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Closed-form American Options Pricing Models on Foreign Assets

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

The closed-form solution for European options on foreign assets did not consider the impact of strike price volatility. Therefore, these models overestimate the value of European options [1,2]. Using the pricing relationships among European, American and Bermudan option proposed by Yan [3-5], the article establishes four kinds closed-form solution for American and Bermudan option pricing models on foreign assets. The article contribution has three aspects. First, the volatility of underlying assets and strike price together determine the value of American options struck in foreign assets or exchange rate. The greater the strike price volatility, the smaller the American options value. Second, if underlying assets volatility is less than the volatility of strike price, the value of these two kinds of American options equals zero. Third, the bigger the correlation coefficient between exchange rate and foreign assets logarithm yields, the greater the value of these two kinds of American options. We can obtain European and Bermudan options pricing models on foreign assets from American options pricing models.

Keywords: American options on foreign assets; European options on foreign assets; Bermudan options on foreign assets; continuous martingale; measure transformation; domestic and foreign currency measure transformation.

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1. INTRODUCTION

The binomial is the mainstream model of American option pricing [6]. Monte Carlo simulation method [7], finite difference methods [8], GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model [9] and semi-infinite linear programming method [10] provides other numerical methods for American options pricing. Geske and Johnson (hereafter GJ) provide an efficient and approximate method to price American options [11]. Ho, Stapleton and Subrahmanyam (hereafter HSS) generalized GJ method [12]. Chung proposed method simplified HSS's three-dimensional solution to a one-dimensional solution [13]. Using these models to price American options the cost is very high.

Early strike European options (warrants) and early termination American options (warrants) are called non-standard options, Yan proposed closed-form non-standard options pricing models [14,15], closed-form dividend American options pricing models [3], closed-form foreign exchange American options pricing models [16], closed-form Bermudian options pricing models, closed-form dividend fractal American options pricing models, and closed-form combinational American options pricing models [4]. Yan established the pricing relationships among American, European and Bermudan options [5], the error between Yan models and binomial is very small [3].

$$c_A = c_E e^{r_d T},\ p_A = c_E e^{r_d T}$$

$$c_B = c_E e^{r_d (T_2 - T_1)},\ p_B = p_E e^{r_d (T_2 - T_1)}$$

Where: c_E is European call options present value; p_E is European put options present value; c_A is the current value of American call options; p_A is the current value of American put options; c_B is the current value of Bermudan call options; p_B is the current value of Bermudan put options; T is the term of options; T_1 is the beginning strike time of Bermudan options; T_2 is the ending strike time of Bermudan options, $0 \le T_1 \le T_2 \le T$; r_d is domestic risk-free interest rate.

The strike price of foreign asset options may be the domestic currency, foreign currency and exchange rates. When the strike price is domestic currency, the strike price is a constant; when the strike price is foreign currency or exchange rates, the strike price is the stochastic process. Reiner proposed four European option pricing method, he ignored the impact on option value of the standard deviation of strike price and the correlation coefficient of underlying asset, and overestimated the value of European options [1,2]. Chung valued American options on foreign assets in a stochastic interest rate using a two-point GJ method [17]. Actually, Yan models are the limit values of the binomial and GJ method when the number of steps and the number of strikes tend to infinity [3,4].

Using measure transformation [18] this article will present closed-form American options pricing models struck price in domestic currency, closed-form quanto American options pricing models, closed-form American options pricing models stuck price in exchange rate, and closed-form American options pricing models struck price in a foreign currency. For quanto American options and American options stuck in a foreign currency we have to change foreign measure into domestic measure [2]. In addition to quanto American options, the larger the correlation coefficient between exchange rate and foreign asset logarithm yields, the greater the value of American options. The larger the volatility of strike price, the smaller the value of American.

The second section will give the pricing models of American options stuck in domestic currency. The third section will propose the pricing models of quanto American options. The fourth section will deduce the pricing models of American options struck price in exchange rate. The fifth section will deduce the pricing model of American options struck price in a foreign currency. The sixth section will present empirical researches that parameters' changes affect on the value of the American options. Finally, concludes the article.

2. OPTIONS STRUCK IN DOMESTIC CURRENCY

If the strike price is domestic currency X_d, the current value of American call options on foreign assets S_T is

$c_A = \mathbb{E}_{\mathbb{Q}} \max(C_T S_T - X_d, 0)$

Where: C_T stands for exchange rate per unit foreign currency at T time.

According measure transformation the arbitrage-free price of stochastic process $C_t S_t$ at time T is

$$C_T S_T = CS \exp[\sigma W_T^* + (r_d - q_f - \frac{1}{2}\sigma^2)T]$$

Where: C is the domestic currency price of per unit foreign currency; S is the current price of foreign assets; σ is the joint distribution standard deviation between foreign assets and exchange rate logarithm yields; r_d is domestic currency risk-free rate, r_f is foreign currency risk-free rate; q_f is the dividend rate of foreign assets; $W_T^* = \phi\sqrt{T}$ is domestic currency measure Brownian motion (or Wiener process), $\phi \sim N(0,1)$, $\mathbb{E}_{\mathbb{Q}}(W_T^*)=0$.

Since the value of American call options on foreign assets is greater than zero, then

$$CS\exp[\sigma\phi\sqrt{T} + (r_d - q_f - \frac{1}{2}\sigma^2)T] - X_d \geq 0$$

The domain of the random variable ϕ at time T is

$$\phi \geq -\frac{\ln(\frac{CS}{X_d}) + (r_d - q_f - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} = a$$

The expectation of American call option on foreign assets is

$$c_A = \frac{1}{\sqrt{2\pi}}\int_a^\infty \{CS\exp[\sigma\phi\sqrt{T} + (r_d - q_f - \frac{1}{2}\sigma^2)T] - X_d\}e^{-\frac{1}{2}\phi^2}d\phi$$

We can immediately obtain American call and put option pricing models.

$$c_A = CSe^{(r_d - q_f)T}N(d_1) - X_d N(d_2)$$

$$p_A = X_d N(-d_2) - CSe^{(r_d - q_f)T}N(-d_1)$$

Where

$$d_{1,2} = \frac{\ln(CS/X_d) + (r_d - q_f \pm \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$\sigma = \sqrt{\sigma_C^2 + \sigma_S^2 + 2\rho\sigma_C\sigma_S}$$

σ_C and σ_S respectively is the standard deviation of exchange rate and foreign assets logarithm yields. The C, S, σ, σ_C, σ_S, r_d, r_f and q_f are all constants.

3. QUANTO OPTIONS

If the strike price is foreign currency, the underlying asset is foreign assets, the exchange rate C is a constant, and the current value of American call options is

$c_A = C\,\mathbb{E}_{\mathbb{Q}}\max(S_T - X_f, 0)$

According measure transformation the arbitrage-free price of stochastic process S_t at time T is

$$S_T = S\exp[\sigma_S W_T^* + (r_f - q_f - \sigma_C\sigma_S - \frac{1}{2}\sigma_S^2)T]$$

Where: $W_T^* = \phi\sqrt{T}$ is domestic currency measure Brownian motion, $\phi \sim N(0,1)$, $\mathbb{E}_{\mathbb{Q}}(W_T^*)=0$.

Since the value of American foreign assets call option is greater than zero, then

$$S\exp[\sigma\phi\sqrt{T} + (r_f - q_f - \sigma_C\sigma_S - \frac{1}{2}\sigma^2)T] - X_f \geq 0$$

The domain of the random variable ϕ at time T is

$$\phi \geq -\frac{\ln(\frac{S}{X_f}) + (r_f - q_f - \sigma_C\sigma_S - \frac{1}{2}\sigma_S^2)T}{\sigma_S\sqrt{T}} = a$$

The expectation of American foreign asset call option is

$$c_A = \frac{Ce^{(r_f - q_f - \sigma_C\sigma_S)T}}{\sqrt{2\pi}}\int_a^\infty [S\exp(\sigma\phi\sqrt{T} - \frac{1}{2}\sigma_S^2 T)]e^{-\frac{1}{2}\phi^2}d\phi - \frac{C}{\sqrt{2\pi}}\int_a^\infty X_f e^{-\frac{1}{2}\phi^2}d\phi$$

We can immediately obtain American quanto call and put option pricing models.

$$c_A = C[Se^{(r_f - q_f - \sigma_C\sigma_S)T}N(d_1) - X_f N(d_2)]$$

$$p_A = C[X_f N(-d_2) - Se^{(r_f - q_f - \sigma_C\sigma_S)T}N(-d_1)]$$

Where

$$d_{1,2} = \frac{\ln(S/X_f) + (r_f - q_f - \sigma_C\sigma_S \pm \frac{1}{2}\sigma_S^2)T}{\sigma_S\sqrt{T}}$$

4. OPTIONS STRUCK IN A FOREIGN CURRENCY

If the strike price of American foreign asset options is foreign currency, the current value of the American call option on foreign assets is

$c_A = \mathbb{E}_{\mathbb{Q}}\max[C_T(S_T - X_f), 0]$

or

$c_A = \mathbb{E}_{\mathbb{Q}}\max[C_T S_T - C_T X_f), 0]$

To facilitate the derivation of American option pricing models, we put stochastic process C_tS_t-C_tX as the underlying asset, the option strike price is zero. The price of the underlying asset at *t* time is

$$G_t = C_t S_t - C_t X_f = CSe^{(\mu_C+\mu_S)t+\sigma W_t} - CX_f e^{\mu_C t+\sigma_C U_t}$$

Where: μ_C and μ_S are the real logarithm yields of exchange rate and foreign assets, they are all constants; $W_t = \phi\sqrt{t}$ is Brownian motion under $\mathbb{P}$ measure; $U_t = \phi\sqrt{t}$ is Brownian motion under $\mathbb{R}$ measure. $\mathbb{P}$ and $\mathbb{R}$ are not equivalent measure. W_t and U_t are all domestic measure Brownian motion.

The current domestic currency price of the underlying assets at *t* time is

$$Z_t = CSe^{(\mu_C+\mu_S-r_d)t+\sigma W_t} - CX_f e^{(\mu_C-r_d)t+\sigma_C U_t}$$

The stochastic differential equation of the stochastic processes Z_t is

$$dZ_t = CS[(\mu_C+\mu_S-r_d+\frac{1}{2}\sigma^2)dt+\sigma dW_t] - CX_f[(\mu_C-r_d+\frac{1}{2}\sigma_C^2)dt+\sigma_C dU_t]$$

If the financial markets have not arbitrage opportunities, the stochastic differential equation must be continuous martingale process.

Assuming W_t^* is the Brownian motion under $\mathbb{P}^*$ measure, U_t^* is the Brownian motion under $\mathbb{R}^*$ measurer. $\mathbb{P}^*$ and $\mathbb{P}$ are equivalent measure, $\mathbb{R}^*$ and $\mathbb{R}$ are equivalent measure. According to measure transformation theorem, let

$$dW_t = dW_t^* - \frac{1}{\sigma}(\mu_C+\mu_S-r_d+\frac{1}{2}\sigma^2)dt$$

$$dU_t = dU_t^* - \frac{1}{\sigma_C}(\mu_C-r_d+\frac{1}{2}\sigma_C^2)dt$$

Where:

$W_t^* = \phi\sqrt{t}$, $U_t^* = \phi\sqrt{t}$, $\phi \sim N(0,1)$, $\mathbb{E}_{\mathbb{P}^*}(W_t^*)=0$, $\mathbb{E}_{\mathbb{R}^*}(U_t^*)=0$.

The martingale process after measure transformation is

$$dZ_t = CS\sigma dW_t^* - CX\sigma_C dU_t^*$$

Use Ito theorem solving the stochastic differential equation, we can get the stochastic process

$$Z_t = CS\exp(\sigma W_t^* - \frac{1}{2}\sigma^2 t) - CX_f\exp(\sigma_C U_t^* - \frac{1}{2}\sigma_C^2 t)$$

The final value of the underlying assets at *t* time is

$$Y_t = e^{(r_d-r_f)t}CSe^{(r_f-q_f)t}\exp(\sigma W_t^* - \frac{1}{2}\sigma^2 t) - e^{(r_d-r_f)t}CX_f\exp(\sigma_C U_t^* - \frac{1}{2}\sigma_C^2 t)$$

Or

$$Y_t = CS\exp[\sigma W_t^* + (r_d-q_f-\frac{1}{2}\sigma^2)t] - CX_f\exp[\sigma_C U_t^* + (r_d-r_f-\frac{1}{2}\sigma_C^2)t]$$

Because the value of American options is greater than zero, so arbitrage-free price of the C_tS_t-C_tX must be greater than zero, the domain of the random variable ϕ at time *T* is

$$\phi \ge -\frac{\ln(\frac{S}{X_f}) + (r_f-q_f-\frac{1}{2}\sigma^2+\frac{1}{2}\sigma_C^2)T}{(\sigma-\sigma_C)\sqrt{T}} = a$$

The value of American call options is

$$c_A = \frac{CSe^{(r_d-q_f)T}}{\sqrt{2\pi}}\int_a^\infty [CS\exp(\sigma\phi\sqrt{T}-\frac{1}{2}\sigma^2 T)]e^{-\frac{1}{2}\phi^2}d\phi - \frac{CXe^{(r_d-q_f)T}}{\sqrt{2\pi}}\int_a^\infty [\exp(\sigma_1\phi\sqrt{T}-\frac{1}{2}\sigma_C^2 T)]e^{-\frac{1}{2}\phi^2}d\phi$$

Or

$$c_A = \frac{CSe^{(r_d-q_f)T}}{\sqrt{2\pi}}\int_a^{\infty} e^{-\frac{1}{2}(\phi-\sigma\sqrt{T})^2}d\phi - \frac{CX_f e^{(r_f-r_f)T}}{\sqrt{2\pi}}\int_a^{\infty} e^{-\frac{1}{2}(\phi-\sigma_C\sqrt{T})^2}d\phi$$

Or

$$c_A = \frac{CSe^{(r_d-q_f)T}}{\sqrt{2\pi}}\int_{a-\sigma\sqrt{T}}^{\infty} e^{-\frac{1}{2}\varphi^2}d\varphi$$

$$-\frac{CXe^{(r_f-r_f)T}}{\sqrt{2\pi}}\int_{a-\sigma_C\sqrt{T}}^{\infty} e^{-\frac{1}{2}\theta^2}d\theta$$

Let $d_1 = -(a-\sigma\sqrt{T})$, and $d_2 = -(a-\sigma_C\sqrt{T})$, then

$$c_A = CSe^{(r_d-q_f)T}N(d_1) - CX_f e^{(r_d-r_f)T}N(d_2)$$

$$p_A = CX_f e^{(r_d-r_f)T}N(-d_2) - CSe^{(r_d-q_f)T}N(-d_1)$$

Where

$$d_{1,2} = \frac{\ln(\frac{S}{X_f}) + [r_f - q_f \pm \frac{1}{2}(\sigma-\sigma_C)^2]T}{(\sigma-\sigma_C)\sqrt{T}}$$

$$\sigma = \sqrt{\sigma_C^2 + \sigma_S^2 + 2\rho\sigma_C\sigma_S}$$

The condition of the above option pricing models established is $\sigma>\sigma_C$. If $\sigma\leq\sigma_C$, the value of European, American and Bermudan option struck in foreign currency is equal to zero. When $\rho<0$, there may be $\sigma\leq\sigma_C$.

5. OPTIONS STRUCK IN EXCHANGE RATE

If the strike price of American options is exchange rate, the current value of American call options is

$c_A= \mathbb{E}_{\mathbb{Q}}\max[(C_T-X_C)S_T,0]$

Or

$c_A= \mathbb{E}_{\mathbb{Q}}\max(C_TS_T- X_CS_T,0)$

To facilitate the derivation of American option pricing models, we put stochastic process C_tS_t- XS_t as the underlying asset, the option strike price is zero. The price of the underlying assets at t time is

$$G_t = C_tS_t - X_CS_t = CSe^{(\mu_C+\mu_S)t+\sigma W_t} - X_CSe^{\mu_S t+\sigma_S\widetilde{V}_t}$$

Where: W_t is Brownian motion under $\mathbb{P}$ measure; V_t is Brownian motion under $\mathbb{Q}$ measure, $\mathbb{P}$ and $\mathbb{Q}$ are not equivalent measure; W_t and V_t are the Brownian motion of domestic currency measure; $\widetilde{V}_t = V_t - \sigma_C t$ is Brownian motion of foreign currency measure. Then

$$G_t = CSe^{(\mu_C+\mu_S)t+\sigma W_t} - X_CSe^{(\mu_S-\sigma_C\sigma_S)t+\sigma_S V_t}$$

The current domestic currency price of the underlying assets at t time is

$$Z_t = CSe^{(\mu_C+\mu_S-r_d)t+\sigma W_t} - X_CSe^{(\mu_S-\sigma_C\sigma_S-r_d)t+\sigma_S V_t}$$

The stochastic differential equation of the stochastic processes Z_t is

$$dZ_t = CS[(\mu_C+\mu_S-r_d+\frac{1}{2}\sigma^2)dt+\sigma dW_t] - X_CS[(\mu_C-\sigma_C\sigma_S-r_d+\frac{1}{2}\sigma_S^2)dt+\sigma_S dV_t]$$

If the financial markets have not arbitrage opportunities, the stochastic differential equation must be continuous martingale process.

Assuming W_t^* is the Brownian motion under $\mathbb{P}^*$ measure, V_t^* is the Brownian motion under $\mathbb{Q}^*$ measurer. $\mathbb{P}^*$ and $\mathbb{P}$ are equivalent measure, $\mathbb{Q}^*$ and $\mathbb{Q}$ are equivalent measure. According to measure transformation theorem, let

$$dW_t = dW_t^* - \frac{1}{\sigma}(\mu_C+\mu_S-r_d+\frac{1}{2}\sigma^2)dt$$

$$dV_t = dV_t^* - \frac{1}{\sigma_S}(\mu_S-\sigma_C\sigma_S-r_d+\frac{1}{2}\sigma_S^2)dt$$

Where: $W_t^* = \phi\sqrt{t}$, $V_t^* = \phi\sqrt{t}$, ϕ ~N(0,1), $\mathbb{E}_{\mathbb{P}^*}(W_t^*)=0$, $\mathbb{E}_{\mathbb{Q}^*}(V_t^*)=0$。

The martingale process after measure transformation is

$$dZ_t = CS\sigma dW_t^* - X_C S\sigma_S dV_t^*$$

Use Ito theorem solving the stochastic differential equation, we can get the stochastic process

$$Z_t = CS\exp(\sigma W_t^* - \frac{1}{2}\sigma^2 t) - X_C S\exp(\sigma_S V_t^* - \frac{1}{2}\sigma_S^2 t)$$

The final value of the underlying assets at t time is

$$Y_t = e^{(r_d - r_f)t} CSe^{(r_f - q_f)t}\exp(\sigma W_t^* - \frac{1}{2}\sigma^2 t) - X_C Se^{(r_f - q_f - \sigma_C\sigma_S)t}\exp(\sigma_S V_t^* - \frac{1}{2}\sigma_S^2 t)$$

Or

$$Y_t = CS\exp[\sigma W_t^* + (r_d - q_f - \frac{1}{2}\sigma^2)t] - X_C S\exp[\sigma_S V_t^* + (r_f - q_f - \sigma_C\sigma_S - \frac{1}{2}\sigma_S^2)t]$$

Because the value of American option is greater than zero, so arbitrage-free price of stochastic process C_tS_t-XS_t must be greater than zero, the domain of the random variable ϕ at T time is

$$\phi \ge -\frac{\ln(\frac{C}{X_C}) + (r_d - r_f + \sigma_C\sigma_S - \frac{1}{2}\sigma^2 + \frac{1}{2}\sigma_S^2)T}{(\sigma - \sigma_S)\sqrt{T}} = a$$

The current value of American call option stuck in exchange rate is

$$c_A = \frac{CSe^{(r_d - q_f)T}1}{\sqrt{2\pi}}\int_a^\infty [\exp(\sigma\phi\sqrt{T} - \frac{1}{2}\sigma^2 T)]e^{-\frac{1}{2}\phi^2} d\phi - \frac{X_C Se^{(r_f - q_f - \sigma_C\sigma_S)T}}{\sqrt{2\pi}}\int_a^\infty [\exp(\sigma_S\phi\sqrt{T} - \frac{1}{2}\sigma_S^2 T)]e^{-\frac{1}{2}\phi^2} d\phi$$

Or

$$c_A = \frac{CSe^{(r_d - q_f)T}}{\sqrt{2\pi}}\int_a^\infty e^{-\frac{1}{2}(\phi - \sigma\sqrt{T})^2} d\phi - \frac{X_C Se^{(r_f - q_f - \sigma_C\sigma_S)T}}{\sqrt{2\pi}}\int_a^\infty e^{-\frac{1}{2}(\phi - \sigma_S\sqrt{T})^2} d\phi$$

Or

$$c_A = \frac{CSe^{(r_d - q_f)T}}{\sqrt{2\pi}}\int_{a-\sigma\sqrt{T}}^\infty e^{-\frac{1}{2}\varphi^2} d\varphi - \frac{X_C Se^{(r_f - q_f - \sigma_C\sigma_S)T}}{\sqrt{2\pi}}\int_{a-\sigma_S\sqrt{T}}^\infty e^{-\frac{1}{2}\theta^2} d\theta$$

Let $d_1 = -(a - \sigma\sqrt{T})$, and $d_2 = -(a - \sigma_S\sqrt{T})$.

Then American call and put option pricing models are respectively

$$c_A = CSe^{(r_d - q_f)T}N(d_1) - X_C Se^{(r_f - q_f - \sigma_C\sigma_S)T}N(d_2)$$

$$p_A = X_C Se^{(r_f - q_f - \sigma_C\sigma_S)T}N(-d_2) - CSe^{(r_d - q_f)T}N(-d_1)$$

Where

$$d_{1,2} = \frac{\ln(\frac{C}{X_C}) + [r_d - r_f + \sigma_C\sigma_S \pm \frac{1}{2}(\sigma - \sigma_S)^2]T}{(\sigma - \sigma_S)\sqrt{T}}$$

$$\sigma = \sqrt{\sigma_C^2 + \sigma_S^2 + 2\rho\sigma_C\sigma_S}$$

The condition of the above option pricing models established is $\sigma>\sigma_S$. If $\sigma\le\sigma_S$, the value of European, American and Bermudan option struck in exchange rate equals to zero. When $\rho<0$, there may be $\sigma\le\sigma_S$.

6. EMPIRICAL RESEARCHES

If the term of an American option is T=1. The current exchange rate is C=1.11, the standard deviation of the exchange rate is σ_C=10%. The current price of foreign stock is S=60, foreign stock dividend payout rate is q_f=0, and the standard deviation of the logarithm yields is σ_S=25%. The risk-free rate of domestic currency is r_d=5%, the risk-free interest rate of foreign currency is r_f=4%. The strike price is domestic

Table 1. The values comparison of American options on foreign assets

Strike prices	X	Strike prices volatility	ρ=-0.5		ρ=0		ρ=0.5	
			c_A	p_A	c_A	p_A	c_A	p_A
Domestic currency	X_d=66.60	σ_d =0.00	7.79	4.37	9.07	5.65	10.29	6.88
Foreign currency	X_S =60.00	σ_C=0.10	4.78	2.03	5.60	2.85	7.28	4.54
Exchange rate	X_C =1.11	σ_S=0.25	0.00	0.00	1.70	0.02	3.18	0.77

currency X_d=66.6, the strike price is foreign currency X_S=60, and the strike price is exchange rate X_C=1.11. The correlation coefficient between exchange rate and foreign stock logarithm yields respectively is ρ=-0.5, ρ=0, ρ=0.5. Calculate the values of the American options.

Solution: In the empirical studies we do not involve quanto American options because its strike price and exchange rate is constant. The values of quanto American options do not relate to the strike price volatility and the correlation coefficient between exchange rate and foreign stock logarithm yields.

If the strike price is domestic currency that is a constant, X_d =66.60, the strike price volatility is equal to zero, σ_d =0.00, and the value of American options is the largest. The larger the correlation coefficient between exchange rate and foreign stock logarithm yields, the greater the value of American options.

If the strike price is foreign currency, X_S =60.00, the strike price is a stochastic process, and its volatility is σ_C=0.10. The larger the volatility of the strike price, the smaller the value of American options. The values of American options are less than formers. American option value is proportional to the correlation coefficient between exchange rate and foreign stock logarithm yields.

If the strike price is exchange rate, X_C=1.11, the strike price is a stochastic process, and its volatility is σ_S=0.25. Because the volatility of strike price is larger than formers, in the case of the correlation coefficient equal，the values of American options are less than the formers. If the correlation coefficient is ρ=-0.5, then the volatility σ=0.2180 of underlying asset is less than the volatility σ_S=0.25 of strike price. So the values of American options are all zero. The values of other American options are shown in Table 1(above).

$$\sigma = \sqrt{\sigma_C^2 + \sigma_S^2 + 2\rho\sigma_C\sigma_S}$$

$$= \sqrt{0.10^2 + 0.25^2 - 2\times 0.5\times 0.10\times 0.25}$$

$$= 0.2180 < \sigma_S = 0.25$$

If the strike price is domestic currency, the volatility of the strike price is zero, and the value of American options is the maximum. If the strike price is foreign currency, the standard deviation of the strike price is in the middle, the values of the American options are also centered. If the strike price is exchange rate, the volatility of the strike price is the largest, and the American option value is the smallest. The greater the correlation coefficient between the exchange rate and foreign assets logarithm yields, the greater the American option value. According to the pricing relationship between European, American and Bermuda option, we can obtain the value of European and Bermudan option from American option value.

7. CONCLUSION

If the logarithm yields of exchange rate prices and foreign assets prices show normal distribution, the joint distribution of the product also follows normal distribution. The strike price of American options on foreign assets may be domestic currency, foreign currency and exchange rate. The volatility of underlying asset and strike price decide the value of American options on foreign assets. If strike price is domestic currency that is a constant, the standard deviation of strike prices is zero, and American options' value is the maximum.

The exchange rate and strike price of quanto American options on foreign assets is a constant. The values of quanto American options do not relate to the strike price volatility and the correlation coefficient between exchange rate and foreign stock logarithm yields. The relationship between foreign and domestic measure leads to American call option value much bigger and American put options value much smaller.

If the strike price of American options on foreign assets is foreign currency, the strike price is the stochastic process of foreign assets. The standard deviation of the stochastic process of foreign assets also affects the value of American options on foreign assets. The greater the standard deviation of strike price, the smaller the value of American options.

If the strike price of American options on foreign assets is exchange rate, the strike price is the stochastic process of exchange rate. Generally, the standard deviation of exchange rate is less than the standard deviation of foreign assets. Therefore, the value of American options struck in exchange rate is less than the value of American options struck in foreign currency.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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