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Mathematical Model of Projective Transformations for Image Normalization Based on the Properties of Projective Geometry

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

This paper focuses on the development of new mathematical model for image normalization base on the projective geometry. For this purpose several decompositions of full projective group and their subgroups were developed that allow technical realization such as: displacement, compressions, oblique displacements, turns, and perspective transformations. The developed decompositions had been proved through perspective transformation which binds the affine and projective geometry using properties of the perspective transformations.

Keywords: Centroaffine; image normalization; transformation group; centroaffine transformation.

1. INTRODUCTION

The task of creating mathematical models for digital image processing attracts researchers; because of the growing number of practical problems that use images, such as robotics, satellite images, medicine, remote sensing etc [1]. In general, images are differ by the presence of geometric transformations [2], so the most appropriate approach to deal with image perception and analysis is the process of normalization, which will help to improve the

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perception of the object by reducing it to a known correlation procedure or comparison features, where such procedure can reduced time complexity to hundreds of times [2,3].

Existing normalization methods of projective transformations are related to the parallel normalization, that is, the parameters of normalizers are determined in a single step [4]. Parallel normalization, in particular includes four points and a method based on the solution of pairs of algebraic quadratic formulas [5]. However, the acceptable time complexity of this method does not make it acceptable for use in practical problems; because its technical implementation needs major improvement [6]. When normalizing using projective transformations we have to find four points in the input image that must be reflexed in the reference image, three of which are collinear, which is also a rather complicated problem [7,8]. In this regard, the urgent task is to find methods of normalization of projective transformations using successive procedure that is sequentially find all the parameters of the projective group, with properties that meet only the projective transformation [9].

The normalization can be defined as a procedure of compensation of geometric transformations that linking inputted image with distorted images and the set of geometric transformations, known as transformation group [8], that forming the difference between the inputted and the distorted image.

Typically, when processing real images, it may not be known beforehand which geometric transformations it could be. Since all simple geometric transformations and their combination *x* contain an affine transformation, so author will assume that investigating the actual image is influenced by the affine transformation group G_a.

Displacement compensation is quite simple to accomplish [10,11], for example, we can find the image center, then carry out the operation of centering, after combining the center of image with the geometric center of the field of the sight. After centering the image it can be investigated under the influence of only centroaffine group G_a^c which can be represented as a real square matrix [12]:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad (1)$$

Where $a_{11}a_{22} - a_{21}a_{12} \neq 0$.

Thus, to normalize the image, it is necessary to determine in advance the unknown parameters of centroaffin transformation, after which the process of normalization can be presented by the reverse mapping centroaffin described by the inverse matrix A^{-1}.

2. DECOMPOSITION OF CENTROAFFINE TRANSFORMATION MATRIX

The parameters of centroaffin matrix can be determined by the decomposition of the matrix *A* in product of matrices of simple groups of geometric transformations [13,14], where each of which is a subgroup of G_a^c. This combined transformation must satisfy the following requirements:

- It should be a group,
- Between the parameters of the combined transformation and the parameters $a_{11}, a_{12}, a_{21}, a_{22}$ must exist unambiguous, i.e., the parameters] must be expressed through the parameters of the combined transformation, and vice versa

In general, any mapping can be described by a real square matrix *A*, represents a combination of self-adjoint (in our case - symmetric) and orthogonal mapping [15].

$$A = A_s A_{ort} \quad (2)$$

Where

A_s – Symmetric mapping matrix;
A_{ort} – Orthogonal mapping matrix.

After the transition to the basis of the eigenvectors of a symmetric mapping A_s then we can get the following:

$$H^{-1} A_s H = L \quad (3)$$

Where

H – Orthogonal matrix consisting of eigenvectors matrix A_s
L – Real diagonal matrix,

Rewrite formula (3) in another form, we obtain:

$$A_s = HLH^{-1} \quad (4)$$

Where $A = HLH^{-1}A_{ort}$

Since the matrix H is orthogonal and, consequently, H^{-1} also orthogonal, then, if we denote $H = K, H^{-1}A_{ort} = L$, then we will get the following:

$$A = KLP \quad (5)$$

Where

L — Real diagonal matrix
R, K — Orthogonal matrix

Let us take into consideration the case when K, L — pure rotation matrix U_1 and U_2 , in addition to L — matrix of an inhomogeneous rescaling. Then $A = U_1 D U_1$, which can be represented by the following:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \gamma & 0 \\ 0 & \mu \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix} \quad (6)$$

Where

α — Parameter transformation rotation U_2
β — Parameter transformation rotation U_1
D — Rescaling matrix
γ, μ — Parameters of the diagonal transformation D.

From the matrix relation (6) we can proceed to four equations:

$$\begin{aligned} a_{11} &= \gamma\cos\alpha\cos\beta - \mu\sin\alpha\sin\beta \\ a_{12} &= \gamma\cos\alpha\cos\beta + \mu\sin\alpha\cos\beta \\ a_{21} &= -\gamma\sin\alpha\cos\beta - \mu\cos\alpha\sin\beta \\ a_{22} &= -\gamma\sin\alpha\sin\beta - \mu\cos\alpha\cos\beta \end{aligned} \quad (7)$$

Equations (7) can be rewritten as:

$$\begin{aligned} \gamma\cos\beta &= a_{11}\cos\alpha - a_{12}\sin\alpha \\ \gamma\sin\beta &= a_{12}\cos\alpha - a_{22}\sin\alpha \\ -\mu\sin\beta &= a_{11}\sin\alpha + a_{21}\cos\alpha \\ \mu\cos\beta &= a_{12}\sin\alpha + a_{22}\cos\alpha \end{aligned} \quad (8)$$

Solving these equations for the parameters α, β, γ and μ, we obtain:

$$\begin{aligned} tg2\alpha &= \frac{-2(a_{11}a_{21} + a_{12}a_{22})}{(a_{11})^2 + (a_{12})^2 - (a_{21})^2(a_{22})^2}, \\ tg2\beta &= \frac{2(a_{11}a_{12} + a_{21}a_{22})}{(a_{11})^2 + (a_{21})^2 - (a_{12})^2(a_{22})^2}, \\ \gamma &= \frac{a_{11}\cos\alpha - a_{21}\sin\alpha}{\cos\beta}, \\ \mu &= \frac{a_{12}\sin\alpha + a_{22}\cos\alpha)}{\cos\beta}, \end{aligned} \quad (9)$$

This verifies that the transformation $U_2 D U_1$ is a centroaffine group.

The condition of associativity will be executed on the basis of the properties of matrices, i.e., $(U_2 D)U_1 = U_2(DU_1)$.

Identity transformation is $U_2 D U_1$ with the parameters $\alpha = 0, \beta = 0, \gamma = \mu = 1$.

$(U_2 D U_1)^{-1} = U_1^{-1} D^{-1} U_2^{-1}$. Since $\det U_1 \neq 0, \det D \neq 0, \det U2 \neq 0$, then the inverse exists.

As a result of superposition transformations of $U_2 D U_1$ and $U_4 D U_3$ should get a transformation consisting of rotation of U_1^*, diagonal U_2^* and another rotation U_2^* which can be represented in the following transformation: $U_2^* D^* U_1^*$, this shows that the parameters of the resulting transformation $\alpha^*, \beta^*, \gamma^*$ and μ^* are:

$$\begin{aligned} tg2\alpha^* &= \frac{-2(a_{11}^* a_{21}^* + a_{12}^* a_{22}^*)}{(a_{11}^*)^2 + (a_{12}^*)^2 - (a_{21}^*)^2 - (a_{22}^*)^2}, \\ tg2\beta^* &= \frac{-2(a_{11}^* a_{12}^* + a_{21}^* a_{22}^*)}{(a_{11}^*)^2 + (a_{21}^*)^2 - (a_{12}^*)^2 - (a_{22}^*)^2}, \\ \gamma^* &= \frac{a_{11}^*\cos\alpha^* - a_{21}^*\sin\alpha^*}{\cos\beta^*}, \\ \mu^* &= \frac{a_{12}^*\sin\alpha^* + a_{22}^*\cos\alpha^*)}{\cos\beta^*}, \end{aligned} \quad (10)$$

Where

$a_{11}^*, a_{12}^*, a_{21}^*, a_{22}^* -$ Parameters of the matrix A^* equal to $(U_2 D_1 U_1) * (U_4 D_2 U_3)$ i.e., it can be expressed by the terms $\alpha_1, \beta_1, \gamma_1, \mu_1$of $(U_2 D_1 U_1)$ transformation and the terms $\alpha_2, \beta_2, \gamma_2, \mu_2$ of $(U_4 D_2 U_3)$ transformation.

As all conditions are satisfied then $(U_4 D_2 U_3)$ is a group and since between the parameters $\alpha, \beta, \gamma, \mu$ and the parameters of the centroaffine matrix $a_{11}, a_{12}, a_{21}, a_{22}$ there exists a mutual unambiguity, then the combined transformation $U_2 D U_1$ is a centroaffine group and, therefore, it contains all subgroups of centroaffine transformation, as well as all their combinations [16].

But the decomposition $U_2 D U_1$ was not too convenient in practice, since there are ambiguities and uncertainties, even with some simple transformations [17]. Ambiguity, for example, occurs when $\gamma = \mu$, which means the same scale in all direction. In this case, there is no sense in the second rotation. Consequently, the angle of one of rotations must be equal to zero. But there is an ambiguity, which consists in the fact that zero may be equal to the angle of first and second rotation. Moreover, with the transformations of rotation, uniform scale, their combination and some other transformations in the first two formulas (10-1,10-2) arises uncertainty of the form $\frac{0}{0}$.

There are also restrictions on the parameters α and β from the first two formulas, where α and β must not equal to $\frac{\pi}{4} \mp \frac{\pi}{2} n$ [18].

Therefore, the use of the specified decomposition for the normalization will cause difficulties. Let’s consider other decomposition, assume that the decomposition of the centroaffine set will be a combined transformation of DXU, where $D -$ the matrix of the inhomogeneous scale, $X -$ matrix of a slanting shift along the X axis and $U -$ rotation matrix. Then $A = DXU$, which can be rewritten as follow:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \gamma & 0 \\ 0 & \mu \end{bmatrix} \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\cos\alpha & \cos\alpha \end{bmatrix} \quad (11)$$

From formula (11) we can write four equations:

$$\begin{aligned} a_{11} &= \gamma(\cos\alpha - h\sin\alpha)\,, \\ a_{12} &= \gamma(\sin\alpha - h\cos\alpha)\,, \\ a_{21} &= -\sin\alpha\,, \\ a_{22} &= \mu\cos\alpha\,. \end{aligned} \quad (12)$$

Solving these equations regarding $\alpha, h, \gamma, \mu,$ we obtain:

$$\begin{aligned} \alpha &= arctg\left(-\frac{a_{21}}{a_{22}}\right) \\ \mu &= \frac{a_{22}}{\cos\alpha} = \frac{a_{22}}{\cos\left(arctg\left(-\frac{a_{21}}{a_{22}}\right)\right)} \\ h &= \frac{a_{21}a_{11} + a_{22}a_{12}}{a_{11}a_{22} - a_{21}a_{21}} \\ \gamma &= \frac{\mu a_{11}}{a_{22} + a_{21}h} = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{22}} \cos\left(arctg\left(-\frac{a_{21}}{a_{22}}\right)\right) \end{aligned} \quad (13)$$

Verifying the performance of requirements to the group, we conclude that the combined transformation DXU is also centroaffine. Therefore, all simple groups, for example, conversions of slanting shift on X axis, on Y axis, the transformation of rotation, transformation of inhomogeneous scale, as well as all their combinations will be subgroups of decomposition DXU, i.e. $X = DXU, Y = DXU, U = DXU, D = DXU,$ … etc. at defined values of the parameters α, β, μ, h. So, $Y = DXU$ only when the parameters of DXU are equal:

$$\begin{aligned} \mu &= \frac{1}{\cos(arctg(-h^*))} \\ \alpha &= arctg(-h^*) \\ h &= h^* \\ \gamma &= \cos(arctg(-h^*)) \end{aligned} \tag{14}$$

Where

$h^* -$ Parameter of transformation of slanting shift along an X axis for which transformation matrix looks like $\begin{bmatrix} 1 & 0 \\ h^* & 1 \end{bmatrix}$.

Suppose further that $A = YDX$, then:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ h_2 & 1 \end{bmatrix} \begin{bmatrix} \gamma & 0 \\ 0 & \mu \end{bmatrix} \begin{bmatrix} 1 & h_1 \\ 0 & 1 \end{bmatrix} \tag{15}$$

Rewrite (15) in the form:

$$\begin{aligned} a_{11} &= \gamma \\ a_{12} &= \gamma h_2 \\ a_{21} &= \gamma h_1 \\ a_{22} &= \gamma h_1 h_2 + \mu \end{aligned} \tag{16}$$

Express the parameters γ, h_1, h_2, μ through $a_{11}, a_{12}, a_{21}, a_{22}$, we obtain:

$$\begin{aligned} \gamma &= a_{11} \\ h_1 &= \frac{a_{21}}{a_{11}} \\ h_2 &= \frac{a_{12}}{a_{11}} \\ \mu &= \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11}} \end{aligned} \tag{17}$$

Based on formula (17) and the results of performance requirements group for the decomposition of YDX we find that YDX is also a decomposition of the matrix A.

Since the decomposition of DXU and YDX are centroaffine, then they are equivalent to each other, and the combined transformation of YDX can be decomposed into DXU, and vice versa, in other words $YDX = DXU$ only when the parameters $\alpha, \beta, \gamma, \mu$ of DXU decomposition equal to:

$$\begin{aligned} \alpha &= arctg\left(-\frac{\gamma^* h_1^*}{\gamma^* h_1^* h_2^* + \mu^*}\right) \\ \mu &= -\frac{\gamma^* h_1^*}{\sin \alpha} \\ \gamma &= \frac{\gamma^*}{\cos \alpha - \sin \alpha h} \\ h &= \frac{h_2^* \cos \alpha - \sin \alpha}{\cos \alpha + h_2^* \sin \alpha} \end{aligned} \tag{18}$$

Where $\gamma^*, \mu^*, h_1^*, h_2^* -$ parameters of YDX decomposition which has the form $\begin{bmatrix} \gamma^* & \gamma^* h_2^* \\ \gamma^* h_1^* & \gamma^* h_1^* h_2^* + \mu \end{bmatrix}$.

But $DXU = YDX$ only when the parameters of YDX decomposition will have the following values:

$$\begin{aligned} \gamma^* &= \gamma(\cos \alpha - h \sin \alpha) \\ h_2^* &= \frac{\sin \alpha + h \cos \alpha}{\cos \alpha - h \sin \alpha} \\ h_1^* &= -\frac{\mu \sin \alpha}{\gamma(\cos \alpha - h \sin \alpha)} \\ \mu^* &= \frac{\mu}{\cos \alpha - h \sin \alpha} \end{aligned} \tag{19}$$

Not every superposition of elementary transformations D, Y, X, U is suitable for the decomposition of the matrix A [18,19]; only the superposition which is a group and contains four parameters, with one-one correspondence with the parameters $a_{11}, a_{12}, a_{21}, a_{22}$ may be a centroaffine decomposition of the matrix A. If these requirements are not met, then such combined transformation is not a decomposition of the matrix A, for example, the combined transformation YXU, YUX are not a decomposition of matrix A.

Depending on the decomposition of matrix A and the properties of the affine transformation it is possible to find the parameters of a simple transformation which are included in the decomposition of matrix A by using one-dimensional normalization method.

This normalization cannot be used to calculate the normalization parameters for the whole image $B(x, y)$, it just can be only used to calculate its restriction $b(\xi)$ on some straight lines [20,21], which is much easier.

Calculating the parameters of one-dimensional normalization and using the decomposition of the matrix A, we obtain the parameters for the normalization of the entire image, i.e., the parameters of the inverse matrix A^{-1}.

We know that the centroaffine transformation of the center of the image remains fixed [3,22]. Since affine transformation translates straight line into straight line [23], then any line passing through the center coordinates under centroaffine transformation will be translated into straight line.

Thus, each line passing through the coordinates (0, 0), corresponds to the inputted images obtained from the centroaffine transformation.

Appropriate straight lines of inputted and distorted images will differ in the angle of slope, while the restriction $b_0(\xi), b(\xi)$ of inputted and distorted images on these straight lines will differ in the scale [24].

To find the parameters of one-dimensional normalization it is necessary to calculate the angle of slope and the scaling factor between the appropriate straight lines.

3. EXAMPLE OF USAGE

Here is an example of one-dimensional normalization and the resulting decomposition of centroaffine matrix into superposition simple transformation to find the parameters of normalized matrix.

Let's take DXU as a decomposition of normalized matrix A, and calculate the parameters of simple transformations, involved in this decomposition, using the methods of one-dimensional normalization.

Fig. 1 shows the inputted image, while Fig. 2 shows the distorted image obtained by acting on the inputted image using any of centroaffine transformation of A.

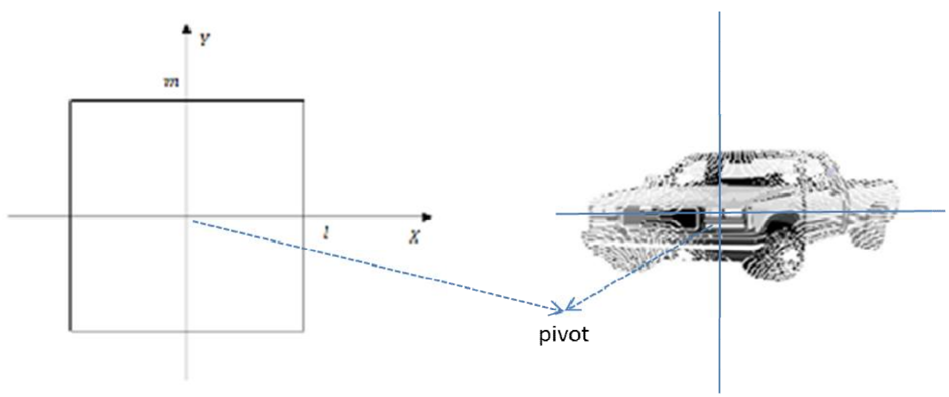

Fig. 1. Inputted image

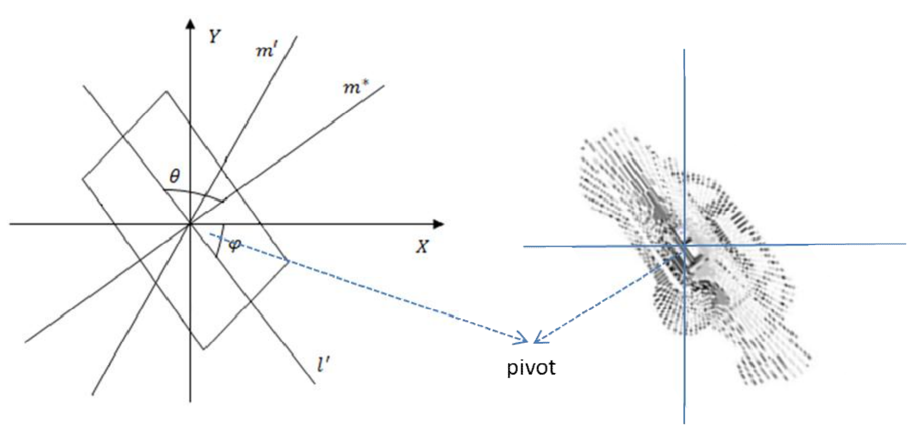

Fig. 2. Distorted image by the centroaffin transformation of A

In Fig. 1, for example, if we select two perpendicular lines l and m, coordinate X and Y axes, we must find the corresponding lines in Fig. 2 $l^{'}$ and $m^{'}$, and calculate the following four parameters:

- $k_1 -$ Scaling factor between the constrains on the line l and $l^{'}$;
- $k_2 -$ Scaling factor between the constrains on the line m and $m^{'}$;
- $\varphi -$ Angle between the old position of line l, (in our case, the X axis, and $l^{'}$;
- $\theta -$ Angle between the line m^{*}, perpendicular to the line $l^{'}$.

So we can use the found parameters $\varphi, \theta, k_1, k_2$, defined by the parameters a, h, γ, μ of simple transformations in the decomposition of DXU. If the perpendicular line l and m in the inputted image were chosen coordinate axes X and Y, then:

$$\begin{aligned} \gamma &= \frac{l}{k_1} \\ \mu &= \frac{l}{(k_2 \cos\theta)} \\ \alpha &= -\varphi \\ h &= -tg\theta \end{aligned} \qquad (20)$$

By substituting (20) in equation (12), we obtain the parameters of normalized matrix A.

4. CONCLUSIONS

The normalization can be understood as the influence on the image obtained by the resulting matrix A. In other words, the simultaneous compensation of all transformations will form the difference between the benchmark and real images.

We can also apply a consistent normalization consistently to calculate the parameters $\gamma, \mu, \alpha, \beta$, and alternately having compensated transformation U, X, D.

In conclusion, we note that all the decompositions of matrix A are equivalent to each other, but in terms of some practical implementation, they are not equal. The criterion for selecting the most suitable can be: ease of implementation, restrictions on the allowed value of parameter of transformation, interference protection, etc.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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