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# Synergies between Machine Learning, Artificial Intelligence, and Game Theory for Complex Decision-Making

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***Authors' contributions***

*This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.*

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## Abstract

The special focus of this paper is to discuss a likely intersection of machine learning, artificial intelligence (AI) technology, and game theory, pointing at the importance of this synthesis both in mathematics and engineering. As these domains develop various means of addressing decision-making problems become more and more sophisticated and can be used in different areas such as economics, security, and social sciences. We will also address selected game-theoretic issues including the concept of decision making in terms of Nash equilibria or in the distinction of games as being cooperative or non-cooperative and how they work in synergy with machine learning approaches bringing in reinforcements and deep learning to leverage forecasting and strategizing. The paper makes references to problem-oriented branches of studies such as

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autonomous systems or market strategies stressing the importance of the novel direction for further studies. Within the scope of machine learning and game theory the goal is to implement better complex models which utilize real world intricacies for enhancing decision making within a populated agent environment.

*Keywords: Machine learning; artificial intelligence; game theory; Nash equilibrium; cooperative games*.

# 1 Introduction

A well-known field of research in science and mathematics is the interface of game theory, artificial intelligence (AI), and machine learning (ML). As each of these fields expands, their interactions will offer novel solutions to complex decision-making problems in a variety of domains, including as the social sciences, information security, and economics. Machine learning is a form of artificial intelligence that makes intelligent systems smarter by using vast amounts of data to identify patterns and make predictions (Jordan & Mitchell, 2015). On the other hand, game theory provides a mathematical framework for assessing the strategic interactions of rational agents, allowing for the modeling of scenarios in which the result is dependent on the choices made by a large number of decision-makers (Von Neumann & Morgenstern, 1944).

Machine learning approaches may serve to inform and augment standard game theory by better understanding players' actions and forecasting results in strategic situations. For example, reinforcement learning makes it possible for agents to devise optimal plans of action simply through trial and error as they adjust their behavior in response to feedback from the environment (Sutton & Barto, 2018). Such a feature is very important, for example, in dynamic games where the set of possible strategies for each player may change based on the actions taken by the other players over time.

Furthermore, gaining more intelligence in our systems by combining AI and game theory helps create intelligent agents that can make decisions under competitive conditions in real time. These agents would then be fitted with ML bounty hunting algorithms that would allow them to observe how their competitors behaved and change how they competed (Wang & Zhang, 2018). This is important in areas such as automated systems for stock exchanges, defense against cybernetic attacks, and in business, strategic maneuvers and planning, (Zhou & Wu, 2023).

The relevance of such multi-faceted geo-spatial strategies is most evident in the recent growth of both AI and Game Theory. It is noted that there is a growing interest to study how ML can be used to address computational issues related to game or economic equilibrium, improve the accuracy of predictions through quantitative measures, and optimize the selection of strategies in various games (Amato & De Santis, 2020). By linking these departments, more effective models corresponding to contemporary reality can be achieved, thereby assisting in the development of intelligent systems which can manage complicated strategic interactions.

The combination of machine learning, artificial intelligence, and game theory is simply the state of the art in mathematics and possibly computational sciences. The concept of this paper includes the combination of various disciplines (ML, AI and GT) and their potential were incorporated, while addressing their benefits, relevance and analysis different players of the game and for future research (Wang & Sun, 2023).

# 2 Literature Review

Yekkehkhany & Nagi (2022) in their paper titled: Risk-averse equilibria for vehicle navigation in stochastic congestion games talked about the requirement of intelligent navigation to manage the uncertainty/stochasticity that exist in network is increasing rapidly specifically for self-driven cars, drones, and many others. They also talked about identifying paths that can accommodate stochastic arc delays is more difficult than the well-known shortest path problem. Three proposed classes of "risk-averse equilibria" for atomic congestion games with load-dependent stochastic delays: - Mean-variance equilibrium (MVE): Reduces variability of path length - Conditional value at risk (CVaR) equilibrium: Minimizes tail risk - Risk-averse equilibrium (RAE): Ensures that the probability of achieving the shortest path is possible where also brought out.

# 3 Key Concepts in Game Theory

## 3.1 Nash equilibriumin game theory

Nash equilibrium is a concept in game theory where the game reaches an optimal outcome. This is a state that gives individual players no incentive to deviate from their initial strategy (Bloembergen et al., 2015). The players know their opponent's strategy and still will not deviate from their initial chosen strategies because it remains the optimal strategy for each player. Machine learning is a useful tool used for addressing Nash equilibria; this can make it difficult to forecast when agents lack sufficient information (Li et al., 2022). A key idea in game theory is the Nash Equilibrium, which is a scenario in which no player can benefit from independently change their strategy as long as the other players' plans stay the same (Bhandari & Chen, 2023;Agrawal & Jaiswal (2012).

In a game with *n* players, let $s_i$ be the strategy chosen by player *i*, and let $s_{-i}$ denote the strategies chosen by all other players. The Nash Equilibrium can be defined mathematically as follows:

A strategy profile $(S_1^*, S_2^*, \dots, S_n^*)$ is a Nash Equilibrium if, for every player *i*:

$$U_i(S_i^*, S_{-i}^*) \geq U_i(S_i', S_{-i}^*) \forall S_i' \in S_i$$

**Where:**

- $U_i$ is the utility function (or payoff) for player *i*.
- $S_i$ is the set of all possible strategies for player *i*.
- $S_i^*$ is the equilibrium strategy for player *i*.
- $S_{-i}^*$is the strategies chosen by all players other than player *i*.

## 3.2 Cooperative and non-cooperative games

Cooperative and non-cooperative games they are the two basic categories into which games in game theory are usually divided. Every category has unique aspect and fallouts for tactics and results. This is a thorough comparison:

### 3.2.1 Cooperative games

Cooperative games are those in which players can form binding agreements and collaborate to achieve better outcomes for all participants (Fudenberg & Tirole, 1991). The focus is on the collective payoff and how to distribute it among the players. Cooperative game theory emphasizes on how players might profit from coalitions and how best to distribute the advantages of cooperation (Osborne & Rubinstein, 1994). One of the fundamental concepts in cooperative games is the Shapley value since it provides a means for the players' efforts to be equally split among them, therefore deciding their overall earnings (Lee & Kim, 2023; Koller & Friedman, 2009).

**Shapley Value Equation**

The Shapley value for a player $i$ in a cooperative game is calculated by:

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!\,(|N| - |S| - 1)!}{|N|!} (v(S \cup \{i\}) - v(S))$$

**Example of a Cooperative Game**

Consider a simple cooperative game with three players A, B and C and a value function *v* define the following

- $v(\phi) = 0$
- $v(\{A\}) = 1$

- $v(\{B\}) = 2$
- $v(\{C\}) = 3$
- $v(\{A, B\}) = 3$
- $v(\{A, C\}) = 4$
- $v(\{B, C\}) = 5$
- $v(\{A, B, C\}) = 6$

With the help of Shapley value formula, each players contributions can be calculated and determine their fair share of the overall gains.

The cooperative game equation, notably the Shapley value, provides an easy mechanism to examine and divide the benefits of collaboration among participants. It underlines the relevance of each player's role while evaluating alternative coalition groupings (Zhang & Zhao, 2023).

### 3.2.2 Non-cooperative games

Non-cooperative games are those in which players make decisions independently and cannot form binding agreements. Every participant acts in his or her interests so they may gain from what they have done. Under non-cooperative game theory, players make their own decisions and the emphasis is on identifying best strategies considering the decisions of others. The Nash Equilibrium is among the fundamental ideas in non-cooperative games (Shoham & Leyton-Brown, 2009).

## 3.3 Nash equilibrium

When no person can gain from changing their approach while the other players keep their strategy, the game is said to be in Nash equilibrium. This shows that, in light of the other players' strategies, each player's approach is ideal.

**Nash Equilibrium Equation:**

The game with $n$ players, each player $i$ has a set of strategies $S_i$, the Nash Equilibrium is:
Let:

- $u_i\ (s_1, s_2, \ldots, s_n)$ be the utility (payoff) function for player $i$ given the strategy profiles $s_1, s_2, \ldots, s_n$

A strategy profile $(s_1^*, s_2^*, \ldots, s_n^*)$ is a Nash Equilibrium if:

$u_i\ (s_1^*, s_2^*, \ldots, s_n^*) \geq u_i\ (s_1^*, \ldots s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*)\ \forall_{si} \in S_i$

**Example of a Non-Cooperative Game:**

Consider a simple game between two players, Alice and Bob, where each has two strategies: Cooperate (C) or Defect (D):

| | **Bob: C** | **Bob: D** |
|---|---|---|
| Alice: C | 2, 2 | 0, 3 |
| Alice: D | 3, 0 | 1,1 |

**Finding the Nash Equilibrium:**

1. **Alice's Best Responses**:
    - If Bob cooperates (C), Alice's best response is to defect (D) (3 > 2).
    - If Bob defects (D), Alice's best response is to defect (D) (1 > 0).
2. **Bob's Best Responses**:
    - If Alice cooperates (C), Bob's best response is to defect (D) (3 > 2).
    - If Alice defects (D), Bob's best response is to defect (D) (1 > 0).

**The Outcome of Nash Equilibrium:**

From the above analysis, the strategy profile (D,D) (D, D)(D,D) is a Nash Equilibrium, as not one player can improve their payoff by changing their strategy single-handedly.

The Nash Equilibrium in non-cooperative games that contribution a framework for appreciating the way players might maximize their strategy in competitive situations. In various domains, including as economics, political science, and behavioral sciences, the formulae and ideas of non-cooperative game theory are fundamental for judging strategic interactions (Tuyls & Parsons, 2007).

**Table 1. Comparison summary**

| Feature | Cooperative Games | Non-Cooperative Games |
|---|---|---|
| **Agreements** | Binding agreements allowed | No binding agreements |
| **Player Interaction** | Players can form coalitions | Players act independently |
| **Outcome Assessment** | Focus on collective payoffs | Focus on individual payoffs |
| **Solution Concepts** | Shapley value, core | Nash Equilibrium |
| **Example** | Joint ventures, cartels | Price competition among firms |

Understanding the distinction between cooperative and non-cooperative games is vital for studying strategic interactions in numerous domains such as economics, political science, and social sciences. Each type of game provides vital insights on how players might attain optimal results based on their capacity (or unwillingness) to interact.

## 3.4 Repeated games

In repeated games, participants engage in the same game numerous times, allowing for strategies that can depend on past actions. Compared to a one-shot game, this structure can have a big impact on the results because each player can use their previous behavior to inform their future methods.

**Key Concepts in Repeated Games**

1. **Payoff Structure**: In repeated games, the total payoff can be a sum of payoffs over each round. For a game repeated TTT times, the total payoff for player 3 is as follow:

$$U_i = \sum_{t=1}^{T} u_i\,(s_{1t,}s_{2t,}\dots, s_{nt,})$$

2. **Discounting Future payoffs:** A discount factor is frequently used in iterative games to lower future payouts to reflect their current worth δ, where 0 < δ < 1. The total payoff is:

$$U_i = \sum_{t=0}^{T-1} \delta u_i\left(s_{1t,}s_{2t,}\dots, s_{nt,}\right)$$

### 3.4.1 Equilibrium in repeated games

Players have the option to use methods in repeated games that depend on their past play. The Subgame Perfect Equilibrium (SPE), a popular equilibrium concept, guarantees that players' strategies create a Nash Equilibrium in each subgame.

**Example of a Repeated Game**

**Consider a simple two-player game where each player can either Cooperate (C) or Defect (D). Below is the matrix:**

| | Player 2: C | Player 2: D |
|---|---|---|
| **Player 1:C** | 3, 3 | 0, 5 |
| **Player 1:C** | 5, 0 | 1,1 |

Compared to one-shot games, the dynamics of play in repeated games are very different, enabling more complicated methods and results, understand how players will optimize their long-term advantage through strategic interactions that requires an understansding of the equations regulating total payoffs and discounting future gains (Wang & Huang, 2023).

## 3.5 Machine learning techniques in game theory

Using a distinct sort of algorithms to analyze methods, each player conduct, and simplify decision-making is important in the integrating machine learning methods into game theory. Below are the principles.

### 3.5.1 Reinforcement learning

In game theory, reinforcement learning is a widely used machine learning approach that allows agents to interact with their surroundings and learn the best course of action.

**Q-Learning Equation**

In Q-learning, the value of a state-action pair is updated using the following the below equation:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\left(r + \gamma\frac{maxQ}{a'}(s',a') - \mathrm{Q(s,a)}\right)$$

### 3.5.2 Nash Q-learning

In multi-agent settings, Q-learning can be extended to account for the strategies of other players leading to Nash Q-learning

**Nash Q-Learning rule**

For a player *i*,

$$Q(s,a) \leftarrow (1{-}\propto)Q_i(s,a) + \alpha\left(r + \gamma\frac{maxQ_i}{a'}(s',a')\right)$$

### 3.5.3 Best response dynamics

In game theory, a best response is a strategy that maximizes a player's payoff given the other players' strategies.

**Best Response Equation**

For player i:

$$BR_i(s_{-i}) = argmax\ u_i(s_i, s_{-i})$$

### 3.5.4 Evolutionary game theory

In evolutionary game theory, strategies evolve over time based on their payoff.

**Replicator Dynamics equation**

The change in the proportion of a strategy $x_i$ canbe modeled as

$$x_i = x_i(u_i - \bar{u})$$

The above named equations, describe how game theory and machine learning methods, especially reinforcement learning, can be used to examine and improve strategic interactions. By taking use of these strategies, players may gradually modify their approach, increasing their understanding of complex games and decision-making techniques.

**3.5.5 Combinations of game theory, artificial intelligence, and machine learning in mathematics**

The convergence of machine learning (ML), artificial intelligence (AI), and game theory (GT) in mathematics presents a rich area of research and application. When these fields work together, they complement each other and produce creative answers for challenging issues. Most of the ideas, highlights, and applications are listed below.

# 4 Foundations of Synergy

## 4.1 Game theory and its relevance

Game theory provides a mathematical framework for analyzing strategic interactions among rational participants, this aid in the understanding how the participants make decisions in competitive and cooperative environments. The concepts include:

- **Nash Equilibrium**: When no player wins by changing their approach while each of the players keeps their plan, this is known as Nash equilibrium.
- **Evolutionary Game Theory (EGT):** Focuses on the evolution of strategies in populations, which is particularly relevant in sciences.

## 4.2 Machine learning and AI

These equations form the backbone of many machine learning and AI algorithms.

**1.** Linear Regression

The linear regression model:

$$y = \beta_0 + \beta_1 x + \epsilon$$

**2.** Logistic Regression

The logistic function used for binary classification

$$P\left(Y = \frac{1}{X}\right) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 X)}}$$

**3.** Cost Function for Linear Regression

$$J(\beta) = \frac{1}{m}\sum_{i=1}^{m}(y_1 - \hat{y}_1)^2$$

**4.** Gradient Descent rule

$$\theta := \theta - \alpha\nabla J(\theta)$$

**5.** Support Vector Machine (SVM)

$$w.x + b = 0$$

**6.** K-means clustering

$$J = \sum_{i=1}^{k}\sum_{x\in C_i}\|x - \mu_i\|^2$$

**7.** Neural Network Output

$$y = \sigma(w.x + b)$$

**8.** Cross-Entropy Loss

$$L = -\frac{1}{m}\sum_{i=1}^{m}[y_i \log(\hat{y}_i) + (1-y_i)\log(1-\hat{y}_i)]$$

The machine learning is a part of Artificial intelligence which has algorithms using that computers can learn how to perform task based on data. The concepts include:

- **Reinforcement Learning (RL):** An example of ML where agent learns to make decisions by receiving rewards or penalties in respect to the actions it performs.
- **Deep Learning** - A neural networks with many layers to model complex patterns in data.

## 4.3 Interconnections between ML, AI, and GT

Integrating concepts from machine learning, artificial intelligence and game theory can lead to interesting mathematical formulations. Below are key equations.

1. **Expected Utility in strategic games**

In game theory, the expected utility of a strategy can be expressed as

$$U(s_i) = \sum_{s_{-i}} P(s_{-i})u_i(s_{i,}s_{-i})$$

2. **Reinforcement Learning with Game Theory**

$$Q_i(s,a) = R_i(s,a) + \gamma \sum_{s'} P(s'|s,a) \max_{a'} Q_{-i}(s',a')$$

3. **Bayesian Game with Machine Learning**

$$E[u_i(s_i\ \theta)] = \sum_{\theta} P(\theta)u_i\big(s_i, s_{-i}(\theta)\big)$$

4. **Evolutionary Game Theory in AI**

$$\dot{x}_i = x_i(f_i(x) - \bar{f}(x))$$

5. **Multi-Agent Optimization**

$$\min_{\theta_i} \mathcal{L}\big(y, f(x;\theta_i)\big) + \sum_{j \neq i} R_{ij}(\theta_i, \theta_j)$$

## 4.4 Learning dynamics

The dynamics of learning in multi-agent environments can be modeled using game theory. For instance:

- **Stochastic Games**: These games incorporate randomness and can be used to model environments where agents learn and adapt over time.
- **Reinforcement Learning Dynamics**: The learning process can be viewed through the lens of game theory, where agents adjust their strategies based on the actions of others Zhang(2020).

## 4.5 Applications

### 4.5.1 Economic and social systems

Game theory combined with machine learning has applications in economics and social sciences, such as:

- **Market Strategies**: Companies can use these models to predict competitor behavior and optimize pricing strategies.
- **Social Dynamics**: Understanding how social norms evolve can be modeled using EGT and ML techniques.

### 4.5.2 Autonomous systems

In autonomous systems, such as self-driving cars or drones, the integration of AI, ML, and GT is crucial for:

- **Decision Making**: Agents must make real-time decisions based on the actions of other agents in their environment.
- **Safety and Efficiency**: Game-theoretic models can help ensure that these systems operate safely and efficiently in shared spaces.

### 4.5.3 Hybrid models

The development of hybrid models that combine game-theoretic principles with advanced machine learning techniques is an emerging area of research. These models can provide deeper insights into complex interactions and improve the robustness of AI systems.

The synergy between machine learning, artificial intelligence, and game theory offers powerful tools for understanding and solving complex problems across various domains. By leveraging the strengths of each field, researchers and practitioners can develop more effective strategies for cooperation, competition, and decision-making in multi-agent environments.

## 4.6 Applications of machine learning, artificial intelligent and game theory

The integration of machine learning (ML), artificial intelligence (AI), and game theory (GT) has led to significant advancements across various fields. Each of these domains contributes unique methodologies and insights that enhance decision-making, optimize strategies, and improve predictive capabilities. Below are some key applications of these technologies.

### 4.6.1 Multi-agent systems

- **Autonomous Vehicles**: Game theory is used to model interactions between multiple autonomous vehicles, allowing them to make decisions that optimize traffic flow and safety. Machine learning algorithms help vehicles learn from their environment and adapt their strategies accordingly (Seyidova & Gojayev, 2023).
- **Resource Management**: In smart grids, AI and ML algorithms manage energy distribution, while game theory optimizes resource allocation among users, ensuring efficient energy use and minimizing costs (Seyidova & Gojayev, 2023).

### 4.6.2 Healthcare

- **Treatment Optimization**: Game theory models can optimize treatment plans by considering the interactions between patients, healthcare providers, and insurers. Machine learning algorithms predict patient outcomes based on historical data, enhancing decision-making in treatment strategies (Seyidova & Gojayev, 2023).

- **Resource Allocation During Crises**: During health emergencies, such as pandemics, game theory can help allocate limited resources (like ventilators) effectively among competing hospitals and patients (Seyidova & Gojayev, 2023).

#### 4.6.3 Economics and market strategies

- **Auction Theory**: Game theory is extensively used in auction design and analysis, where machine learning can predict bidder behavior and optimize bidding strategies. This is particularly relevant in online advertising auctions, where companies bid for ad placements.
- **Market Competition**: Companies can use game-theoretic models to analyze competitive behaviors and develop strategies that maximize their market share while using machine learning to forecast market trends and consumer preferences.

#### 4.6.4 Robotics and automation

- **Collaborative Robots (Cobots):** In industrial settings, game theory can model the interactions between multiple robots working together, while machine learning enables these robots to learn from their experiences and improve their collaborative strategies over time.
- **Service Robots**: AI algorithms enhance the decision-making capabilities of service robots, allowing them to interact effectively with humans and adapt to changing environments.

### 4.7 Game development

Non-Player Character (NPC) Behavior: AI and machine learning are used to create more realistic and adaptive NPCs in video games. Game theory can help design strategies for NPCs that respond to player actions in a competitive or cooperative manner.

The synergy between machine learning, artificial intelligence, and game theory offers powerful tools for addressing complex problems across various domains. By leveraging the strengths of each field, researchers and practitioners can develop more effective strategies for cooperation, competition, and decision-making in multi-agent environments (Khan & Liu, 2023).

#### 4.7.1 Numerical results

Using the game theory equation for machine learning that meets AI, below are the analysis with two players:

**Table 2. Game theory equation for machine learning that meets AI with two players**

| Player 1 Strategy | Player 2 Strategy | Player 1 Payoff | Player 2 Payoff | Round | Learning Rate |
|---|---|---|---|---|---|
| Cooperate | Cooperate | 3 | 3 | 1 | 0.1 |
| Cooperate | Defect | 0 | 5 | 1 | 0.1 |
| Defect | Cooperate | 5 | 0 | 1 | 0.1 |
| Defect | Defect | 1 | 1 | 1 | 0.1 |
| Cooperate | Cooperate | 3 | 3 | 2 | 0.1 |
| Cooperate | Defect | 0 | 5 | 2 | 0.1 |
| Defect | Cooperate | 5 | 0 | 2 | 0.1 |
| Defect | Defect | 1 | 1 | 2 | 0.1 |
| Cooperate | Cooperate | 3 | 3 | 3 | 0.05 |
| Cooperate | Defect | 0 | 5 | 3 | 0.05 |
| Defect | Cooperate | 5 | 0 | 3 | 0.05 |
| Defect | Defect | 1 | 1 | 3 | 0.05 |
| Cooperate | Cooperate | 3 | 3 | 4 | 0.05 |
| Cooperate | Defect | 0 | 5 | 4 | 0.05 |
| Defect | Cooperate | 5 | 0 | 4 | 0.05 |
| Defect | Defect | 1 | 1 | 4 | 0.05 |

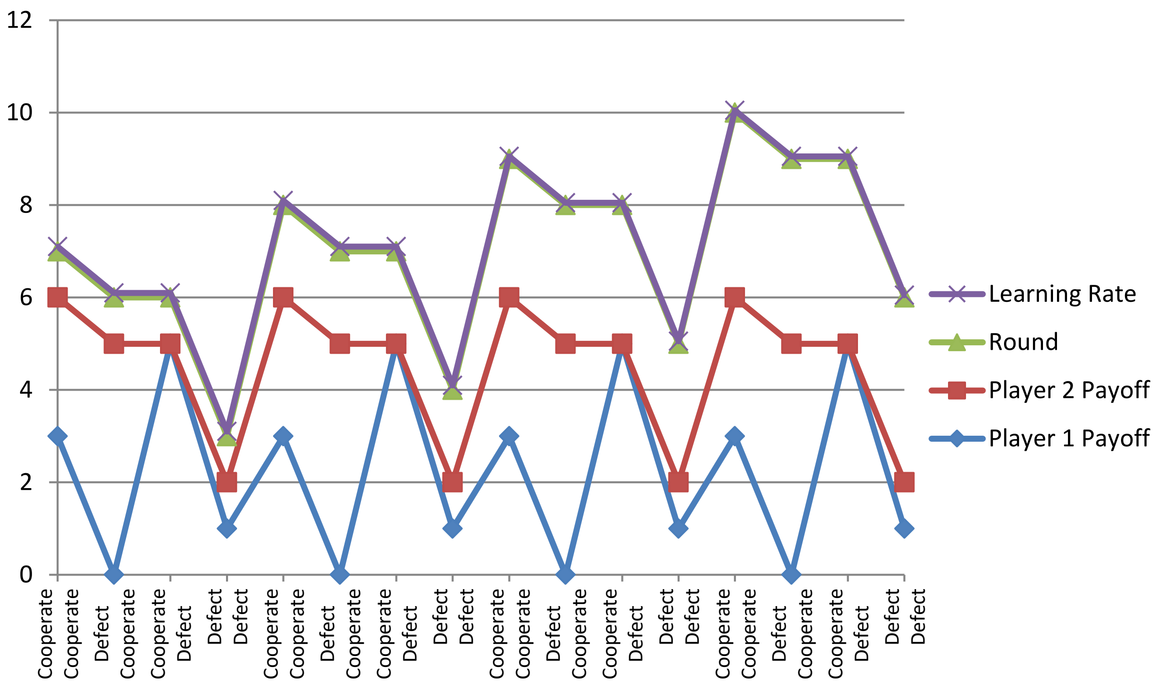


**Fig. 1. Graphical presentation of game theory equation for machine learning that meets AI with two players**

### 4.7.2 Key components

- **Player Strategies**: Two strategies, "Cooperate" and "Defect," representing the choices available to each player.
- **Payoffs:** Numeric values indicating the outcome for each player based on their chosen strategies.
- **Rounds:** Simulates multiple rounds to observe changes in strategy and payoffs over time.
- **Learning Rate:** Indicates how quickly players adjust their strategies based on previous outcomes.

Using the game theory equation for machine learning that meets AI, below are the analysis with three players:

**Table 3. Game theory equation for machine learning that meets AI with three players**

| Player 1 Strategy | Player 2 Strategy | Player 3 Strategy | Player 1 Payoff | Player 2 Payoff | Player 3 Payoff | Round | Learning Rate |
|---|---|---|---|---|---|---|---|
| Cooperate | Cooperate | Cooperate | 3 | 3 | 3 | 1 | 0.1 |
| Cooperate | Cooperate | Defect | 1 | 1 | 5 | 1 | 0.1 |
| Cooperate | Defect | Cooperate | 5 | 0 | 3 | 1 | 0.1 |
| Cooperate | Defect | Defect | 0 | 4 | 4 | 1 | 0.1 |
| Defect | Cooperate | Cooperate | 4 | 5 | 0 | 1 | 0.1 |
| Defect | Cooperate | Defect | 2 | 2 | 4 | 1 | 0.1 |
| Defect | Defect | Cooperate | 1 | 1 | 5 | 1 | 0.1 |
| Defect | Defect | Defect | 0 | 0 | 0 | 1 | 0.1 |
| Cooperate | Cooperate | Cooperate | 3 | 3 | 3 | 2 | 0.05 |
| Cooperate | Cooperate | Defect | 1 | 1 | 5 | 2 | 0.05 |
| Cooperate | Defect | Cooperate | 5 | 0 | 3 | 2 | 0.05 |
| Cooperate | Defect | Defect | 0 | 4 | 4 | 2 | 0.05 |
| Defect | Cooperate | Cooperate | 4 | 5 | 0 | 2 | 0.05 |
| Defect | Cooperate | Defect | 2 | 2 | 4 | 2 | 0.05 |
| Defect | Defect | Cooperate | 1 | 1 | 5 | 2 | 0.05 |
| Defect | Defect | Defect | 0 | 0 | 0 | 2 | 0.05 |

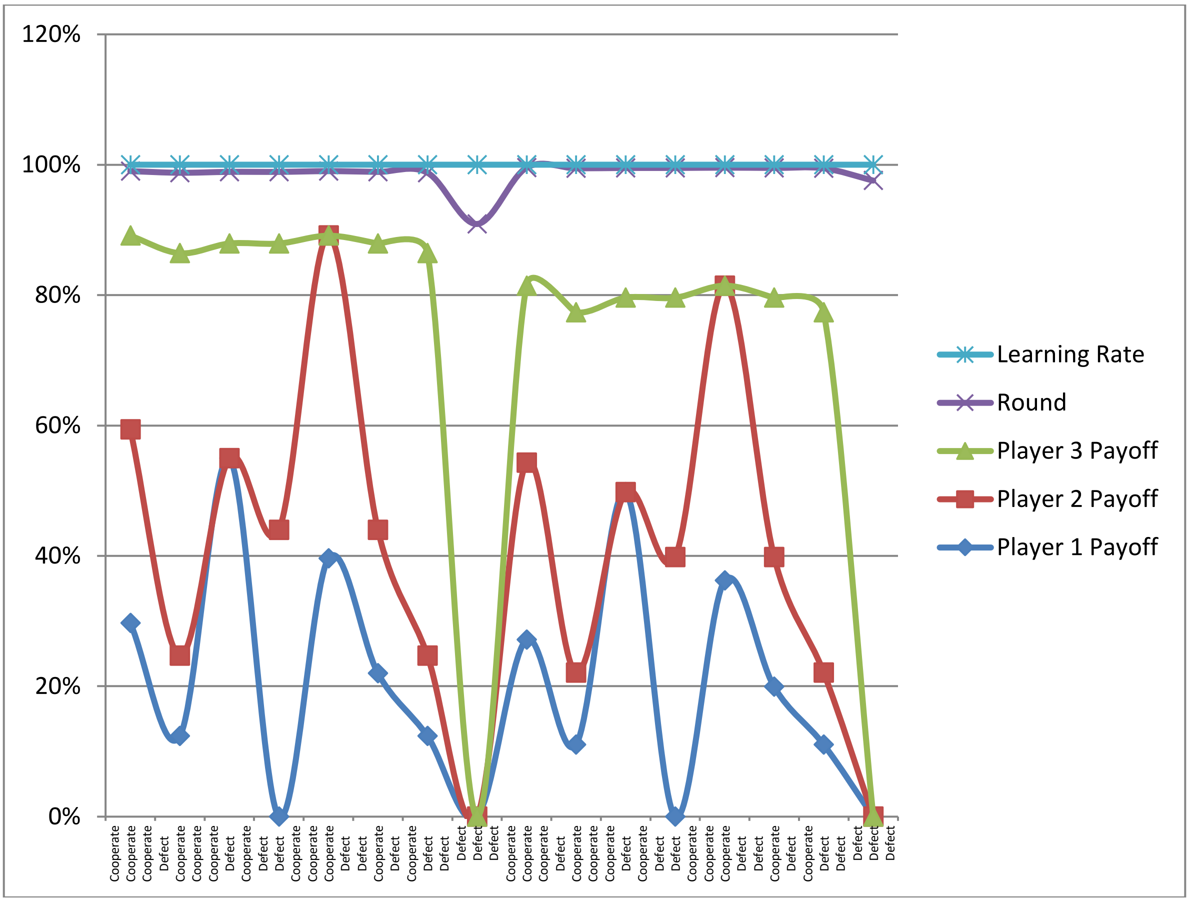


**Fig. 2. Graphical presentation of game theory equation for machine learning that meets AI with three players**

Using the game theory equation for machine learning that meets AI, below are the analysis with five players:

**Table 4. Game theory equation for machine learning that meets AI with five players**

| Player 1 Strategy | Player 2 Strategy | Player 3 Strategy | Player 4 Strategy | Player 5 Strategy | Player 1 Payoff | Player 2 Payoff | Player 3 Payoff | Player 4 Payoff | Player 5 Payoff | Round | Learning Rate |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Cooperate | Cooperate | Cooperate | Cooperate | Cooperate | 4 | 4 | 4 | 4 | 4 | 1 | 0.1 |
| Cooperate | Cooperate | Cooperate | Cooperate | Defect | 3 | 3 | 3 | 3 | 5 | 1 | 0.1 |
| Cooperate | Cooperate | Defect | Cooperate | Cooperate | 3 | 3 | 5 | 3 | 3 | 1 | 0.1 |
| Cooperate | Defect | Cooperate | Cooperate | Cooperate | 5 | 0 | 3 | 3 | 3 | 1 | 0.1 |
| Defect | Cooperate | Cooperate | Cooperate | Cooperate | 5 | 3 | 3 | 3 | 3 | 1 | 0.1 |
| Defect | Defect | Cooperate | Cooperate | Cooperate | 1 | 1 | 3 | 3 | 3 | 1 | 0.1 |
| Cooperate | Cooperate | Cooperate | Defect | Cooperate | 3 | 3 | 3 | 5 | 3 | 1 | 0.1 |
| Cooperate | Defect | Defect | Cooperate | Cooperate | 0 | 4 | 4 | 3 | 3 | 1 | 0.1 |
| Defect | Cooperate | Defect | Cooperate | Cooperate | 4 | 0 | 4 | 3 | 3 | 1 | 0.1 |
| Defect | Defect | Defect | Cooperate | Cooperate | 1 | 1 | 1 | 3 | 3 | 1 | 0.1 |
| Defect | Defect | Defect | Defect | Cooperate | 0 | 0 | 0 | 0 | 5 | 1 | 0.1 |
| Cooperate | Cooperate | Cooperate | Cooperate | Cooperate | 4 | 4 | 4 | 4 | 4 | 2 | 0.05 |
| Cooperate | Cooperate | Cooperate | Cooperate | Defect | 3 | 3 | 3 | 3 | 5 | 2 | 0.05 |
| Cooperate | Cooperate | Defect | Cooperate | Cooperate | 3 | 3 | 5 | 3 | 3 | 2 | 0.05 |
| Cooperate | Defect | Cooperate | Cooperate | Cooperate | 5 | 0 | 3 | 3 | 3 | 2 | 0.05 |
| Defect | Cooperate | Cooperate | Cooperate | Cooperate | 5 | 3 | 3 | 3 | 3 | 2 | 0.05 |

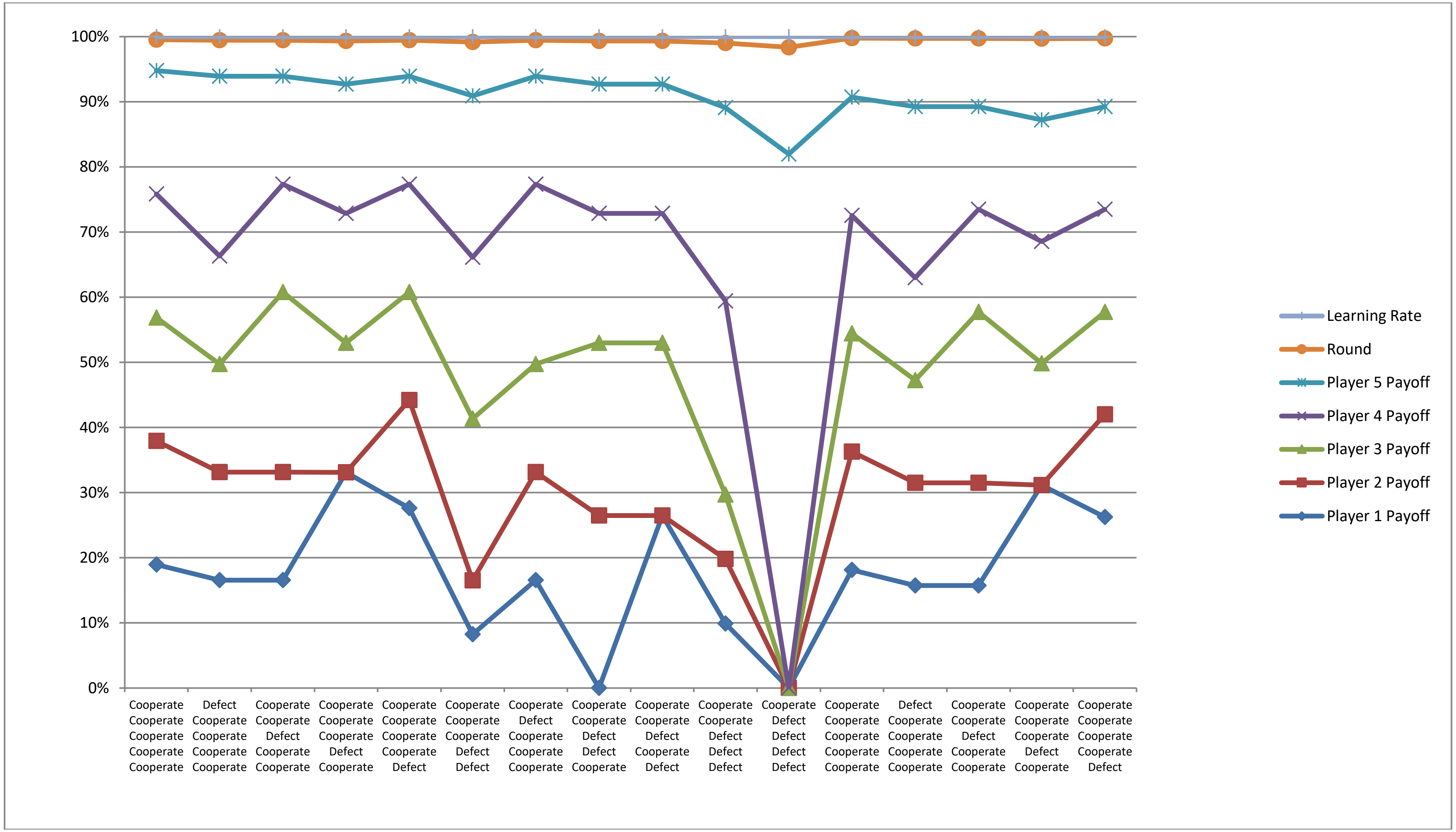


**Fig. 3. Graphical presentation of game theory equation for machine learning that meets AI with five players**

The above analysis models show the lead to the prediction of each player's behavior and optimize strategies. It gives better strategies of each player which leads to higher payoffs for each player over a period of time. The learning rates of each players affects them in the long run and the Nash Equilibrium of the strategy pairs lead to where no player has an incentive to deviate. The graphs show the payoffs over rounds and changes in strategies.

# 5 Conclusion

Through the integration of artificial intelligence, game theory, and machine learning, a new avenue opens up on comprehending the strategies and their application across various spheres. By leveraging professionals from different disciplines, scientists can build reliable models and systems which are capable of operating in complex environments and making reasonable decision-making. While game theory offers insight about human willingness to cooperate or be competitive, machine learning allows for predicting responses and events in AI systems. These range from the development of advanced autonomously operating systems, improved economic management systems to better management of healthcare resources. Forthcoming research will most likely target improving such combinations while also making ethical considerations and addressing certain barriers that are present in practical applications. Such an integration enhances artificial intelligence decision-making possibilities and offers solutions to the strengthening of social and economic systems by integrating various specialists in the field. Given the increasing cross-linkage of the world today, solving the problems of the present requires a disciplinary encompassing approach.

# Disclaimer (Artificial Intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of this manuscript.

# Competing Interests

Authors have declared that no competing interests exist.

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