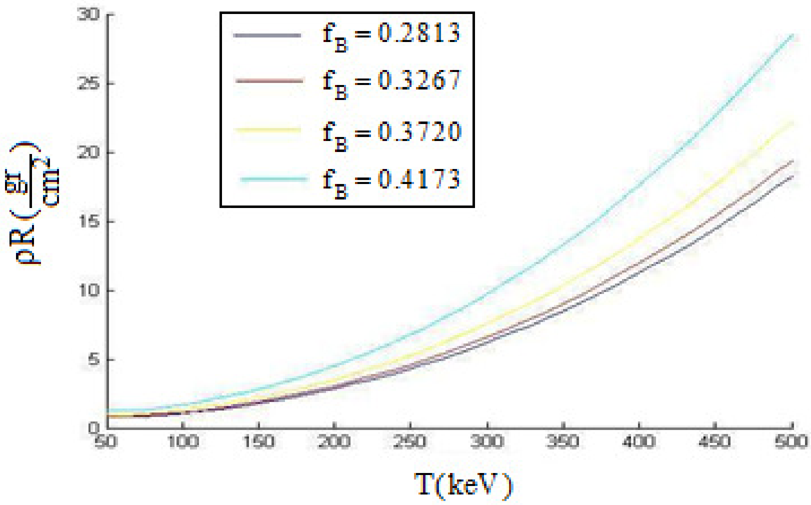

Fig. 13. Two dimensional variations of minimum of ρR parameter in terms of T for several values of f_B at $\eta_H = 0.3$

From this figure, we see clearly that, by increasing temperature at a fixed f_B and η_H the minimum value of ρR is increased. Also our calculations show that minimum value of ρR is increased by decreasing η_H because η_H is in the denominator of ρR experession.

4. FUSION ENERGY GAIN FOR P-^{11}B REACTION IN THE PRESENCE OF PROTON AND BORON SOURCES

We define fusion energy gain of the system with the following relation:

$$G = \frac{E_{out}}{E_{inp}} \tag{22}$$

By inserting equations (20) and (21) inside above equation we can calculate G.

ε parameter is given by: $\varepsilon = \frac{n_B}{n_P}$. We must be notice that G is a function of f_B, ρR, η_H and T. Therefore for plotting energy gain two parameters can be selected fix. (See Figs. 14, 15, 16). Also, for having more information about fusion energy gain you can see Table 3. From this table, we see that at $f_B = 0.1247$, $f_p = 0.1253$, $\varepsilon = 0.9947$ and $T = 63.59\,\text{keV}$ the energy gain is maximized with the value of $G = 19.2163$.

Table 3. Our calculated numerical values of energy gain for P-11B fusion reaction in terms of different parameters ($<\sigma v>$ $(cm^3.s^{-1})$ and T (KeV))

	$\langle \sigma v \rangle$	1.15×10^{-18}	1.69×10^{-18}	2.22×10^{-18}	2.09×10^{-18}	1.44×10^{-18}	9.84×10^{-19}	7.20×10^{-19}
ε	T / f_B	40	51.79	63.59	146.15	264.10	382.05	500
0.0475	0.0113	2.9845	3.3553	3.3707	1.5616	0.5636	0.2690	0.1522
0.1574	0.0340	7.9607	8.9997	9.0617	4.2007	1.5087	0.7154	0.4018
0.4648	0.0793	14.6060	16.5363	16.6597	7.2740	2.7707	1.3116	0.7352
0.6892	0.1020	16.2752	18.4292	18.5679	8.6088	3.0877	1.4613	0.8190
0.9947	0.1247	16.8424	19.0724	19.2163	8.9094	3.1454	1.5122	0.8474
1.4351	0.1473	16.3079	18.4660	18.6050	8.6260	3.0939	1.4642	0.8206
2.1250	0.1700	14.6708	16.6098	16.7338	7.7583	2.7830	1.3174	0.7385

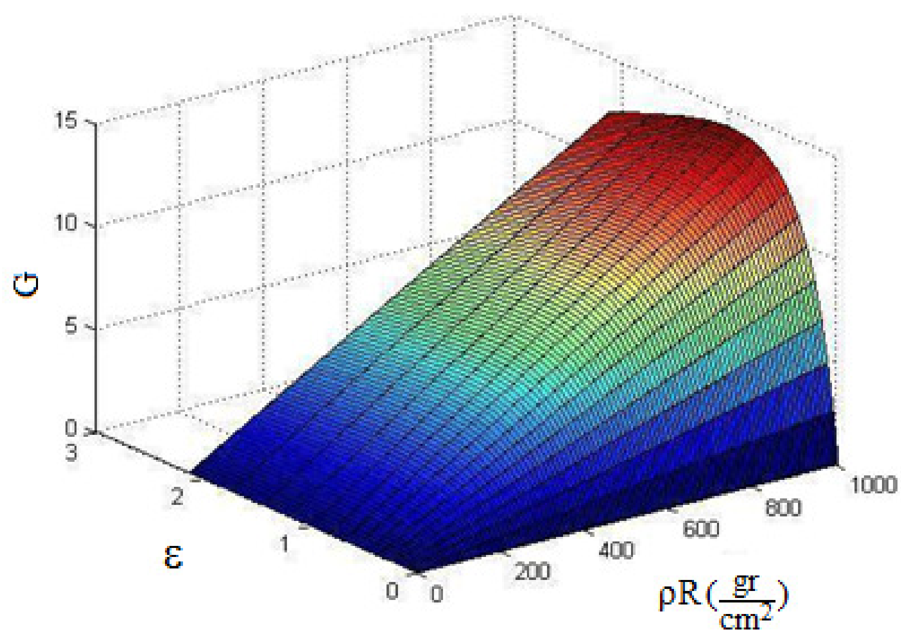

Fig. 14. Three dimensional fusion energy gain in terms of ε and ρR at $T = 200\,\text{keV}$ **and** $\eta_H = 0.5$

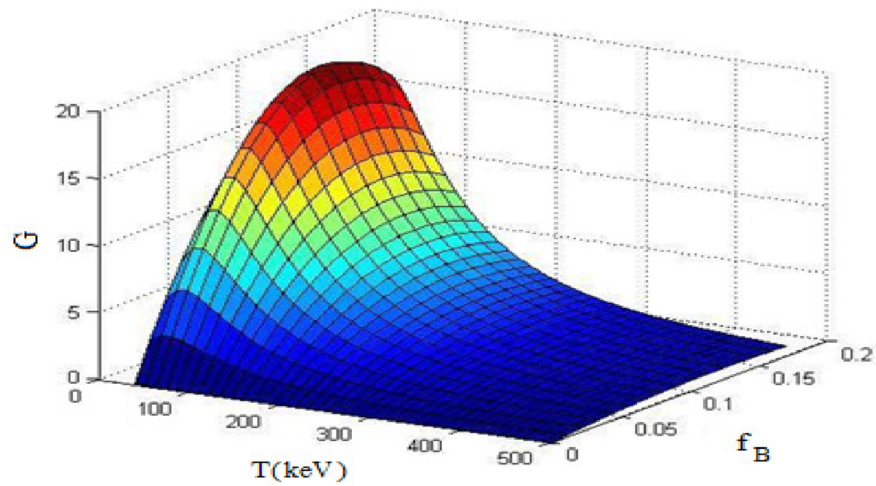

Fig. 15. Three dimensional fusion energy gain in terms of T and f_B at $\rho R = 38\frac{kg}{m^2}$ and $\eta_H = 0.5$

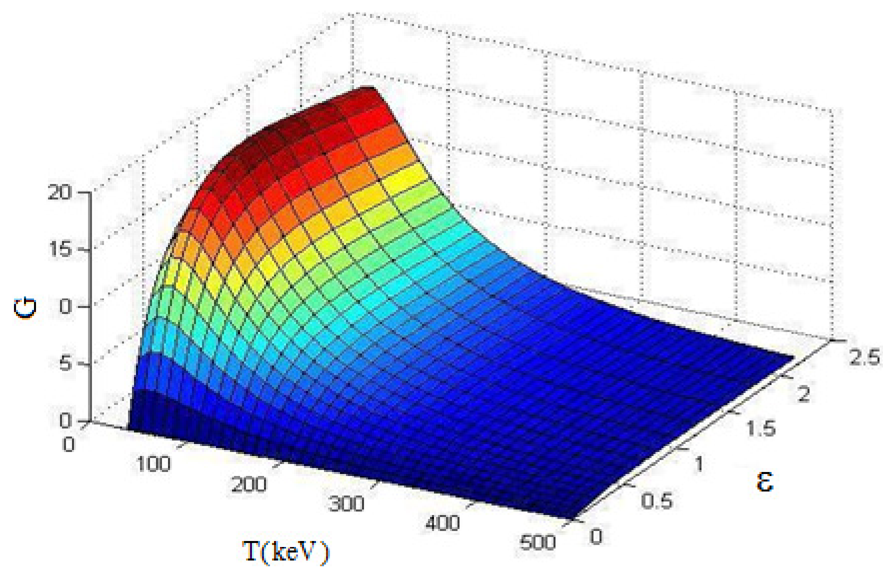

Fig. 16. Three dimensional fusion energy gain in terms of T and ε at $\rho R = 38\frac{kg}{m^2}$ and $\eta_H = 0.5$

5. CALCULATION OF POWER AND ENERGY DENSITY FOR P-^{11}B FUSION REACTION IN THE PRESENCE OF THE PROTON AND BORON SOURCES

Another important issue that should be consider bout fusion plasma of deuterium-tritium, is behavior of total power in fusion system.

Total power includes P_α, P_{oh} , P_{ext} , P_{berms} , P_{loss} , P_{sync} which are defined in the following,respectively:

a) P_α: is the portion of transmitted power to high energy alpha particles produced from P-^{11}B reaction that is deposited into the plasma. This power is given by: $\mathrm{P}_\alpha = \mathrm{n_p n_B} \langle \sigma \mathrm{v} \rangle_{\mathrm{P-^{11}B}} \mathrm{Q}_\alpha$, where Q_α is the energy released from P-^{11}B fusion reaction and is equal to $8.7\,\mathrm{MeV}$ [14-15]. We rewrite this relation as a form of : $\mathrm{P}_\alpha = 13.9 \times 10^{-13}\ \mathrm{n_i^2(0)\ f_p f_B} \langle \sigma \mathrm{v} \rangle_{\mathrm{P-^{11}B}} \quad \left(\frac{\mathrm{W}}{\mathrm{m}^3}\right)$

b) P_{oh}: is a thermal power, but here there is no any current in the plasma, therefore this power is equal zero.

c) P_{ext}: the total power that is given to the system by an external factor.

d) P_{berms}: is the portion of loss power that is due to Bremsstrahlung radiation ,that is given by :

$$\mathrm{P_{brems}} = 5.4 \times 10^{-37}\ \mathrm{n_i^2(0) Z_{eff}} \sqrt{\mathrm{T_i(0)}} \qquad \left(\frac{\mathrm{W}}{\mathrm{m}^3}\right)$$

e) P_{loss}: is the portion of transmitted power to alpha particle, during the P-^{11}Breaction because of escaping alpha particle from fusion plasma that is not deposited in the chamber. That is given by:

$$\mathrm{P_{loss}} = \frac{3}{2} \times 1.6 \times 10^{-19} \times \mathrm{n_i(0) T_i(0)} \frac{(1+\mathrm{f_p}+\mathrm{f_B})}{\tau_\mathrm{E}} \qquad \left(\frac{\mathrm{W}}{\mathrm{m}^3}\right)$$

f) P_{sync}: is the portion of loss power that is due to synchrotron radiation.

In these relations temperature is in KeV and we have [14,15]:

$$T_e(x) = \frac{T_e(o)}{1+\alpha_T} \tag{23}$$

$$n_e(x) = \frac{n_e(o)}{1+\alpha_n} \tag{24}$$

Where $\alpha_\mathrm{T} = 1$, $\alpha_\mathrm{n} = 0.5$, $\mathrm{Z_{eff}} = 3$, $\tau_\mathrm{E} = 1.4$ [10].

So due to being loss , P_{sync}, P_{berms}, P_{loss} and being productive P_α, P_{oh}, P_{ext}, we have the following balance equation of energy density [10]:

$$\frac{dw(x)}{dt} = \frac{P_{ext}}{V} + P_{oh} + P_\alpha - P_{loss} - P_{brems} - P_{sync} \tag{25}$$

We plotted the variations of P_α , P_{berms} and P_{loss} in terms of time and f_B in Figs. 17 to 19.

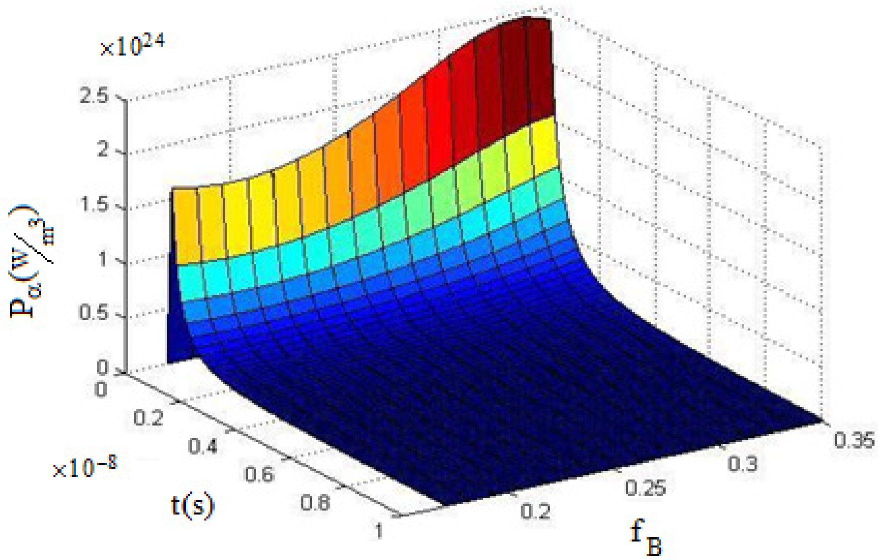

Fig. 17. Three dimensional variations of P_α versus time and f_B

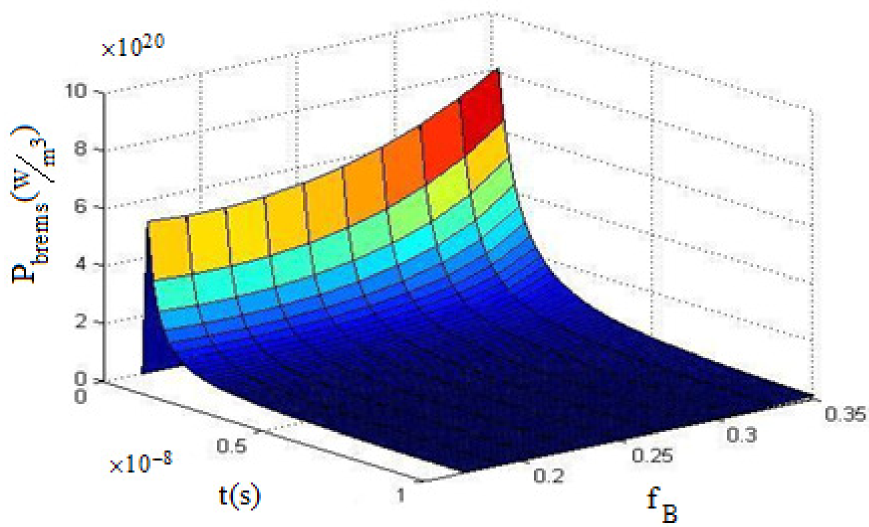

Fig. 18. Three dimensional variations of P_{berms} versus time and f_B

From these figures we see that at each time by increasing f_B, the value of P_α , P_{berms} and P_{loss} are increased. Also at a fixed f_B, with increasing time at first the values of P_α , P_{berms} and P_{loss} are increased so much in a short time interval then by increasing time decreased slowly.

Also in Fig. 20 we plotted the variations of energy density in terms of time at several values of f_B.

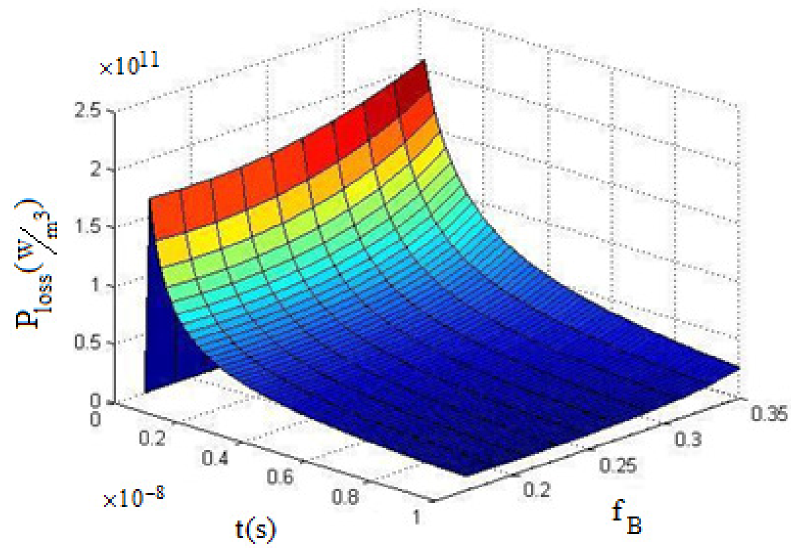

Fig. 19. Three dimensional variations of P_{loss} versus time and f_B

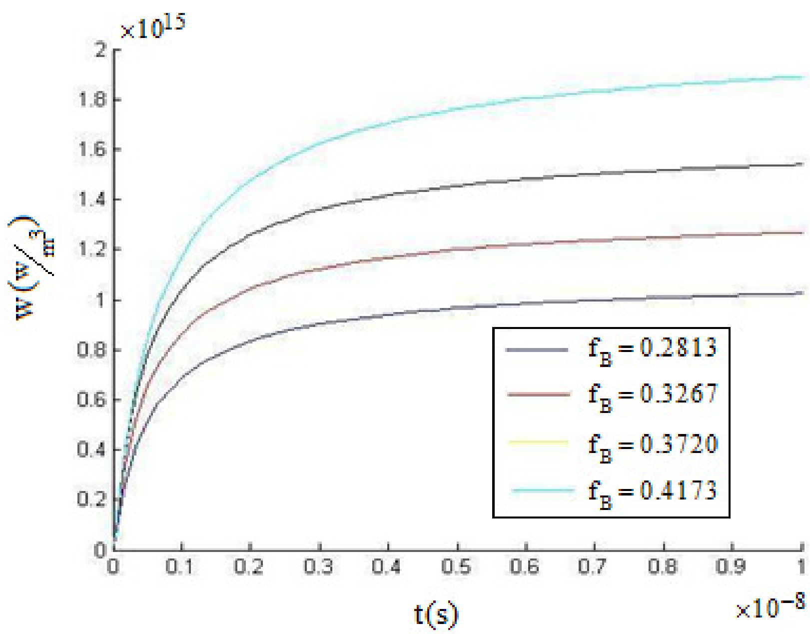

Fig. 20. Variations of energy density versus time at several values of f_B

From this figure you can see that at each time by increasing f_B energy density also increased.

6. STUDY ON THE TEMPERATURE EFFECT ON THE P-^{11}B FUSION REACTION

Another point that should be referred to in proton-boron fusion is the effect of temperature changes in terms of time, to achieve this goal, by doing time derivative from equation n_i(x)=n_P(x)+n_B(x)+3n_α(x) we have .

$$\frac{dn_i}{dt} = \frac{dn_p}{dt} + \frac{dn_B}{dt} + 3\frac{dn_\alpha}{dt} \tag{26}$$

By inserting equations (1),(2),(3), in this relation we have:

$$\frac{dn_i}{dt} = S_{PB} + n_p n_B \langle \sigma v \rangle_{P-^{11}B} - \frac{n_p + n_B}{\tau} - \frac{3n_\alpha}{\tau_\alpha} \tag{7}$$

Where $S_{PB} = S_P + S_B$. However, we know that energy density for a system with density n and temperature T, is:

$$w = {}^{3}/_{2} nT \tag{28}$$

where is T is in Kev. So the plasma energy density is given by:

$$W = \frac{3}{2}\left[n_i T_i + (n_p + n_B + n_\alpha)T_j\right] \tag{29}$$

In which T_i is ionic temperature and T_j is the temperature of proton, boron and alpha particles. By derivation of this equation respect to time we obtain:

$$\frac{dW}{dt} = \frac{3}{2}\left[\frac{dn_i}{dt}T_i + n_i\frac{dT_i}{dt} + \left(\frac{dn_p}{dt} + \frac{dn_B}{dt} + \frac{dn_\alpha}{dt}\right)T_j + (n_D + n_T + n_\alpha)\frac{dT_j}{dt}\right] \tag{30}$$

and with assuming that quantity $\Theta = \frac{T_j(x)}{T_i(x)}$ is constant and using relation

$$\frac{dw}{dt} = \frac{3}{2}T_j\left((1+\frac{1}{\Theta})\frac{dn_i}{dt} - \frac{dn_\alpha}{dt}\right) + \frac{3}{2}\frac{dT_j}{dt}\left((1+\frac{1}{\Theta})n_i - n_\alpha\right) \tag{31}$$

With combination above equation and equation (25) we obtain

$$\frac{dT_j}{dt} = \frac{1}{\frac{3}{2}\left(1+\frac{1}{\Theta} - f_\alpha\right)n_i}(p_\alpha - p_{brems} - p_{loss}) - \frac{T_j}{\left(1+\frac{1}{\Theta} - f_\alpha\right)n_i} \times \left((1+\frac{1}{\Theta})\frac{dn_i}{dt} - \frac{dn_\alpha}{dt}\right) \tag{32}$$

First from solving this equation T_j is given then by substituting T_j into relation $T_i = \frac{T_j}{\Theta}$, we can obtain T_i. In Fig. 21, variations of temperature of particles in terms of time and f_B is shown.

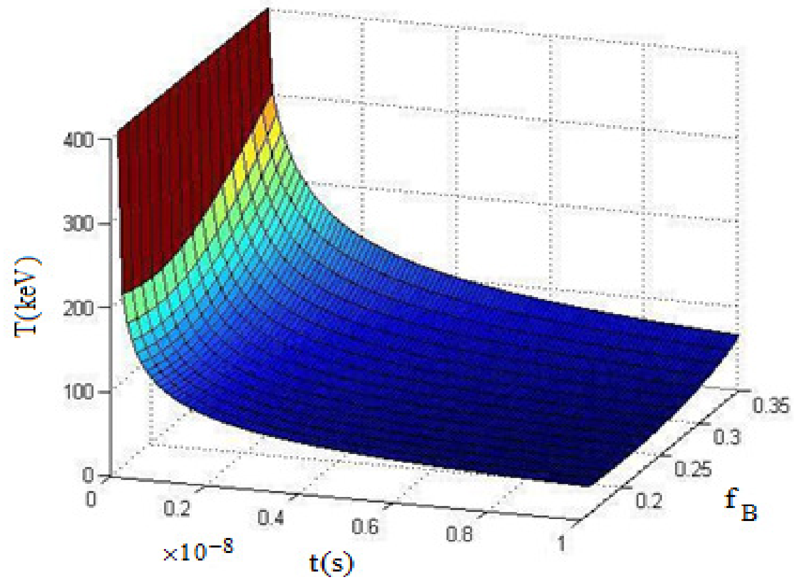

Fig. 21. Three dimensional variations of temperature of particles in terms of time and f_B

From this figure we see that at a certain time by increasing f_B the temperature of particles is increased. Because by enhancement f_B boron consumption fraction in the plasma is increased and this is the cause of enhancement of number of fusion reactions. Thus by increasing fusion rate more fusion energy is released in the plasma and finally increases the temperature of the system.

7. CONCLUSION

In this paper, by solving the balance equation of P-^{11}B fusion reaction in terms of time we see that by decreasing boron consumption fraction, density, power and energy density of plasma are decreased. Also at a fixed f_B by increasing time, the plasma density is reduced and therefore the minimum value of ρR parameteris decreased at certain value of f_B and T by increasing η_H. Also at a fixed value of f_B and $\eta_H$by increasing temperature the minimum value of ρR parameter is increased. Also we see that the energy gain is a function of f_B, ρR, η_H and T. And the optimum value of energy gain at $f_B = 0.1247$, $f_p = 0.1253$, $\varepsilon = 0.9947$ and $T = 63.59\,\mathrm{keV}$ nearly is equal to 20.

COMPETING INTERESTS

Authors have declared that there are no competing interests.

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