Journal of Scientific Research & Reports
3(15): 2110-2117, 2014; Article no. JSRR.2014.15.010

SCIENCEDOMAIN *international*
www.sciencedomain.org

A New Method for Decision Making Based on Soft Matrix Theory

Zhiming Zhang[1*]

[1]College of Mathematics and Computer Science, Hebei University, Baoding 071002, Hebei Province, P.R. China.

Author's contribution

This whole work was carried out by the author ZZ.

Short Research Article

Received 30th March 2014
Accepted 28th May 2014
Published 3rd July 2014

ABSTRACT

Aims: The aim of this paper is to provide a note on "Soft matrix theory and its decision making".
Study Design: In a recent paper [N. Cagman, S. Enginoglu, Soft matrix theory and its decision making, Computers and Mathematics with Applications 59 (10) (2010) 3308-3314], Çaðman and Enginoðlu constructed a soft max-min decision making method which selected optimum alternatives from the set of the alternatives.
Place and Duration of Study: In this paper, we show by an example that Çagman and Enginoglu's method is very likely to get an empty optimum set.
Methodology: Furthermore, we present a new approach to soft set based decision making.
Results: We give an illustrative example to show the advantage of the developed method.
Conclusion: The developed method in this paper can effectively improve the method proposed in Cagman and Enginoglu' paper.

Keywords: Soft sets; soft matrix; products of soft matrices; soft max-min decision making.

**Corresponding author: E-mail: zhimingzhang@ymail.com;*

1. INTRODUCTION

The soft set theory, originally proposed by Molodtsov [1], is a general mathematical tool for dealing with uncertainty. Since its appearance, soft set theory has a wide application in many practical problems, especially the use of soft sets in decision making. Maji and Roy [2] first introduced the soft set into the decision making problems with the help of rough sets [3]. By using a new definition of soft set parameterization reduction, Chen et al. [4] improved the soft sets based decision making in [2]. Çaðman and Enginoðlu [5] defined soft matrices and constructed a soft max-min decision making method which selected optimum alternatives from the set of the alternatives. It should be noted that the Çaðman and Enginoðlu's method has its inherent limitation. There exist some soft set based decision problems in which Çaðman and Enginoðlu's method is very likely to get an empty optimum set. The purpose of this paper is to point out the limitation of Çaðman and Enginoðlu's method by using an example. Moreover, to overcome this limitation, we present a new approach to soft set based decision making problems and give an illustrative example.

2. PRELIMINARIES

In the current section, we will briefly recall the notions of soft sets [1] and soft matrices [5]. Throughout this paper, let U be an initial universe of objects and E the set of parameters in relation to objects in U. Parameters are often attributes, characteristics, or properties of objects. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2.1 [5]. A soft set (f_A, E) on the universe U is defined by the set of ordered pairs

$$(f_A, E) = \{(e, f_A(e)) : e \in E, f_A(e) \in P(U)\},$$

where $f_A : E \to P(U)$ such that $f_A(e) = \varnothing$ if $e \notin A$.

Definition 2.2 [5]. Let (f_A, E) be a soft set over U. Then a subset of $U \times E$ is uniquely defined by

$$R_A = \{(u, e) : e \in A, u \in f_A(e)\}$$

which is called a relation form of (f_A, E). The characteristic function of R_A is written by

$$\chi_{R_A} : U \times E \to \{0,1\}, \ \chi_{R_A}(u, e) = \begin{cases} 1, & (u,e) \in R_A, \\ 0, & (u,e) \notin R_A. \end{cases}$$

If $U = \{u_1, u_2, \cdots, u_m\}$, $E = \{e_1, e_2, \cdots, e_n\}$ and $A \subseteq E$, then the R_A can be presented by a Table as in the following form

R_A	e_1	e_2	$\cdots$	e_n
u_1	$\chi_{R_A}(u_1,e_1)$	$\chi_{R_A}(u_1,e_2)$	$\cdots$	$\chi_{R_A}(u_1,e_n)$
u_2	$\chi_{R_A}(u_2,e_1)$	$\chi_{R_A}(u_2,e_2)$	$\cdots$	$\chi_{R_A}(u_2,e_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\chi_{R_A}(u_m,e_1)$	$\chi_{R_A}(u_m,e_2)$	$\cdots$	$\chi_{R_A}(u_m,e_n)$

If $a_{ij}=\chi_{R_A}(u_i,e_j)$, we can define a matrix

$$\left[a_{ij}\right]_{m\times n}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

which is called an $m\times n$ soft matrix of the soft set (f_A,E) over U.

According to this definition, a soft set (f_A,E) is uniquely characterized by the matrix $\left[a_{ij}\right]_{m\times n}$. It means that a soft set (f_A,E) is formally equal to its soft matrix $\left[a_{ij}\right]_{m\times n}$. The set of all $m\times n$ soft matrices over U will be denoted by $SM_{m\times n}$.

Definition 2.3 [5]. Let $\left[a_{ij}\right],\left[b_{ik}\right]\in SM_{m\times n}$. Then *And*-product of $\left[a_{ij}\right]$ and $\left[b_{ik}\right]$ is defined by

$$\wedge : SM_{m\times n}\times SM_{m\times n}\to SM_{m\times n^2}, \quad \left[a_{ij}\right]\wedge\left[b_{ik}\right]=\left[c_{ip}\right]$$

where $c_{ip}=\min\left\{a_{ij},b_{ik}\right\}$ such that $p=n(j-1)+k$.

Definition 2.4 [5]. Let $\left[a_{ij}\right],\left[b_{ik}\right]\in SM_{m\times n}$. Then *Or*-product of $\left[a_{ij}\right]$ and $\left[b_{ik}\right]$ is defined by

$$\vee : SM_{m\times n}\times SM_{m\times n}\to SM_{m\times n^2}, \quad \left[a_{ij}\right]\vee\left[b_{ik}\right]=\left[c_{ip}\right]$$

where $c_{ip}=\max\left\{a_{ij},b_{ik}\right\}$ such that $p=n(j-1)+k$.

Definition 2.5 [5]. Let $\left[a_{ij}\right],\left[b_{ik}\right]\in SM_{m\times n}$. Then *And-Not*-product of $\left[a_{ij}\right]$ and $\left[b_{ik}\right]$ is defined by

$$\overline{\wedge} : SM_{m\times n}\times SM_{m\times n}\to SM_{m\times n^2}, \quad \left[a_{ij}\right]\overline{\wedge}\left[b_{ik}\right]=\left[c_{ip}\right]$$

where $c_{ip}=\min\left\{a_{ij},1-b_{ik}\right\}$ such that $p=n(j-1)+k$.

Definition 2.6 [5]. Let $\left[a_{ij}\right],\left[b_{ik}\right]\in SM_{m\times n}$. Then *Or-Not*-product of $\left[a_{ij}\right]$ and $\left[b_{ik}\right]$ is defined by

$$\underline{\vee}:SM_{m\times n}\times SM_{m\times n}\to SM_{m\times n^2},\quad \left[a_{ij}\right]\underline{\vee}\left[b_{ik}\right]=\left[c_{ip}\right]$$

where $c_{ip}=\max\left\{a_{ij},1-b_{ik}\right\}$ such that $p=n\left(j-1\right)+k$.

3. ÇAĐMAN AND ENGINOĐLU'S METHOD AND ITS LIMITATION

In [5], Çaðman and Enginoðlu constructed a soft max-min decision making (*SMmDM*) method by using soft max-min decision function. The method selected optimum alternatives from the set of the alternatives. In the current section, we introduce the Çaðman and Enginoðlu's method and show its limitation by an example.

Definition 3.1 [5]. Let $\left[c_{ip}\right]\in SM_{m\times n^2}$, $I_k=\left\{p:\exists i,c_{ip}\neq 0,\left(k-1\right)n<p\leq kn\right\}$ for all $k\in I=\left\{1,2,\cdots,n\right\}$. Then soft max-min decision function, denoted Mm, is defined as follows

$$Mm:SM_{m\times n^2}\to SM_{m\times 1},\quad Mm\left[c_{ip}\right]=\left[\max_{k\in I}\left\{t_k\right\}\right]$$

where

$$t_k=\begin{cases}\min\limits_{p\in I_k}\left\{c_{ip}\right\}, & \text{if } I_k\neq\varnothing,\\ 0, & \text{if } I_k=\varnothing.\end{cases}$$

The one column soft matrix $Mm\left[c_{ip}\right]$ is called max-min decision soft matrix.

Definition 3.2 [5]. Let $U=\left\{u_1,u_2,\cdots,u_m\right\}$ be an initial universe and $Mm\left[c_{ip}\right]=\left[d_{i1}\right]$. Then a subset of U can be obtained by using $\left[d_{i1}\right]$ as in the following way

$$\mathrm{opt}_{\left[d_{i1}\right]}\left(U\right)=\left\{u_i:u_i\in U,d_{i1}=1\right\}$$

which is called an optimum set of U.

By using above definitions, Çaðman and Enginoðlu constructed a *SMmDM* method by the following algorithm.

Algorithm 3.1 [5].

Step 1: Choose feasible subsets of the set of parameters,
Step 2: construct the soft matrix for each set of parameters,
Step 3: find a convenient product of the soft matrices,
Step 4: find a max-min decision soft matrix,
Step 5: find an optimum set of U.

It is worth noting that Çaðman and Enginoðlu's method has its inherent limitation. There exist some soft set based decision problems in which Algorithm 3.1 is very likely to get an empty optimum set. To illustrate this limitation, we consider the following example.

Example 3.1. Suppose that a married couple, Mr. X and Mrs. X, come to the real estate agent to buy a house. Assume that $U=\{u_1,u_2,u_3,u_4,u_5\}$ is a set of five houses under the consideration of Mr. X and Mrs. X to purchase, which may be characterized by a set of parameters $E=\{e_1,e_2,e_3,e_4\}$. For $i=1,2,3,4$, the parameters e_i stand for "expensive", "beautiful", "located in the green surroundings" and "convenient traffic", respectively. Mr. X and Mrs. X consider set of parameters, $A=\{e_1,e_2,e_4\}$ and $B=\{e_1,e_3,e_4\}$, respectively, to evaluate the candidates. After a careful evaluation, Mr. X and Mrs. X construct the following two soft matrices over U according to their own parameters, respectively,

$$\left[a_{ij}\right]=\begin{bmatrix}0&1&0&1\\1&1&0&0\\1&1&0&0\\0&0&0&0\\1&0&0&1\end{bmatrix}\qquad\left[b_{ik}\right]=\begin{bmatrix}1&0&1&0\\0&0&0&1\\1&0&0&1\\0&0&1&1\\0&0&1&1\end{bmatrix}$$

Following we shall select a house by using the *SMmDM* method. Here, we use *And*-product since both Mr. X and Mrs. X's choices have to be considered. We can obtain a product of the soft matrices $\left[a_{ij}\right]$ and $\left[b_{ik}\right]$ by using *And*-product as follows

$$\left[a_{ij}\right]\wedge\left[b_{ik}\right]=\begin{bmatrix}0&0&0&0&1&0&1&0&0&0&0&0&1&0&1&0\\0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0\\1&0&0&1&1&0&0&1&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&1&1&0&0&0&0&0&0&0&0&0&0&1&1\end{bmatrix}$$

We can find a max-min decision soft matrix as

$$Mm\left(\left[a_{ij}\right]\wedge\left[b_{ik}\right]\right)=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

Finally, we can find an empty optimum set of U according to $Mm\left(\left[a_{ij}\right]\wedge\left[b_{ik}\right]\right)$

$$\operatorname{opt}_{Mm\left(\left[a_{ij}\right]\wedge\left[b_{ik}\right]\right)}(U)=\varnothing .$$

Following let us analyze the algorithm 3.1. Suppose that $U=\{u_1,u_2,\cdots,u_m\}$ is a set of m objects and $E=\{e_1,e_2,\cdots,e_n\}$ is a set of parameters. Let $\left[a_{ij}\right],\left[b_{ik}\right]\in SM_{m\times n}$. $\left[c_{ip}\right]$ is a product of $\left[a_{ij}\right]$ and $\left[b_{ik}\right]$. It is clear that $\operatorname{opt}_{Mm\left(\left[c_{ip}\right]\right)}(U)=\left\{u_i \in U \,\middle|\, \exists k \in \{1,2,\cdots,n\}, I_k \neq \varnothing, \text{and } \forall p \in I_k, c_{ip}=1\right\}$. Thus, we can obtain that $\operatorname{opt}_{Mm\left(\left[c_{ip}\right]\right)}(U)$ is a nonempty set if and only if there exist an object $u_i \in U$ and $k \in \{1,2,\cdots,n\}$ such that $I_k \neq \varnothing$ and $c_{ip}=1$ for all $p \in I_k$. It is easy to see that the condition, under which $\operatorname{opt}_{Mm\left(\left[c_{ip}\right]\right)}(U)$ is a nonempty set, is so restrictive that it may limit the application of algorithm 3.1 in some practical problems. In other words, Çagman and Enginoglu's method is very likely to get an empty optimum set in some decision making problems.

4. A NEW APPROACH TO SOFT SET BASED DECISION MAKING

To overcome the limitation of the algorithm 3.1, in the current section we shall present a new approach to soft set based decision making problems. This approach is based on the following concept called the union of soft matrices.

Definition 4.1 [5]. Let $\left[a_{ij}\right],\left[b_{ij}\right]\in SM_{m\times n}$. Then the soft matrix $\left[c_{ij}\right]$ is called the union of $\left[a_{ij}\right]$ and $\left[b_{ij}\right]$, denoted $\left[a_{ij}\right]\tilde{\cup}\left[b_{ij}\right]$, if $c_{ij}=\max\left\{a_{ij},b_{ij}\right\}$ for all i and j.

Algorithm 4.1.

Step 1: Input the (resultant) soft matrices $\left[a_{ij}\right]$ and $\left[b_{ij}\right]$.
Step 2: Compute the union $\left[c_{ij}\right]$ of $\left[a_{ij}\right]$ and $\left[b_{ij}\right]$, where $\left[c_{ij}\right]=\left[a_{ij}\right]\tilde{\cup}\left[b_{ij}\right]$.
Step 3: Compute the choice value $c_i=\sum_{j=1}^{n}c_{ij}$, $i=1,2,\cdots,m$.
Step 4: The optimal decision is to select u_k if $c_k=\max_{1\le i\le m}c_i$.
Step 5: If k has more than one value then any one of u_k may be chosen.

To illustrate this idea, let us reconsider the example 3.1.

Example 4.1. Let $U=\left\{u_1,u_2,u_3,u_4,u_5\right\}$ be a set of five houses under the consideration. The parameter sets $E=\left\{e_1,e_2,e_3,e_4\right\}$, $A=\left\{e_1,e_2,e_4\right\}$ and $B=\left\{e_1,e_3,e_4\right\}$. Two soft matrices $\left[a_{ij}\right]$ and $\left[b_{ik}\right]$ are shown as the example 3.1. The union of $\left[a_{ij}\right]$ and $\left[b_{ik}\right]$ are given as follows.

$$\left[a_{ij}\right]\tilde{\cup}\left[b_{ik}\right]=\begin{bmatrix}1&1&1&1\\1&1&0&1\\1&1&0&1\\0&0&1&1\\1&0&1&1\end{bmatrix}.$$

Following we can compute the choice value c_i $(1\le i\le 5)$ as follows:

Table 1. Choice values

U	**Choice values**
u_1	$c_1 = 4$
u_2	$c_2 = 3$
u_3	$c_3 = 3$
u_4	$c_4 = 2$
u_5	$c_5 = 3$

From the above Table 1, it is clear that the maximum choice value is $\max_{1 \le i \le 5}\{c_i\} = c_1 = 4$. Therefore, according to the algorithm 4.1, u_1 could be selected as the optimal house.

Maji and Roy [5] presented an approach to soft set based decision making. It is noted that Maji and Roy's method differs from our method. Maji and Roy's method works only for a soft set, in contrast, our method can work for several soft sets. Our method first performs an union operation on these soft sets and then compute the choice value of each object from the associated soft set. Therefore, our paper has a broader range of applications than Maji and Roy's method.

5. CONCLUSION

In this paper, we show by an example that Çagman and Enginoglu's method in [5] is very likely to get an empty optimum set. Furthermore, we present a new approach to soft set based decision making and give an illustrative example.

ACKNOWLEDGEMENTS

The authors thank the anonymous referees for their valuable suggestions in improving this paper. This work is supported by the National Natural Science Foundation of China (Grant Nos. 61073121 and 61375075) and the Natural Science Foundation of Hebei Province of China (Grant Nos. F2012201020 and A2012201033).

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Molodtsov D. Soft set theory-first results. Computers & Mathematics with Applications. 1999;37:19-31.
2. Maji PK, Roy AR. An application of soft sets in a decision making problem. Computers & Mathematics with Applications. 2002;44:1077-1083.
3. Pawlak Z. Rough sets. International Journal of Computer & Information Sciences. 1982;11:341-356.

4. Chen D, Tsang ECC, Yeung DS, Wang X. The parameterization reduction of soft sets and its applications. Computers & Mathematics with Applications. 2005;49:757-763.
5. Çaðman N, Enginoðlu S. Soft matrix theory and its decision making. Computers & Mathematics with Applications. 2010;59:3308-3314.

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sciencedomain.org/review-history.php?iid=578&id=22&aid=5174