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Image Zooming Algorithms Based on Granular Computing with l_∞-norm

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Authors' contributions

This work was carried out in collaboration between all authors. Author CL designed the study, performed the analysis of experimental results and wrote the first draft of the manuscript. Authors CL, JY and HL managed the analyses of the study and literature searches. All authors read and approved the final manuscript.

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ABSTRACT

The granular computing with l_∞-norm is used to zoom the image. Firstly, a granule is represented by l_∞-norm and has the form of hypercube. Secondly, the bottle-up computing model is adopted to transform the microcosmic world into the macroscopic world by the designed join operation between two hypercube granules. The proposed granular computing is used to zoom the image and achieves the super-resolution image for the input low-resolution image. Experimental results show that the granular computing with l_∞-norm reduces the error between the original image and the reconstructed super-resolution image compared with bicubic interpolation and sparse representation.

Keywords: Super-resolution; Image reconstruction; granular computing; l_∞-norm.

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1. INTRODUCTION

Image zooming is an image processing technology of the high resolution version from a low resolution image [1]. The image zoom in and out in the digital image processing plays a very important role, in order to meet the special application situation or get a better visual effect, for example to highlight some of the details, they usually need an effectively change the size of the image, the image is magnified and reduced when there is still a high quality. A variety of interpolation technology is a common method of image magnification [2]. The image processing theory and commonly existing is the interpolation method for image zooming in and out, such as translation repeated interpolation, bilinear interpolation and spline interpolation etc [3,4]. These interpolation methods have many advantages for image interpolation [5], and also achieved good results.

Granular computing (GrC) is a transformation method between the universe and the parts, and widely used in pattern recognition, information system, etc. L.A. Zadeh identified three fundamental concepts of the human cognition process, namely, granulation, organization, and causation [6,7]. In recent years, the granular computing is the wide attention of people in the view of set theory. The granule is induced by the training datum, the transformation between two granules is realized by the operation between two granules. The relation between two granules is compounded by the positive valuation function of granules [8-10].

2. GrC CLUSTERING WITH l_∞-NORM

For the data set $S=\{\boldsymbol{x}_i|i=1,2,...,n\}$ in N-dimensional space, GrC algorithm is formed in terms of the following steps. Firstly, the representation method of granule is proposed. Secondly, operations between two granules are designed. Thirdly, the fuzzy inclusion relation between two granules is measured by fuzzy inclusion measure. Finally, the GrC algorithms are designed by operations between two granules.

2.1 Representation of Granules

A granule is represented as the set including the points which distances from the center are less than or equal to the given threshold, so a granule can be represented as the form of vector $\boldsymbol{G}=(\boldsymbol{C},R)$, where $\boldsymbol{C}$ is the center of granule, R is radii of granule, and refers to the granularity of granule $\boldsymbol{G}$ which is measured by the maximal distance between center and the data included in granule. Particularly, a point $\boldsymbol{x}$ is represented by a atomic granule with the center $\boldsymbol{x}$ and granularity 0 in N-dimensional space. The distance between center $\boldsymbol{C}=(c_1,c_2,...,c_N)$ and datum $\boldsymbol{x}=(x_1,x_2,...,x_N)$ can be defined as follows

$$d_p(\boldsymbol{x},\boldsymbol{C})=||x-C||_p=((x_1-c_1)^p+(x_2-c_2)^p+...+(x_N-c_N)^p)^{1/p}$$

The distance between two points denotes Manhattan distance and called as l_1-norm when p=1, and the corresponding granule has the form hyperdiamond in N-dimensional space. The distance between two points denotes Euclidean distance and called as l_2-norm when p=2, and the corresponding granule has the form hypersphere in N-dimensional space. The distance between two points denotes Chebyshev distance and called as l_∞-norm when $p=\infty$, and the corresponding granule has the form hypercube in N-dimensional space.

2.2 Operations between Two Hypercube Granules

The operations between two hypercube granules reflect the transformation between macroscopic and microcosmic of human cognitions. When a person want to observe the object more carefully, the object is partitioned into some suitable sub-objects, namely the universe is transformed into some parts in order to study the object in detail in the view of microscopic. Conversely, there is the same attributes of some objects, we regard the objects as a universe to simple the process in the view of macroscopic. The operations between two hypercube granules are designed to realize the transformation between macroscopic and microscopic. Set-based models of granular structures are special cases of lattice-based models, where the lattice join operation $\vee$ coincides with set union operation $\cup$ and lattice meet operation $\wedge$ coincides with set intersection operation $\cap$.

The aim of join operation $\vee$ of two sets S1 and S2 is to obtain the minimal closure which is a set with the minimal granularity and including S1 and S2. In GrC by l_∞-norm, the join operation between two hypercube granules G1 and G2 is to obtain the join hypercube granule G with minimal granularity and including G1 and G1. Any points are regarded as atomic hypercube granules which are indivisible, the join process are the key to obtain the larger granules compared with atomic granules.

The key issue of the join hypercube granule is to determine the center and the l_∞-distance between center and the farthest point from the center.

For two hypercube granules $\boldsymbol{G}_1=(\boldsymbol{C}_1, R_1)$ and $\boldsymbol{G}_2=(\boldsymbol{C}_2, R_2)$ in N-dimensional space, suppose the join hypercube granule is

$$G=G_1 \vee G_2=(C, R)$$

Firstly, the vector from $\boldsymbol{C}_1$ to $\boldsymbol{C}_2$ and vector from $\boldsymbol{C}_2$ to $\boldsymbol{C}_1$ are computed. If $\boldsymbol{C}_1=\boldsymbol{C}_2$, then $\boldsymbol{C}_{12}=0$ and $\boldsymbol{C}_{21}=0$. If $\boldsymbol{C}_1 \neq \boldsymbol{C}_2$, then $\boldsymbol{C}_{12}=(\boldsymbol{C}_2-\boldsymbol{C}_1)/d(\boldsymbol{C}_1,\boldsymbol{C}_2)$ and $\boldsymbol{C}_{21}=(\boldsymbol{C}_1-\boldsymbol{C}_2)/d(\boldsymbol{C}_2,\boldsymbol{C}_1)$.

Secondly, the crosspoints of $\boldsymbol{G}$ and $\boldsymbol{G}_1$ are $\boldsymbol{P}_1=\boldsymbol{C}_1-\boldsymbol{C}_{12}R_1$ and $\boldsymbol{P}_2=\boldsymbol{C}_1+\boldsymbol{C}_{12}R_1$. The crosspoints of $\boldsymbol{G}$ and $\boldsymbol{G}_2$ are $\boldsymbol{Q}_1=\boldsymbol{C}_2-R_2\boldsymbol{C}_{21}$ and $\boldsymbol{Q}_2=\boldsymbol{C}_2+R_2\boldsymbol{C}_{21}$.

Thirdly, the center C and granularity R of the join hypercube granule $\boldsymbol{G}$ is computed by algorithm 1.

Algorithm 1. Computing C and R of join hypercube granule G between G_1 and G_2

```
Input: G1=(C1,R1) and G2=(C2,R2)
Output: G=(C,R)
if R1>=R2
    if d(C1,C2)<=R1-R2   C=C1             R=R1
    else                 C=(P1+Q1)/2      R=d∞(P1,Q1)/2
    end
else
    if d(C1,C2)<=R2-R1   C=C2             R=R2
    else                 C=(P1+Q1)/2      R=d∞(P1,Q1)/2
    end
end
```

Fig. 1 shows the join process of the hypercube granule $\boldsymbol{G}_1$ = [0.2 0.15 0.1] and the hypercube granule $\boldsymbol{G}_2$ = [0.3 0.1 0.06]. The cross points between hypercube granules $\boldsymbol{G}_1$ and the line crossing vector $\boldsymbol{C}_{12}$=[1,-0.5] and the cross points between hypercube granule $\boldsymbol{G}_2$ and the line crossing vector $\boldsymbol{C}_{21}$=[-1 0.5], which $l\infty$-norm distance is maximal, are $\boldsymbol{P}$=[0.1, 0.2] and $\boldsymbol{Q}$=[0.36,0.07]. According to algorithm1, the central vector and granularity of the join hypercube granule $\boldsymbol{G}$ are $\boldsymbol{C}$=[0.23,0.135] and $\boldsymbol{R}$=0.13, namely $\boldsymbol{G}$=[0.23 0.135 0.13].

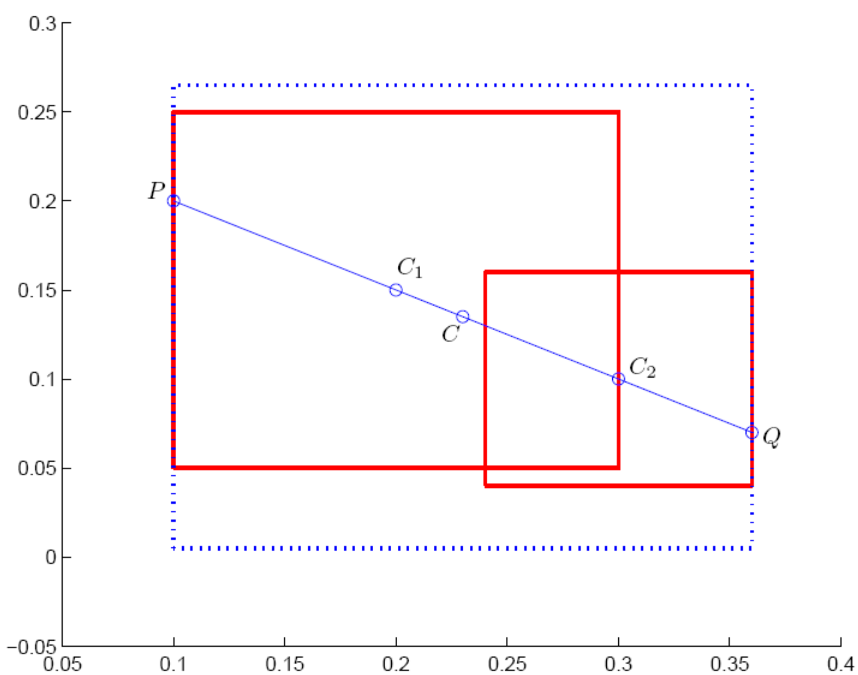

Fig. 1. The join hypercube granule of two hypercube granules

2.3 GrC Clustering with λ_∞-norm

For data set *S*, the granular computing clustering algorithms are proposed by the following steps. Firstly, the samples are used to form the atomic granule. Secondly, the threshold of granularity is introduced to conditionally union the atomic granules by the aforementioned join operation, and the granule set is composed of all the join granules. Thirdly, if all atomic granules are included in the granules of *GS*, the join process is terminated, otherwise, the second process is continued. The GrC clustering algorithms are described as follows.

Suppose the atomic hypercube granules induced by *S* are g_1, g_2, g_3, g_4, g_5. The training process can be described as the following tree structure shown in Fig. 2, leafs denote the atomic hypercube granules, root denotes *GS* including its child nodes G_1, G_2, and g_3. G_1 is induced by join operation of child nodes g_1 and g_2, G_2 is the join hypercube granule of g_4 and g_5, g_3 is the atomic hypercube granule. The whole process of obtaining *GS* is the bottle up process.

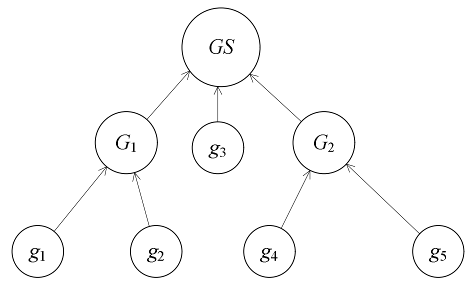

Fig. 2. The framework of GrC

The GrC framework is described as algorithm 2.

Algorithm 2. GrC clustering process
Input: Data set *S*, threshold ρ of granularity
Output: Granule set *GS*

S1. initialize the granule set $GS=\varnothing$
S2. i=1
S3. for the ith sample $\boldsymbol{x}_i$ in *S*, form the corresponding atomic granule $\boldsymbol{G}_i$
S4. j=1
S5. form the join granule $\boldsymbol{G}_i \vee \boldsymbol{G}_j$ of $\boldsymbol{G}_i$ and $\boldsymbol{G}_j \in GS$, if the granularity of $\boldsymbol{G}_i \vee \boldsymbol{G}_j$ is less than or equal to ρ, then $\boldsymbol{G}_j=\boldsymbol{G}_i \vee \boldsymbol{G}_j$, else
S6. $j=j+1$
S7. if all the granularities of $\boldsymbol{G}_i \vee \boldsymbol{G}_j$ are greater than ρ, then $GS=GS\cup\{G_i\}$
S8. remove $\boldsymbol{x}_i$ until *S* is empty.

Suppose training set S includes 10 training data in 2-dimensional space, 10 atomic hypercube granules induced by l_∞-norm are shown in Fig. 3(a), the achieved hypercube granule set including 4 hypercube granules are shown in Fig. 3(b) if algorithm 2 is performed and the threshold ρ is set to 0.25.

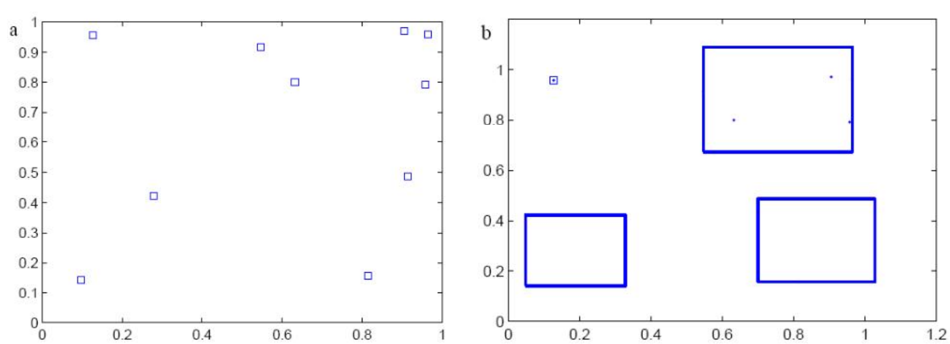

Fig. 3. Numerical example of GrC, (a) atomic hypercube granules, (b) the achieved hypercube granule set (ρ=0.25)

3. EXPERIMENT

91 training images are selected to form the training set which includes 999910 patches extracted from the training images [11]. The reconstruction strategy is adopted to achieve the SR image for the LS image [11]. Four images shown in Fig. 4 are used to evaluate the performance of image zooming algorithm, such as the root mean square error (RMSE) between the SR reconstruction image and the original SR image listed in Table 1, and the input image, original image, and the reconstructed image are shown in Fig. 5, Fig. 6, Fig. 7, and Fig. 8. From the table, we can see the image zooming algorithm based on GrC with l_∞-norm achieves the less RMSE compared with bicubic interpolation and sparse representation. Sparse representation has caused over sparse, and the interpolation method only considers the local information without considering the global information, so the bicubic interpolation and sparse representation are worse than GrC with l_∞-norm.

Table 1. The RMSE of different methods for super-resolution with magnification factor 3, respect to the original images

Images	Bicubic interpolation	Sparse representation	GrC with l_∞-norm
Image a (321×481)	15.5377	12.2445	12.0372 (ρ=0.1)
Image b (321×481)	16.8497	16.5612	16.3122 (ρ=0.1)
Image c (481×321)	16.1873	15.8615	15.3912 (ρ=0.1)
Image d (481×321)	10.0712	9.4427	8.6423 (ρ=0.1)

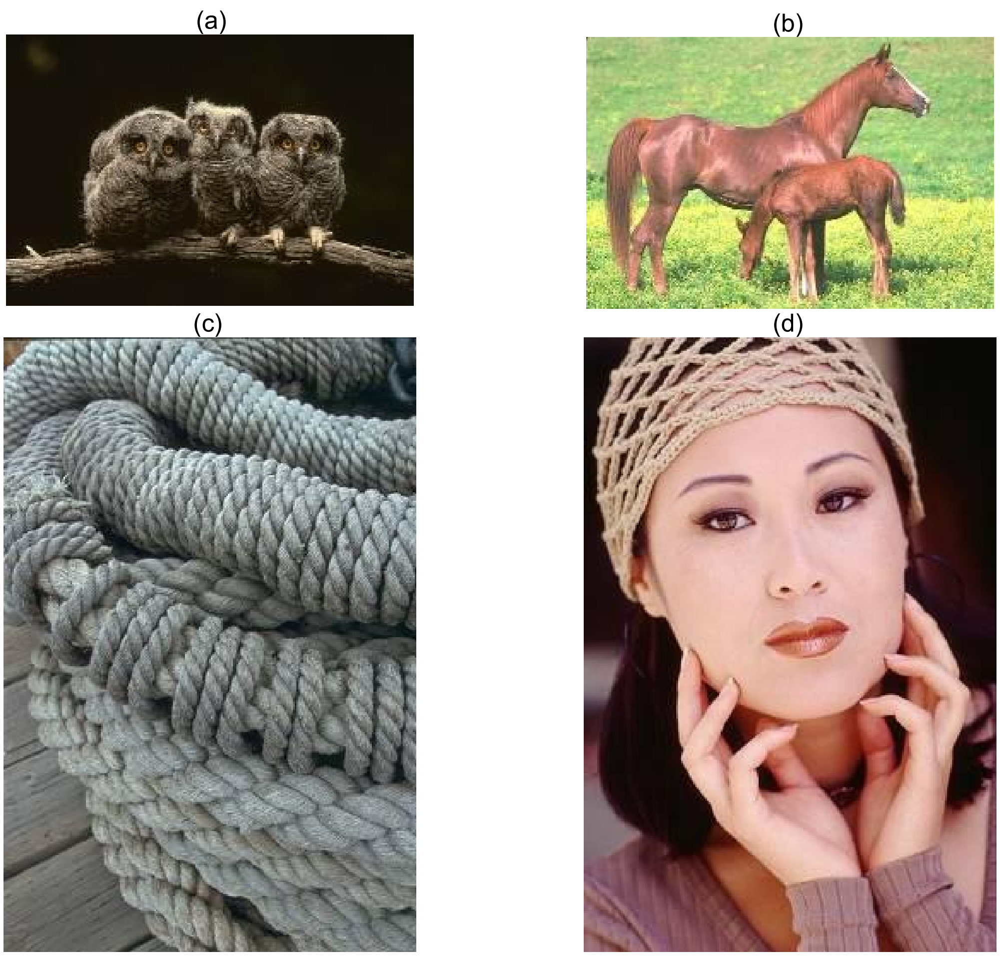

Fig. 4. The four testing images selected from BSD500

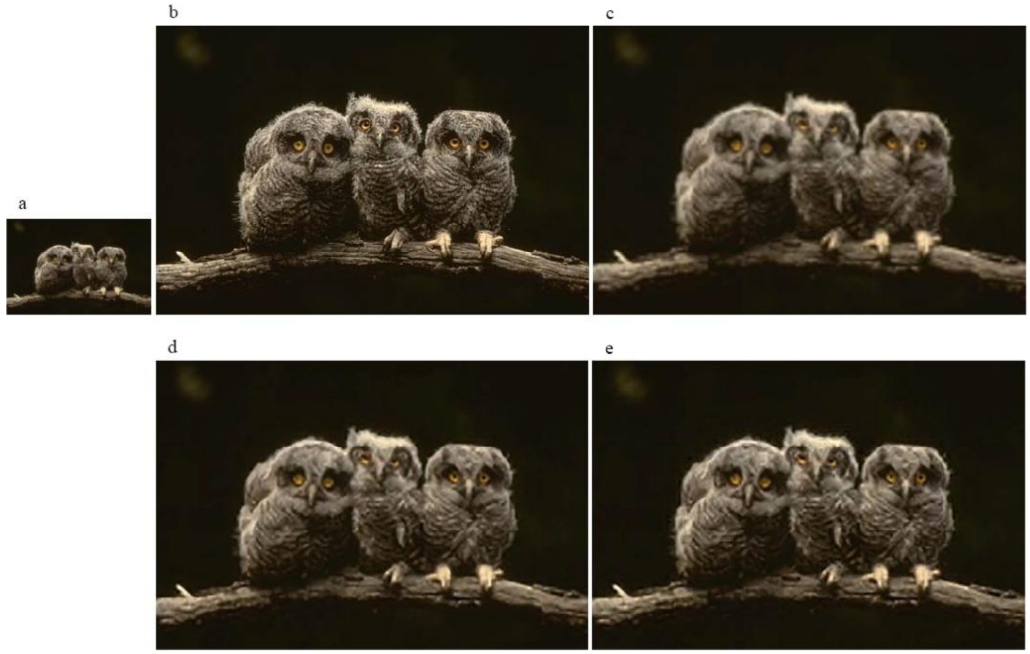

Fig. 5. The image (a) zoomed by a factor of 3. (a) low-resolution image, (b) the original image, (c) SR image by bicubic interpolation, (d) SR image by sparse representation, (e) SR image by GrC

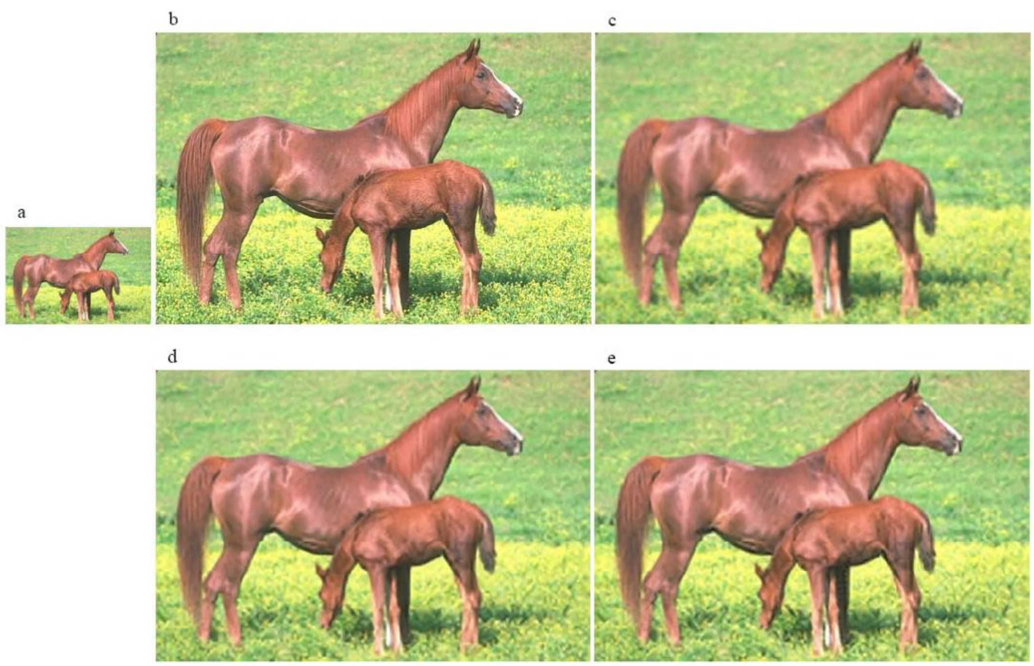

Fig. 6. The image (b) zoomed by a factor of 3. (a) low-resolution image, (b) the original image, (c) SR image by bicubic interpolation, (d) SR image by sparse representation, (e) SR image by GrC

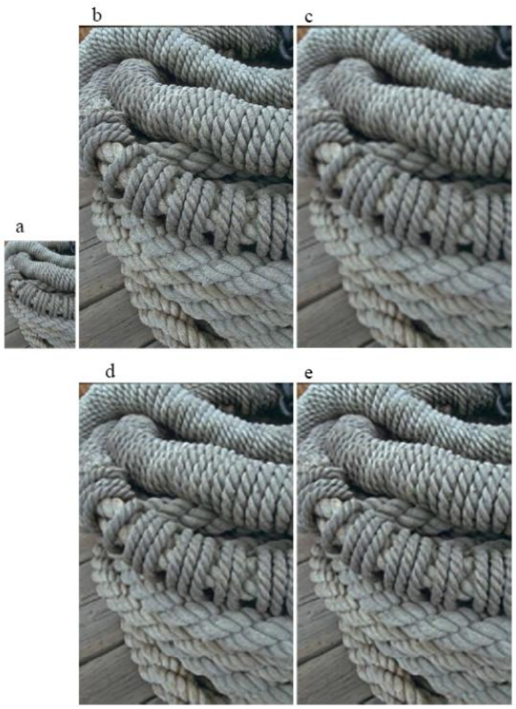

Fig. 7. The image (c) zoomed by a factor of 3. (a) low-resolution image, (b) the original image, (c) SR image by bicubic interpolation, (d) SR image by sparse representation, (e) SR image by GrC

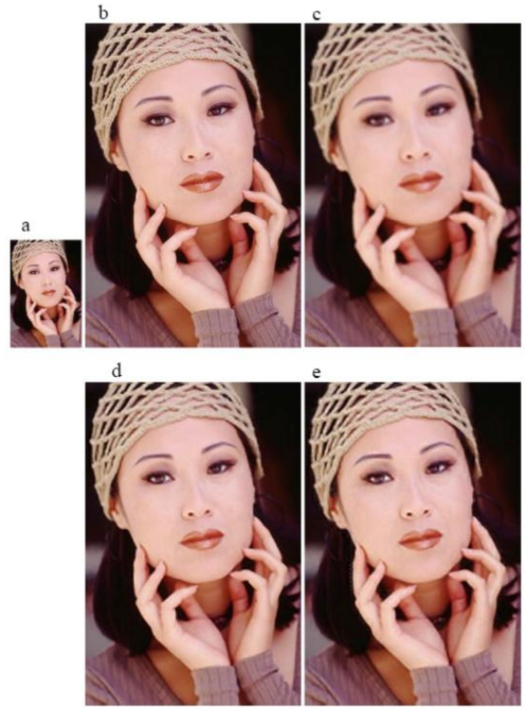

Fig. 8. The image (d) zoomed by a factor of 3. (a) low-resolution image, (b) the original image, (c) SR image by bicubic interpolation, (d) SR image by sparse representation, (e) SR image by GrC

4. CONCLUSION

The image zooming algorithm is proposed based on granular computing with l_∞-norm. The experimental results demonstrate the effectiveness of image zooming via GrC with l_∞-norm compared with bicubic interpolation and sparse representation. However, the effectiveness of super-resolution image by image zooming algorithm is demonstrated by RMSE, the other evaluation methods of SR image reconstruction will be discussed in the future works. To compared with sparse representation, image magnified by a factor of 3 is performed in the paper, and the other magnification factors must retrain the sparse dictionary, it is a time consuming process, and discussed in the future works.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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