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# Social Accounting Matrix Balanced Based on Mathematical Optimization Method and General Algebraic Modeling System

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## Authors' contributions

*This work was carried out in collaboration between all authors. Author MCL designed the study, performed the general algebraic modeling system and wrote the first draft of the manuscript. Author LES managed the analyses of this study. All authors read and approved the final manuscript.*

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## ABSTRACT

**Aims:** Social accounting matrix (SAM) is a kind of important form in the national accounts calculation. The requisite data are typically collected from a number of different sources. Therefore, much of data used will not be immediately consistent. The aim of this paper present the various methods of balancing applied to social accounting matrix.

**Design:** These methods (RAS, Cross-entropy (CE), least square, linear programming with  $L_1$  and linear programming with  $L_\infty$  method) are applied widely for develop and keep and time consistent multi-sector datasets.

**Methodology:** This paper solves the social accounting matrix balanced problem using iterative method (in RAS approach) and using the computer program about General Algebraic Modeling system (GAMS) (in other approach). The sum square error of the estimated table from the initial table is used as the main instrument to measure deviation of the updated matrices from the matrix.

**Results:** We found that CE method is more accurate than the other methods.

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**Keywords:** Social accounting matrix (SAM); System of National Accounts (SNA); Cross entropy (CE); least square method; Linear programming norm  $L_1$  ; linear programming norm  $L_\infty$

## 1. INTRODUCTION

SAM is simply defined as a single entry accounting system whereby each account or transactor is represented by a column for recording outgoings and a corresponding row for recording incomings [1]. It is a framework for organization and reconciliation of data or economic flows in a country with a particular period (usually a year). SAM database is used for a descriptive analysis of economy and provides statistical basis for construction of general equilibrium models that can be used to answer various economic policy questions [2,3] and for analysis of both income distribution and growth issues. A SAM extends the national accounts by classifying institutions according to their socio-economic backgrounds instead of their functional or economic activities [4]. Lahr and Mesnard [5] discuss the RAS algorithm and the various formal formulations of other biproportional approaches as well as discuss what defines an algorithm as “biproportional”. Bellu [6] offers a methodological introduction to SAM for analyzing socio-economic impact of agricultural and rural development policies. Schneider and Zenios [7] describe five applications of matrix balancing and compare the algorithmic and computational performance of balancing procedures that represent the two primary approaches for matrix balancing—matrix scaling and nonlinear optimization. The five applications of matrix balancing are the RAS algorithm, a diagonal similarity scaling algorithm and a truncated Newton algorithm for network optimization.

The contribution of this paper is in extending the work of [8 and 9] who assessed the performance of only two – the RAS and Cross entropy methods. This paper reports the results obtained by balancing a SAM using 5 different methods that are implemented using an iteration program and GAMS software. It explains that why the CE method is the best method (See section 5.2, 5.3 and 5.4).

SAM is constructed using a large number of statistical data sets that comes different sources, such as national accounts, trade data, input-output tables or supply-use tables, the data are not consistent [10]. The data from the different sources is inconsistent. Therefore, the data has to be rendered consistent. This is an important and difficult task. When the data from different sources is compiled the resultant imbalanced, i. e. the column sum and row sum will not equal and hence, will not satisfy one of the defining features of a SAM, i. e. income must equal expenditure for each account. Table 1 is shown a schematic Macro Social Accounting Matrix.

**Table 1. An example of the Macro Social Accounting Matrices**

<b>Receipts</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>Total</b>
1. Suppliers		C	G	I	X	Demand
2. Households	Y					Income
3. Government		T				Receipts
4. Capital Acc.		$S_h$	$S_g$		$S_f$	Savings
5. Rest of world	Z					Imports
<b>Total</b>	<b>Supply</b>	<b>Expenditure</b>	<b>Expenditure</b>	<b>Investment</b>	<b>Row</b>	

Source from Trap et al. [11], Definition of variables:

$S_h$  =Private Savings,  $S_g$  = Government Savings,  $S_f$  = Foreign Savings, T = Tax payments, G = Government Spending, M = Imports, X = Exports, Accounting Identities

$$\begin{aligned}
 Y + Z &= C + G + I + X \quad (\text{GNP}) \\
 C + T + S_h &= Y \quad (\text{Income}) \\
 G + S_g &= T \quad (\text{Government, Budget}) \\
 I &= S_h + S_g + S_f \quad (\text{Savings-Investment}) \\
 X + S_f &= Z \quad (\text{Trade Balance})
 \end{aligned}$$

Estimates from primary or disparate secondary source are often inconsistent. There are several alternative matrix balancing methods used to adjust the initial estimates in order to achieve consistency [9,12,13]. This paper uses mathematical optimization method and GAMS software to estimate and balance a SAM.

## 2. SOCIAL ACCOUNTING MATRIX AND SYSTEM OF NATIONAL ACCOUNTS

### 2.1 Constructing a SAM

The main features of a SAM are threefold. First, it is a *square matrix* in that each account has a both row and corresponding column of the matrix. The transactions are shown in the cells, so the matrix displays the interconnections between agents in an explicit way. Second, it is *comprehensive*, in the sense that it portrays all the economic activities of the system (consumption, production, accumulation and distribution), although not necessarily in equivalent detail. Thirdly, the SAM is *flexible*, in that, although it is usually set up in a standard, basic framework there is a large measure of flexibility both in the degree of desegregation and in the emphasis placed on different parts of the economic system. As it is an accounting framework not only is the SAM square but also the corresponding row and column totals must be equal. Clearly, at one extreme, any set of macroeconomic aggregates can be set out in a matrix format. Therefore, Keuning [14] suggested that SAM is a basis of new system national accounts (SNA). In an open macroeconomic system, any of the implementation of the policy will have an impact on economic activity at all levels. Social Accounting Matrix has become a standard data structure of the computable general equilibrium (CGE) model.

The construction of a SAM with any significant degree of disaggregation of the principle accounts includes: (1) Supply and use tables (input-output tables), or the necessary principle survey data to compile them. (2) Household survey incorporating a labor force survey (a multi-purpose, integrated household survey). (3) Government budget accounts, trade statistics and balance of payments statistics.

Many compilers begin by assembling a macro SAM from the national accounts. This defines a set of control totals for the subsequent disaggregation and means that the SAM is consistent with any macro analysis. Often macro SAMs available for a more recent year than the detailed datasets are input-output tables, household survey. Gallardo and Mardones [15] uses information from I-O table 2008, national account household survey and environmental pollutant emissions to elaborate an environmentally extended SAM for Chile. Payatt et al. [16] have pointed out that compiling detailed SAMs can be part of a process to improve the national accounts estimates. Many countries now re-base their national accounts periodically in accordance with a set of commodity balances (input-output table such as a Ugandan social accounting matrix [17]. Otherwise, household survey data is not always fully in estimating the national accounts (e.g. consumer expenditure is obtained as a residual in the commodity balances), so there might be a case of adjusting the macro SAM in some circumstances.

## 2.2 Basic Social Accounting Matrix Structure

A simple, stylized SAM framework is shown in Table 2. It is a square matrix that represents the transactions taking place in economy during an accounting period, usually one year. Table 2 shows clearly the three basic forms of economic activity, production (accounts, 1, 2, and 3), consumption (accounts 4, 5 and 6), accumulation (account 7) and the transactions with the rest of the world (account 8).

**Table 2. A Basic Social Accounting Matrix (SAM)**

Account		(1)	(2)	(3)	(4)	
production	commodities	(1)	Intermediate consumption		Household consumption	
	Activities	(2)	Domestic sale			
Factors of production		(3)	Gross value added payments to factors			
Institution	Households	(4)		Labor and mixed income	Inter-household transfers	
	Corporate enterprise	(5)		Operating surplus		
	Government	(6)	Net taxes on product		Direct Taxes	
Combined capital accounts		(7)	Enterprise savings			
Rest of World (RoW)		(8)	Imports			
Total			Supply of products	Cost of production activities	Factor income payments	Current household outlays

**Table 2. A Basic Social Accounting Matrix (SAM) (continuous)**

Account		(5)	(6)	(7)	(8)	Total	
production	commodities	(1)	Government consumption	Fixed capital formation and change in stocks	Exports	Demand for products	
	Activities	(2)				Sales of commodities	
Factors of production		(3)			Net factor incomes from RoW	Factor income receipts	
Institution	Households	(4)	Distributed profits to household	Current transfers to household	Labor and mixed income	Net current transfers from RoW	Current household receipts
	Corporate enterprise	(5)		Current transfers to enterprise	Operating surplus	Net current transfers from RoW	Current enterprise receipts
	Government	(6)	Direct Taxes			Net current transfers from RoW	Current government receipts
Combined capital accounts		(7)	Enterprise savings	Government savings	Capital transfers	Net capital transfers from RoW	Capital receipts
Rest of World (RoW)		(8)			Current external balanced		Aggregate outlays from RoW
Total			Current enterprise outlays	Current government outlays	Capital outlays	Aggregate outlays to RoW	

Source: Round [1]

### 3. VARIOUS METHODS OF BALACING SAM

#### 3.1 The RAS Method [18]

RAS is a widely used methodology to balance SAMs. The RAS method is an iterative method of biproportional adjustment of rows and column that has been independently developed by various researches.

Each element in matrix T is indicated  $t_{ij}$ , where  $i = 1, 2, 3, n$ ; is the row index,  $j = 1, 2, 3, n$ ; is the column index.

$$y_j = \sum_{i=1}^n t_{ij} \qquad x_i = \sum_{j=1}^n t_{ij}$$

Column sums  $y_j$  are the column totals. Row sums  $x_i$  are the row totals. As already mentioned, for a given k account, expenditure is equal to receipts and is shown by the fact the sum of row is equal to the sum of the column.

$$\sum_{j=1}^n t_{ij} = x_i \qquad \sum_{i=1}^n t_{ij} = y_j \qquad (1)$$

$$A = (a_{ij})_{n \times n} \text{ is a SAM matrix,} \qquad a_{ij} = \frac{t_{ij}}{y_j} \qquad (2)$$

$$\sum_i a_{ij} = 1 \qquad (3)$$

$$\text{and } Y = AY, \text{ that is} \qquad \sum_{j=1}^n a_{ij} y_j = y_i \qquad (4)$$

A classic approach to solve this problem is to generate a new matrix  $A^*$ , from the old matrix  $A$  by means of "biproportional" row and column operations:

$$a_{ij}^1 = r_i a_{ij} s_j \text{ in matrix notation : } A^1 = \tilde{R} A \tilde{S} \qquad (5)$$

Where  $(\sim)$  indicate a diagonal matrix of elements  $r_i$  and  $s_j$

$$\tilde{R} = \text{diag} [r_1, r_2, \dots, r_n], \quad \tilde{S} = \text{diag}[s_1, s_2, \dots, s_n],$$

$$a_{ij}^1 = r_i a_{ij} s_j \qquad (6)$$

The iterative process is as follows and can be seen below [1]. The original matrix of coefficients is multiplied by the row of column total  $y_j^*$  to obtain the matrix  $f_{ij}$

$$f_{ij} = a_{ij} y_j^* \qquad (7)$$

The row totals of this matrix are represented in the vector  $x_i$ . Let the ration of  $x_i^*$  to  $x_i$  be the multiplier  $r_i$ . Multiplying  $r_i$  and  $f_{ij}$ , we obtain a new  $f_{ij}$ . Row vector  $y_j$  of column totals is obtained and used to calculate the multiplier  $s_j$ .  $f_{ij}$  and  $s_j$  are then multiplied. The entire sequence of operations can be seen in EQ. 8.

$$x_i = \sum_j f_{ij} \quad r_i = x_i^*/x_i \quad f_{ij} = r_i f_{ij} \quad (8)$$

$$y_j = \sum_i f_{ij} \quad s_j = y_j^*/y_j \quad f_{ij} = s_j f_{ij}$$

The iterative process in equation (8) then continuous until the conditions  $x_i^* = x_i$  and  $y_j^* = y_j$  are met.

The RAS method has advantage of being simple to apply iteration steps. The advantages and disadvantages of RAS method described in section 5.4. In the following section, we introduce the cross-entropy method.

### 3.2 Cross-entropy (CE) Method

Some researcher used cross-entropy approach to SAM balancing, such as [9,19,20,21]. The estimation procedure is to minimize the [22] cross-entropy measure of the distance between the new and the prior estimated probability. Golan et al. [23] use cross-entropy method to balance an I-O table.

Applied to the procedure of updating a SAM, the problem is to find a new SAM  $A^* = (a_{ij}^*)$ , close to an existing SAM  $A = (a_{ij})$

The formulas are:

$$\text{Min} \sum_{i,j=1}^n a_{ij}^* \log \frac{a_{ij}^*}{a_{ij}} = \text{Min} \sum_{i,j=1}^n a_{ij}^* (\log a_{ij}^* - \log a_{ij}) \quad (9)$$

$$s.t. \begin{cases} \sum_{j=1}^n a_{ij}^* y_j^* = y_i^* \\ \sum_{i=1}^n a_{ij}^* = 1 \end{cases}$$

The solution is obtained by setting the Lagrange for the above problem and solving it. The outcome combines the information from the data and the prior.

$$a_{ij}^* = \frac{a_{ij} \exp(\lambda_i y_{ij}^*)}{\sum_{i,j} a_{ij} \exp(\lambda_i y_{ij}^*)}$$

Where is the Lagrange is multiplies  $\lambda_i$  is associated with the information on row and column sums and the denomination is a normalization factors.

From the comparison of these two methods, they found that if the focus is on column coefficients, then the CE method appears to be superior to RAS. However, if the focus is on SAM flows, then the two methods are very similar, with RAS performing slightly better than CE [8]. The advantages and disadvantages of CE method described in section 5.4.

### 3.3 Method of Least Squares (LS)

The method is to find a new matrix  $A^*$  from an existing matrix  $A$ , by minimizing the sum of square of deviations between the new values and the prior value in percentage terms.

Suppose that the elements  $a_{ij}$  and  $a_{ij}^*$  are the value of the initial SAM and the values of the estimated SAM respectively.

The formula is:

$$\begin{aligned} & \text{Min} \sum_{i,j=1}^n \left( \frac{a_{ij}}{a_{ij}^*} - 1 \right)^2 \\ & \text{s.t.} \quad \sum_{i=1}^n a_{ij}^* = \sum_{j=1}^n a_{ij}^* \end{aligned} \quad (10)$$

By least squares method, this approach assumes that there is a linear relationship between the dependent variables  $a_{ij}$  and the explanatory variables  $a_{ij}^*$ .

### 3.4 Method of Linear Programming by Minimizing with Norm $L_1$ (LP- $L_1$ )

We define that  $d_{ij}^-$  is negative difference between  $a_{ij}$  and  $a_{ij}^*$ ,  $d_{ij}^+$  is positive difference between  $a_{ij}$  and  $a_{ij}^*$ . Where  $d_{ij}^+ = \max [(a_{ij} - a_{ij}^*), 0]$ ,  $d_{ij}^- = \max [-(a_{ij} - a_{ij}^*), 0]$

The formulas are:

$$\begin{aligned} & \text{Min} \sum_{i,j=1}^n [(d_{ij}^+ + d_{ij}^-) / a_{ij}] \\ & \text{s.t.} \quad \begin{cases} \sum_{j=1}^n a_{ij}^* = \sum_{i=1}^n a_{ij}^* \\ d_{ij}^+ - d_{ij}^- = a_{ij} - a_{ij}^* \end{cases} \end{aligned} \quad (11)$$

### 3.5 Method of Linear Programming by Minimizing with Norm $L_\infty$ (LP- $L_\infty$ )

$$\begin{aligned} & \text{Min} \sum_{i,j=1}^n (d_{ij}^+ + d_{ij}^-) \\ & \text{s.t.} \quad \begin{cases} \sum_{j=1}^n a_{ij}^* = \sum_{i=1}^n a_{ij}^* \\ d_{ij}^+ - d_{ij}^- = a_{ij} - a_{ij}^* \end{cases} \end{aligned} \quad (12)$$

## 4. GAMS SOFTWARE AND PROGRAM FOR BALACING A SAM

### 4.1 GAMS for Balancing a SAM

General Algebraic Modeling System (GAMS) is a high-level modeling system for mathematical programming and optimization [24]. GAMS is designed for modeling and solving linear, nonlinear and mixed-integer optimization problems. The system is tailored for complex, large-scale modeling applications and allows the user to build large maintainable models that can be adapted to new situations

There are ten steps to balance an unbalanced SAM [18].

- Step 1: Define matrix accounts
- Step 2: Input data
- Step 3: Normalize initial matrix cell value
- Step 4: Treatment of negative values
- Step 5: Treatment of zero values
- Step 6: Definition of variables used in optimization process
- Step 7: Definition and initialization of model estimations
- Step 8: Initialization of Variables
- Step 9: Model solving
- Step 10: Results copied to a new matrix

### 4.2 Choose the Appropriate Method

In order to choose the best optimization method, we carry out an approach between the four methods (EQ 8~EQ 12). This approach is:

(1) This approach consists in calculating the value D for each method. Sum of squared errors (SSE) is defined as:

$$D = \sum_{i=1}^n \sum_{j=1}^n (a_{ij}^* - a_{ij})^2 \quad (13)$$

(2) We exclude the condition. The value of element cell (i, j) in A (initial SAM) is not null, but the value of element cell (i, j) in A\* (final SAM) is null.

(3) We choose best method according to the minimum value of D

## 5. ILLUSTRATION

We use the following IO tables to compare the accuracy of the RAS, CE, LS, LP-  $L_1$  and LP-  $L_\infty$  method. We use iterative program to solve the social accounting matrix balanced problem (in RAS approach) and use the computer programmer about General Algebraic Modeling system (GAMS) to solve the social accounting matrix balanced problem (in CE, LS, LP-  $L_1$  and LP-  $L_\infty$  approach).



### 5.1 Define Matrix Accounts and Input Data Matrix

(1) Define matrix accounts

PRO	Production
ACT	Activities
ALAB	Labor value added
ACAP	Capital value added
ENT	Enterprise
HS	Household
GOV	Government
GGOV	Government grants
CAP	Capital
DEP	Depreciation
R0W	Rest of World
TOT	Total

(2) Input data matrix

Use Zhou [25] social account matrix data as the input data matrix in this research (see Table 3)

**Table 3. Input data matrix**

	PRO	ACT	ALAB	ACAP	ENT	HS	GOV
PRO		329.85				207.18	21.30
ACT	629.23						
ALAB		155.96				3.6	50.91
ACAP		134.04	208.28	131.24	76.95		2.08
ENT				128.61			20.28
HS				277.64	0.23	3.46	16.37
GOV	3.28	14.37		6.14	12.49	60.45	47.56
GGOV					0.20	0.28	8.25
CAP					21.76	12.79	
INV							
DEP					37.64	12.39	4.08
R0W	72.87		2.29	9.42	0.20	0.18	0.71
TOTAL	705.38	634.22	210.57	553.05	149.47	300.33	171.54
<b>+</b>	<b>GGOV</b>	<b>CAP</b>	<b>INV</b>	<b>DEP</b>	<b>R0W</b>	<b>TOTAL</b>	
PRO	2.21	53.11	3.41		90.54	707.60	
ACT	2.79					632.02	
ALAB						210.47	
ACAP		3.02			2.25	557.86	
ENT						148.89	
HS	3.15				0.60	301.45	
GOV	0.81	28.48	1.15		0.47	175.2	
GGOV					0.33	9.06	
CAP				54.11		88.66	
INV		4.56				4.56	
DEP						54..11	
R0W		6.05				91.72	
TOTAL	8.96	95.22	4.56	54.11	94.19		

## 6. RESULTS

### 6.1 GAMS Program

The estimated table may have zero elements where the initial SAM has no zero elements. It is possible that we will get final results that are very inaccurate, especially if the initial table's values for the corresponding zero cell, the estimated table is very large [8]. But GAMS program has been specially treated the initial table's values for the corresponding zero cell and the corresponding negative cell.

GAMS program have ten steps to balance an unbalanced SAM. Only steps 1, 2 and 10 require modifications by the user, although modifications are also possible at step 8 [18]. The balanced SAM does not allow negative values. To solve this problem, GAMS program simply transpose these values to their counterpart cell before balancing the SAM. Indeed, as the SAM represents flows from one account to another, a negative flow from account A to account B is equivalent to an equal positive flow from account B to account A. For example, if  $a(6,5)$  is negative, an equal account is added to cell  $a(5,6)$  and set cell  $a(6,5)$  to zero. To avoid having to take the log of zero in the CE method, the GAMS program adds a small amount to each cell value. In GAMS program, matrix cell values are limited between 0 and infinity and empty cells remain empty. No user modifications are necessary, but the user can fix any cell values, as desired. Be careful not to fix too many values, as the program may not find an optimal solution. GAMS solve the optimization using a non linear algorithm in CE method. GAMS solve the optimization using a linear algorithm in LS method, LP-  $L_1$  method and LP-  $L_\infty$  method. The values of element in balanced matrix may have zero. Therefore, we recommend using of CE method and also saying that GAMS program is easy use than Mat Lab software.

#### 6.1.1 The balanced SAM by RAS method

We use an iteration program to obtain the balanced SAM. The balanced SAM by RAS method denoted as Table 4.

**Table 4. The balanced SAM by RAS method**

	<b>PRO</b>	<b>ACT</b>	<b>ALAB</b>	<b>ACAP</b>	<b>ENT</b>	<b>HS</b>	<b>GOV</b>
PRO		331.52				207.15	21.71
ACT	630.55						
ALAB		155.45				3.6	51.53
ACAP		133.79	207.92	126.46	75.71		2.15
ENT				129.45			19.73
HS				278.35	0.21	3.15	15.9
GOV	3.14	12.59		7.16	12.35	60.21	47.54
GGOV					0.2	0.28	8.26
CAP					23.06	15.05	
INV							
DEP					37.42	11.5	5.12
ROW	73.05		2.66	10.03	0.22	0.34	0.78
TOTAL	706.74	633.35	210.58	551.45	149.17	301.28	172.72
<b>+</b>	<b>GGOV</b>	<b>CAP</b>	<b>INV</b>	<b>DEP</b>	<b>ROW</b>	<b>TOTAL</b>	
PRO	2.22	51.2	3.43		89.51	706.74	

ACT	2.8					633.35
ALAB						210.58
ACAP		3.2			2.22	551.45
ENT						149.17
HS	3.12				0.55	300.28
GOV	0.81	27.31	1.15		0.46	172.72
GGOV					0.32	9.07
CAP	0.12			54.04		92.27
INV		4.58				4.58
DEP						54.04
ROW		5.98				93.06
TOTAL	9.07	92.27	4.58	54.04	93.06	

**6.1.2 The balanced SAM by Cross-entropy (CE) method**

We use GAMS program to obtain the balanced SAM. The balanced SAM by CE method denoted as Table 5.

**Table 5. The balanced SAM by Cross-entropy (CE) method**

	PRO	ACT	ALAB	ACAP	ENT	HS	GOV
PRO		330.18				208.15	20.64
ACT	628.25						
ALAB		155.45				3.6	51.53
ACAP		133.52	207.92	126.42	75.9		2.18
ENT				129.45			19.88
HS				278.27	0.21	3.18	15.9
GOV	3.18	12.59		7.18	12.32	60.2	46.51
GGOV					0.2	0.28	8.86
CAP					23.06	13.85	
INV							
DEP					37.42	11.64	5.14
ROW	72.87		2.66	10.03	0.22	0.34	0.82
TOTAL	704.48	631.74	210.58	551.35	149.33	301.24	171.46
<b>+</b>							
	GGOV	CAP	INV	DEP	ROW	TOTAL	
PRO	2.35	51.2	3.45		88.51	704.48	
ACT	3.5					631.74	
ALAB						210.58	
ACAP		3.2			2.21	551.35	
ENT						149.33	
HS	3.2				0.48	302.24	
GOV	0.81	27.15	1.07		0.46	171.46	
GGOV					0.64	9.98	
CAP	0.12			54.22		91.25	
INV		4.52				4.52	
DEP						54.22	
ROW		5.18				92.30	
TOTAL	9.98	91.25	4.52	54.22	92.30		

**6.1.3 The balanced SAM by least squares (LS) method**

We use GAMS program to obtain the balanced SAM. The balanced SAM by least squares (LS) method denoted as Table 6. We find that cell (8, 5) = cell (GGOV, ENT) = 0 in final matrix, but cell (8, 5) = cell (GGOV, ENT) = 0.20 in initial SAM.

**Table 6. The balanced SAM by least squares (LS) method**

	PRO	ACT	ALAB	ACAP	ENT	HS	GOV
PRO		331.24				208.3	20.64
ACT	629.64						
ALAB		155.45				3.6	51.53
ACAP		133.52	207.9	126.42	75.85		2.18
ENT				129.45			19.88
HS				278.1	0.32	3.18	15.9
GOV	3.12	12.59		7.28	12.32	60.2	46.44
GGOV					0	0.28	8.86
CAP					25.58	13.74	
INV							
DEP					35.04	11.64	5.19
ROW	72.93		2.68	10.03	0.22	0.24	0.88
TOTAL	705.69	632.80	210.58	551.28	149.33	301.18	171.50
<b>+</b>	<b>GGOV</b>	<b>CAP</b>	<b>INV</b>	<b>DEP</b>	<b>ROW</b>	<b>TOTAL</b>	
PRO	2.35	51.2	3.45		88.51	705.69	
ACT	3.16					632.80	
ALAB						210.58	
ACAP		3.2			2.21	551.28	
ENT						149.33	
HS	3.2				0.48	301.18	
GOV	0.81	27.15	1.07		0.52	171.50	
GGOV					0.51	9.64	
CAP	0.12			51.88		91.32	
INV		4.52				4.52	
DEP						51.88	
ROW		5.25				92.23	
TOTAL	9.64	91.32	4.52	51.88	92.23		

**6.1.4 The balanced SAM by minimizing with norm  $L_1$**

We use GAMS program to obtain the balanced SAM. The balanced SAM by minimizing with norm  $L_1$  denoted as Table 7.

**Table 7. The balanced SAM by minimizing with norm  $L_1$  method**

	PRO	ACT	ALAB	ACAP	ENT	HS	GOV
PRO		331.34				207.39	2160
ACT	630.33						
ALAB		155.40		130.79		3.59	51.53
ACAP		133.79	207.92	129.45	75.71		2.10
ENT				227.91			19.73
HS				7.07	0.21	3.12	15.907
GOV	3.28	12.59			12.35	59.03	47.54
GGOV					0.20	0.28	8.26
CAP					23.06	14.72	
INV							
DEP					37.42	12.53	4.16
R0W	72.87		2.60	10.23	0.22	0.21	0.78
TOTAL	706.48	633.12	210.52	555.45	149.17	300.82	171.60
<b>+</b>							
	<b>GGOV</b>	<b>CAP</b>	<b>INV</b>	<b>DEP</b>	<b>R0W</b>	<b>TOTAL</b>	
PRO	2.22	51.15	3.41		89.37	706.48	
ACT	2.80					633.13	
ALAB						210.52	
ACAP		2.91			2.22	555.44	
ENT						149.18	
HS	3.15					300.787	
GOV	0.81	27.31	1.15		0.58	171.59	
GGOV					0.46	9.07	
CAP				54.11		91.98	
INV		4.56				4.56	
DEP						54.11	
R0W		6.05				92.96	
TOTAL	8.07	91.98	4.56	54.11	92.96		

**6.1.5 The balanced SAM by minimizing with norm  $L_\infty$**

We use GAMS program to obtain the balanced SAM. The balanced SAM by minimizing with norm  $L_\infty$  denoted as Table 8. We find that cell (8, 5) = cell (GGOV, ENT) = 0 in final matrix, but cell (8, 5) = cell (ENT, GGOV) = 0.20 in initial SAM. The cell (8, 6) = cell (GGOV, HS) = 0 in final matrix, but cell (8, 6) = cell (GGOV, HS) = 028 in initial SAM.

**Table 8. The balanced SAM by minimizing with norm  $L_\infty$  method**

	PRO	ACT	ALAB	ACAP	ENT	HS	GOV
PRO		331.44				208.3	20.64
ACT	630.34						
ALAB		155.45				3.6	51.53
ACAP		133.52	207.9	126.3	75.85		2.18
ENT				129.45			19.88
HS				278.1	0.32	3.18	15.9
GOV	2.95	13.04		7.28	12.32	60.23	45.24
GGOV					0	0	6.85
CAP					25.58	13.13	
INV							
DEP					35.04	9.29	5.65
ROW	72.8		2.68	10.03	0.22	0.24	0.69
TOTAL	706.09	633.45	210.58	551.16	149.33	297.97	168.56
<b>+</b>							
	<b>GGOV</b>	<b>CAP</b>	<b>INV</b>	<b>DEP</b>	<b>ROW</b>	<b>TOTAL</b>	
PRO	2.35	51.2	3.55		88.61	706.09	
ACT	3.11					633.45	
ALAB						210.58	
ACAP		3.2			2.21	551.16	
ENT						149.33	
HS					0.48	297.97	
GOV	2.81	23.15	1.07		0.47	168.56	
GGOV					1.65	8.51	
CAP	0.24			49.98		88.93	
INV		4.62				4.62	
DEP						49.98	
ROW		6.76				93.42	
TOTAL	8.51	88.93	4.62	49.98	93.42		

## 6.2 Result

The five final matrices of coefficients are compared. These matrices are basically the full matrices with the cell in a column divided by the column total. Table 9 is denoted as the comparison of SSE values from matrices coefficient for each method.

**Table 9. Comparison of SSE values from matrices coefficient for each method**

methods	Root mean square error (D)	zero cell
RAS	1.146	
CE	1.017	
LS	0.895	Cell (8 , 5)
LP- $L_1$	1.021	
LP- $L_\infty$	0.917	Cell (8 ,5) and cell (8, 6)

### Compare of SSE value of matrices of coefficients for each estimation methods.

The choice of the best method, we use method of “choose the appropriate method” (section 4.2). We obtain the following inequality between the five values of D.

$$D_{LS} < D_{LP-L_{\infty}} < D_{CE} < D_{LP-L_1} < D_{RAS}$$

From the final matrix of SAM through the LS and LP-  $L_{\infty}$  , we notice the values of the elements are null: Cell (8, 5) in LS method, Cell (8, 5) and cell (8, 6) in LP-  $L_{\infty}$  method.

Where as these values is not null in initial matrix of SAM. The LS and LP-  $L_{\infty}$  method do not reflect the latter. Therefore, the value of D is very low in the method of CE.

### 6.3 Discussion

The disadvantages of RAS method are: (1) a lack of economic foundations, (2) inability to accommodate other sources of data than those on row and column table, e. g. we can not fix new cell values that we suppose are accurately measured ([9], [18]). (3) We must write an iteration program. The advantages of RAS method are: (1) the value of element cell (i, j) in final SAM  $A^*$  is not null, if the value of element cell (i, j) in initial SAM  $A$  is not null. (2) The  $(2n-1)$  unknown multipliers are determined by the  $(2n-1)$  independent row and column restrictions using iterative adjustment procedure. It is easily use RAS method.

The advantages of CE method are: (1) we can fix cell values in addition to row and column tables. (2) the value of element cell (i, j) in final SAM  $A^*$  is positive, if the value of element cell (i, j) in initial SAM  $A$  is positive [13]. Fofana et al. [18] argued that because of the above disadvantages, many researchers prefer use the CE method. Since LS method, LP-  $L_1$  method and LP-  $L_{\infty}$  method are linear relationship between the dependent variables  $a_{ij}$  and the explanatory variables  $a_{ij}^*$ . In 10 step of GAMP program, the negative values are re-transposed to their original position to the new matrix. For example, cell (5, 6) is negative value; GAMP program set cell (5, 6) to zero and set cell (6, 5) to cell (6, 5) + cell (5, 6). Therefore, the element value in the final SAM  $A^*$  may have zero.

### 7. CONCLUSION

A SAM is a single accounting framework with the rows and columns, arranging income and expenditure accounts of various economic agents in a country. A SAM framework is not only a statistical tool but also a framework for macroeconomic analysis. It provides a framework for the organization of information about economic and social structure of a country and a service as dataset for a model of the economy. Since estimating cell value from primary or disparate secondary source are often inconsistent. There are several alternative matrix balancing methods are available to adjust the initial estimates for consistency. We detail describe five types the matrix balancing methods. The RAS method is an iterative method of “biproportional” adjustment of rows and column that has been independently developed by various researches.

In this paper, we present a GAMS code and CE, LS, LP- $L_1$  and LP- $L_\infty$  to balance an unbalanced SAM in ten steps. We obtained the five results (see section 5.2.1 to 5.2.5). We present a comparison of SSE values from matrices coefficient for each method. According to section 5.3 and section 5.4, we also explain that the CE method is the best method in five types the matrix balancing methods.

## **COMPETING INTERESTS**

Authors have declared that no competing interests exist.

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