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Profit-Maximizing Strategies for an Artificial Payment Card Market. Is Learning Possible?

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ABSTRACT

In this paper, we study the dynamics of competition in the payment card market. This is done through a multi-agent based model, which captures explicitly the commercial transactions at the point of sale between consumers and merchants. Through simulation, we attempt to model the demand for payment instruments on both sides of the market. Constrained by this complex demand, a Generalised Population Based Incremental Learning (GPBIL) algorithm is applied to find a profit-maximizing strategy, which in addition has to achieve an average number of card transactions. In the present study we compare the performance of a profit-maximizing strategies obtained by the GPBIL algorithm versus the performance of randomly selected strategies. We found that under the search criteria used, GPBIL was capable of improving the price structure and price level over randomly selected strategies.

Keywords*: *Competition in Payment Card markets*, *Multi-Agent Modeling*, *Evolutionary Computation

1. Introduction

The payment cards, known as credit and debit cards, in the last two decades have become an important element of the modern economies. The growing importance of these electronic payment instruments is among the reasons why economists and policymakers have put a lot of efforts to understand the way in which the payment card market works [1-4]. This is built over so called two-sided platform [5], in which for a successful transaction with an electronic instrument the consumers have to hold a card and the merchants have to accept it as a payment. Consumers and merchants prefer the use of plastic cards over cash for security reasons or because it could save them time at the counter. These, so called convenience externalities arise among the different retail payment methods used. Further, the higher the number of establishments that accept a particular card, the higher the benefits to the card holders. Similarly, the merchants obtain higher benefits if the number of consumers using a particular card increases. These, known as network externalities, are crucial element of the competition among payment card purveyors.

Platform operators, as Visa and Mastercard, organize their business in a four party scheme: consumers, merchants, issuers (banks that provide cards to the consumers) and acquirers (financial institutions that provide electronic terminals to merchants). Each network establishes a specific level of interchange fees, which is charged per transaction and usually flows from acquirers to issuers. For a long time, the main focus of the literature has been on the fee structure of payment cards, with the emphasis laid on the interchange fee [6,7]. The research in the field can be divided into models studying the problematic of a single card [1,2,8,9] and [10], and models that allow the competition between payment card platforms [11-14].

Nevertheless, given that the possibility to incorporate the complex market dynamics into an analytical model is limited, the results of the theoretical studies strongly depend on the assumptions regarding the relationships among the market participants. In addition, these models are unable to incorporate the heterogeneity in the preferences of consumers and merchants, neither the externalities arising from the complex dynamics of two-sided demand for electronic payment cards.

Following a different approach, in order to gain better insight of the market, in [15] the authors have developed an agent-based model, which was motivated from [8]. The model simulates the interactions at the point of sale among consumers and merchants. This artificial market

reproduces the demand of payment cards on both sides and opens the possibilities to study the market dynamics, using computational environment. Further, this work was extended in [16], in which evolutionary computational techniques were applied to find a profit-maximizing strategy. In that environment, instead of representing explicitly issuers and acquirers, the model captures the competition among different platform operators, which payment cards are not compatible. More specifically, each competitor decides his own price level and structure as well as the amount to spend in publicity. The price structure consists in variable and fixed fees on both sides of the market[1], but each card provider decides which particular fees to apply. The four elements that determined the price, together with the publicity cost form the payment card purveyor's strategy, which is modelled as a vector. Given the complex domain of this vector, there exist multiple possibilities of sampling it. For instances, the authors applied a Generalised Population Based Incremental Learning (GPBIL) [17] algorithm in order to obtain profit-maximizing strategies, which in addition have to achieve an average number of card transactions.

Under this line of research, here we present a study that tests how efficient the GPBIL-strategies reported in [17] are. At this point the Box's statement: "all models are wrong, but some are useful" (p.424 in [20]) could not be more appropriate. Motivated by Box, our aim is to find out if this model is purposeful? To this end, we subject the five elements of a card provider's strategy to training. These are the variable and fixed fees on both sides of the market and the publicity cost. The simplest mechanism of sampling the strategy is to follow a random selection. Alternatively, an instantiation of the vector could be the result of an extensive search over the strategy space constrained by the complex shape of the modelled demand. The space of possible strategies is enormous, due to combinatorial explosion. To improve search efficiency, this search has to be guided by particular criteria of interest, e.g. to find profit-maximizing strategies. The search method chosen could make substantial difference in finding better strategies, in terms of both the quality of the strategies found and search efficiency [21]. In order to find the advantage and disadvantage of using the GPBIL, in the present study our aim is to compare the performance of the GPBIL-strategy and the randomly generated strategy. To that end, we have created an artificial market with nine competitors, in which local interactions among consumers and merchants are reproduced. We report the performance of five different GPBIL-strategies against ten different random strategies. The test is organized in five exercises, structured in the following way. Each exercise consists of ten independent instantiations of the model, also refereed as cases. In each independent case, eight of the nine competitors use the same profit-maximizing strategy, whereas one competitor applies a randomly generated strategy. Therefore, the exercise consists in comparing the performance of one GPBIL-strategy against ten different random strategies. For each exercise we compare the strategies in two aspects: profit and number of card transactions. The randomly generated strategy needs to obtain higher number of card transactions and better profit in comparison to the GPBIL-strategies, in order for the former to be considered more efficient.

The paper is organized as following: in the next section we briefly introduce the elements' definition and the decisions of the agents interacting at the artificial payment card market. Following, in Section 3 we present the insides of the GPBIL algorithm, whereas in Section 4 we explain the structure of the experiment. Next, Section 5 is dedicated to present the results of the experiment with our observations and finally in Section 6 the related conclusions and suggestions for future work are given.

2. Artificial Payment Card Market

The artificial payment card market is a model that simulates commercial transactions among consumers and merchants, following as a framework the model presented in [8]. In this section we introduce formally the elements' definitions and the decisions of the agents.

2.1. Elements' Definition

2.1.1. Merchants

Suppose we have a set of merchants $\mathcal{M}$ with $|\mathcal{M}|=N_{\mathcal{M}}$, who are offering a homogeneous good at a common price and face marginal cost of production lower than this price. In other words, we eliminate the price competition among merchants in order to concentrate our analysis on the competition among payment cards. The merchants are located at random intersections of a $N \times N$ lattice, where $N^2 \gg N_{\mathcal{M}}$. Let the top and bottom edges as well as the right and left edges of this lattice be connected.

2.1.2. Consumers

Consumers occupy all remaining intersections of the $N \times N$ lattice. The set of consumers is denoted C with $|C|=N_C$, where $N_C \gg N_{\mathcal{M}}$ and $N^2 = N_C + N_{\mathcal{M}}$. Each consumer has a budget constraint that allows him to buy in a single interaction exactly one unit of the good offered by the merchants. The utility gained from the consumption of this good is bigger than its price. In order to

[1]The variable fees are charged for each transactions paid with a card, whereas fixed fees are membership fees, which are charged regularly regardless the use of the card.

obtain the good any consumer $c \in C$ has to travel to a merchant $m \in \mathcal{M}$. The distance is measured by the Manhattan distance $d_{c,m}$ between the locations on the lattice and it imposes travel costs on consumers. The longitude between two adjacent intersections is normalized to unity. Let $\mathcal{M}_c$ denotes the set of merchants a consumer considers to go to, given that we restrict him to the nearest merchants[2].

2.1.3. Payment Cards

There exists a set of payment methods $\mathcal{P}$ with $|\mathcal{P}|=N_{\mathcal{P}}+1$ and $N_{\mathcal{P}} \ll N_{\mathcal{M}}$. All payment forms are card payments offered by network operators, with the exception of the first element of the set, which is the benchmark and can be interpreted as cash payment method. The cash is used by all consumers and is accepted by all merchants. Furthermore, in order for a card payment to occur, the consumer as well as the merchant must have a subscription to the card in question. We assume that consumers prefer card payments over cash payments.

A fixed subscription fee of $F_p \geq 0$ could be charged per each transaction to the consumer, whereas $\Gamma_p \geq 0$ could be charged per each transaction to the merchant. The domains of those fees, $\mathbb{D}_{F_p}$ and $\mathbb{D}_{\Gamma_p}$ are subsets of real numbers. Cash payments do not attract any fees. For each unit of goods sold using a payment card $p \in \mathcal{P}_m$, a merchant $m \in \mathcal{M}$ receives net benefits of β_p. Such benefits may include reduced costs from cash handling and could differ across payment methods. These are identical for all merchants for a given card. The domain $\mathbb{D}_{\beta_p}$ is a subset of real numbers. Note that the benefits β_p could have a negative value, which means that the variable fee paid by the merchant to the card issuer is bigger than the benefits he receives from that particular electronic payment method. Cash payments do not produce any benefits.

Consumers also receive net benefits from paying by card, b_p, but no benefits from cash payments. Here, the benefits may arise from the delayed payment, insurance cover or cash-back options. The benefits are the same for all consumers, but could differ across card purveyors. The $\mathbb{D}_{b_p}$ is a subset of real numbers and as in the case of the merchants could also include negative values.

Finally, the issuer of the payment method has to decide how much it should spend in publicity $l_p \in \mathbb{D}_{l_p}$, in order to increase the number of consumers and merchants using the electronic card that he is providing. The publicity domain, $\mathbb{D}_{l_p}$, is a subset of real numbers. Thus, the variables controlled by the card purveyors are F_p, Γ_p, β_p, b_p and l_p and those form its strategy.

[2]We have modelled local *network connections.*

2.2. Decisions of the Agents

2.2.1. Merchants' Decisions

After certain period of interactions[3] at the point of sale the merchants have elements to decide to which new cards to subscribe and which old subscriptions to keep. In order to do so, at the beginning of the simulation, merchants start with certain number of cards assigned to them. Then for each commercial transaction the establishments keep track of the cards presented at the counter. Every time a card $p \in \mathcal{P}$ is presented to the merchant $m \in \mathcal{M}$ and he has a subscription to this card $p \in \mathcal{P}_m$ with $|\mathcal{P}_m|=N_{\mathcal{P}_m}$ he increases the score of the card $\theta^-_{m,p}$ by one. Here, $\theta^-_{m,p}$ is an element of the vector defined as

$$\left(\theta^-_{m,1},\cdots,\theta^-_{m,N_{\mathcal{P}_m}}\right)$$

On the other hand, if the merchant does not have subscription to the card, *i.e.* $p \in \mathcal{P}^1_m$, the score of the card $\theta^+_{m,p}$ is increased by one, given that $\theta^+_{m,p}$ is an element of the vector:

$$\left(\theta^+_{m,1},\cdots,\theta^+_{m,N_{\bar{\mathcal{P}}_m}}\right)$$

The merchant decides to cancel the subscription of a card with probability[4]

$$\pi^-_{m,p} = \frac{x^-_m q}{x^-_m q + \exp\left(\frac{\theta^-_{m,p}}{\theta_m}\right)} \tag{1}$$

where θ_m denotes the number of cards presented. Similarly he decides to subscribe to a new card with probability

$$\pi^+_{m,p} = \frac{\exp\left(\frac{\theta^+_{m,p}}{\theta_m}\right)}{x^+_m q + \exp\left(\frac{\theta^+_{m,p}}{\theta_m}\right)} \tag{2}$$

where $x^-_m q$ and $x^+_m q$ represent the inertia to add or drop a card; $q = \left(1+\Gamma_p + N_{\mathcal{P}_m} + \frac{\varepsilon}{\beta_p}\right)$, whereas x^-_m and x^+_m are constants.

2.2.2. Consumers' Decision

In the model the consumers take three decisions: which merchant to visit, which card to use and similar to the merchants' decision, to which card to subscribe?

[3]The number of interactions is dissimilar across merchants and it is determined by Poisson distribution specific for each *m*.

[4]The probabilities defined in Equations (1) and (2) are affected by the publicity applied by each payment card provider.

Regarding the first decision, we assume that when deciding which merchant to visit, the consumer has not yet decided which of the cards he holds will be used. Suppose $\mathcal{P}_{c,m}$ is the set of cards the consumers and merchants have in common. Given that $\left|\mathcal{P}_{c,m}\right| = N_{\mathcal{P}_{c,m}}$, we assume that the more common payment cards the merchant m and the consumer c have, the more attractive a merchant becomes. This is due to the fact that the consumer always carries all his cards with him and he decides which card to use at the moment of the payment. Additionally, the smaller the distance $d_{c,m}$ between the consumer and the merchant, the higher the possibility for this merchant to be chosen by the consumer. From these deliberations we propose to use a preference function for consumer to visit merchant:

$$v_{c,m} = \frac{\frac{N_{\mathcal{P}_{c,m}}}{d_{c,m}}}{\sum_{m' \in \mathcal{M}_c} \frac{N_{\mathcal{P}_{c,m'}}}{d_{c,m'}}} \tag{3}$$

Each consumer $c \in C$ chooses a merchant $m \in \mathcal{M}$ with probability $v_{c,m}$ as defined in 3. The consumers will continuously update their beliefs regarding the number of common payment cards for all merchants they may visit. With respect to the second decision, the consumer decides which payment card he wants to use at the counter of the chosen merchant. We assume a preferred card choice, given that the consumer chooses the card with the higher benefits b_p, alternatively if the merchant does not accept any of the consumers' cards the transaction is settled using cash payment.

Finally, after certain periods of interactions[5] the cardholders decide which new cards to subscribe to and which old subscriptions to keep.

This decision has the following structure. Similarly to the merchants, initially consumers have certain number of cards $\mathcal{P}_c$ with $\left|\mathcal{P}_c\right| = N_{\mathcal{P}_c}$. Every consumer $c \in C$ keeps track whether the cards he owns are accepted by the merchant or not. If card $p \in \mathcal{P}_c$ is accepted by the visited merchant $m \in \mathcal{M}_c$, the consumer increases the score of the card $\omega^-_{c,p}$ by one. Here $\omega^-_{c,p}$ is an element of the vector specified as

$$\left(\omega^-_{c,1}, \cdots, \omega^-_{c,N_{\mathcal{P}_c}}\right)$$

Assume that he cancels his subscription with probability[6] defined in 4, given that the number of merchants visited is ω_c.

$$\pi^-_{c,p} = \frac{x^-_c k}{x^-_c k + \exp\left(\frac{x^-_{c,p}}{\omega_c}\right)} \tag{4}$$

Here $x^-_c k$ accounts for the inertia of the consumer to change cards; $k = \left(1 + F_p + N_{\mathcal{P}_c} + \frac{\varepsilon}{b_p}\right)$, whereas ε and x^-_c are constants.

At the other hand, let $\mathcal{P}^-_c$ with $\left|\mathcal{P}^-_c\right| = N_{\mathcal{P}^-_c}$ be the set of payment cards, to which the consumer does not have subscription. Suppose consumer c visits a merchant m and they are faced with the situation, in which they do not have cards in common. The set of cards the merchant accepts is $\mathcal{P}_m \neq \{\varnothing\}$. In that case the consumer increases the score $\omega^+_{c,p}$ by one $\forall p \in \mathcal{P}_m \subset \mathcal{P}^-_c$. Here $\omega^+_{c,p}$ is an element of the vector, which is defined as

$$\left(\omega^+_{c,1}, \cdots, \omega^+_{c,N_{\mathcal{P}_c}}\right).$$

Given that ω^+_c is a constant, the probability of subscribing to these cards is then determined by

$$\pi^+_{c,p} = \frac{\exp\left(\frac{\omega^+_{c,p}}{\omega_c}\right)}{\omega^+_c k + \exp\left(\frac{\omega^+_{c,p}}{\omega_c}\right)}. \tag{5}$$

2.2.3. Payment Cards Providers' Decisions

The payment card providers' decision is to define what strategy they are going to use. For that reason we define the solution space of the payment card's strategy as

$$\mathbb{S} = \mathbb{D}_{F_p} \times \mathbb{D}_{\Gamma_p} \times \mathbb{D}_{b_p} \times \mathbb{D}_{\beta_p} \times \mathbb{D}_{l_p}$$

rewritten as

$$\begin{aligned} &\mathbb{S} = \mathbb{D}_1 \times \cdots \times \mathbb{D}_5 \text{ with} \\ &\mathbb{D}_1 = \mathbb{D}_{Fp}, \cdots, \mathbb{D}_5 = \mathbb{D}_{l_p}. \end{aligned} \tag{6}$$

In addition we assume that the cost of publicity, l_p, spend by the card issuer in each interaction, has a direct impact in the consumers' and the merchants' decisions to subscribe/cancel a card. The probabilities, $\pi^-_{m,p}$, $\pi^+_{m,p}$, $\pi^-_{c,p}$, $\pi^+_{c,p}$, defined in Equations (1), (2), (4) and (5) are then adjusted according to the rule presented in the following equation

$$\Delta\pi = \tau\pi\left(2 - \pi\right). \tag{7}$$

Here π substitutes any of the above probabilities, Δ represents the differences between the original value of π and the adjusted π, and finally $\tau = \alpha\left(\varphi - \exp\left(-l_p\right)\right)$. The constants α and φ satisfy the constrains

[5]The number of interactions is different across consumers and is defined by individual Poisson distribution.

[6]The probabilities defined in Equations (4) and (5) are affected by the publicity applied by each payment card provider.

$\pi - \Delta\pi \geq 0$ and $\pi + \Delta\pi \leq 1$.

Let $\mathbf{S} = \left(s_1, \cdots, s_{N_\mathcal{P}}\right)$ be the vector of sample strategies for all payment methods. The decisions of a particular payment card provider are an instantiation of this vector. Sampling s_p from $\mathbb{S}$ could follow a random selection process. Alternatively, the vector $\left(s_1, \cdots, s_{N_\mathcal{P}}\right)$ could be the result of an extensive search over the strategy space constrained by the complex shape of the modelled demand. In the next section we briefly explain how we have applied the GPBIL algorithm, which incorporates positive and negative learning, and is able to approximate versatile distributions.

3. Applying GPBIL in Finding Profit Maximizing Strategy

The strategy's domain $\mathbb{D}_i$ is interval of real numbers. Assume a probability distribution functions $\mathbb{F}_{\mathbb{D}_i} : \mathbb{R} \to [0,1]$ for unconditional random variables over the ranges $\mathbb{D}_i$, we define the joint probability distribution $\mathbb{F}_\mathbb{S}$ over S by

$$\mathbb{F}_\mathbb{S} = \mathbb{F}_{\mathbb{D}_1} \cdots \mathbb{F}_{\mathbb{D}_5} . \quad (8)$$

All electronic cards providers have the same joint probability distribution and we are using it firstly to sample individual strategies from the space, and secondly to modify $\mathbb{F}_\mathbb{S}$ through learning.

We have defined $S = \left(s_1, \cdots, s_{N_\mathcal{P}}\right)$ as the vector of strategies of all payment methods in one execution of the artificial payment card market. Additionally, we define $\phi_p = \left(\Phi_p, N_{T_p^*}, p\right)$ as the measurement of the performance achieved in one execution of the model for one payment method. The three elements that compose it are the profit of the card issuer Φ_p, the number of transactions obtained in the market $N_{T_p^*}$ and the corresponding index of the card *p*. The vector $\left(\phi_1, \cdots, \phi_{N_\mathcal{P}}\right)$ represents the performance of all payment cards in one execution of the artificial market, after certain number of interactions among consumers and merchants.

In **Figure 1** we present the MARKET-GPBIL process used to find a profit-maximizing strategy. In our application the strategy should fulfill the following main objective: obtain the highest possible profit ϕ_p under the constraint that the number of transactions obtained $N_{T_p^*}$ should be equal or above average.

The first step is to initialize the joint probability function. This is performed by the function *initialisation*, which receives as a parameter the solution space $\mathbb{S}$ and returns the initialized joint probability function, $\mathbb{F}_\mathbb{S}$. The main part of the algorithm consists of a loop over *R* runs. At the beginning of each run every payment card provider selects a strategy S_p. This process is carried out by the function *sampling* (line 5 of the GPBIL algorithm **Figure 1**), which returns a strategy S_p for each one of the payment cards, following the probability distribution function $\mathbb{F}_\mathbb{S}$.

```
MARKET-GPBIL( )
1  I = ℑ; 𝕊 = 𝔖; N_𝒫 = ℜ_𝒫; R = ℜ
2  𝔽_𝕊 = initialisation(𝕊)
3  FOR r = 1,⋯,N_𝒫 DO
4      FOR p = 1,⋯,N_𝒫 DO
5          s_p = sampling(𝔽_𝕊)
6      (φ_1,⋯,φ_{N_𝒫}) = MARKET((s_1,⋯,s_{N_𝒫}), I)
7      φ⃗′ = profitDescendingSort(φ_1,⋯,φ_{N_𝒫})
8      𝔽_𝕊 = learning(𝔽_𝕊, φ⃗′, (s_1,⋯,s_{N_𝒫}))
9  RETURN 𝔽_𝕊
```

Figure 1. The process MARKET-GPBIL for profit-maximizing strategies.

Thereafter, in line 6, we instantiate the process MARKET with the strategy vector $\left(s_1, \cdots, s_{N_\mathcal{P}}\right)$ and the number of interactions *I*. This process reproduces the transactions at the point of sale among consumers and merchants. At the end it returns a vector of all payment cards performance measures $\left(\phi_1, \cdots, \phi_{N_\mathcal{P}}\right)$.

Before the learning function is carried out, the performance of the payment cards providers $\left(\phi_1, \cdots, \phi_{N_\mathcal{P}}\right)$ are sorted (line 7) according to the profit Φ_p achieved at the MARKET. The new vector is denoted $\vec{\phi}'$. Following this step, the joint probability function F_S is modified by a learning process (line 8). This task is accomplished considering the market share $N_{T_p^*}$ obtained in the MARKET. More specifically, the function receives as parameters the current values of the joint probability distribution $\mathbb{F}_\mathbb{S}$, the profit ordered according the performance $\vec{\phi}'$ and the vector of strategies $\left(s_1, \cdots, s_{N_\mathcal{P}}\right)$. The strategies that have obtained better than or equal to the average number of transactions are considered examples of positive learning, whereas the rest of the strategies form the set of negative learning examples.

Finally, in line 9, the GPBIL algorithm returns the resulting joint probability distribution. This function is used as a probabilistic model to generate strategies that fulfill the main objective: to achieve the highest possible profit, given the additional restriction of obtaining above or equal average number of card transactions. In the next section we compare the performance of randomly generated strategies over the performance of the strategies resulting from the joint probability distribution found by the MARKET-GPBIL.

4. The Setting of the Experiment

In this section we present the setting of the model's parameters and the structure of the efficiency test. We have

defined the number of interactions among consumers and merchants as I and it is settled at I = 3000; the Poisson distribution, used to determine the decision period of consumers and merchant, has a mean $\lambda = 20$. The sets of consumers C and merchants $\mathcal{M}$ are instantiate with N_C = 1100 and $N_{\mathcal{M}}$ = 125. The rest of the user defined parameters are divided in two tables. In **Table 1** we have listed the values of the constants, which impact the decisions of consumers and merchants, whereas in **Table 2** the domain of each element of the strategy space is presented. The efficiency test consists of comparing the performance of the profit-maximizing strategy with the performance of randomly generated strategy.

In order to do so, we have settled a market with nine competitors. Using the parameter setting described above, in the simulation eight of the nine competitors use the same profit-maximizing strategy, whereas one competitor applies a randomly generated strategy. We test five of the ten GPBIL-strategies presented in [17]. Each of the five exercises consists in comparing the evolved strategy against ten random strategies in independent executions of the model. We use the same randomly generated strategies in the five comparisons.

In **Tables 3** and **4** the set of random and GPBIL strategies are presented. In the first column of the tables we list the identifier of the strategy[7], whereas in the rest of the columns we present the strategy's elements.

Finally, we list the measurements used to evaluate the performance of the different kinds of strategies.

- Φ_p^{gp} is the profit of the competitor with the randomly generated strategy;
- Φ_p^{rm} is the average profit of the eight competitors using the strategy obtained by the GPBIL;
- $\Phi_{I_p}^{gp}$ is the average number of card transactions of the eight payment card providers, using the strategy obtained by the GPBIL;
- $\Phi_{I_p}^{rm}$ is the number of transactions of the competitor, using the randomly generated strategy.

5. Results

In this section we present the results obtained from the evaluation of the performance of the GPBIL strategies in maximizing profit and obtaining on average number of transactions. The section is divided in two parts, both of them based on the data shown in **Tables 5-9**[8].

In the first part in five graphics the individual performance of each GPBIL strategy tested is shown against

[7]This number is used latter to relate each strategy to its performance.

[8]The tables are organized in the following way: in the first column the random strategy's identifier is presented, in the second and third columns the number of transactions and the profit of the random strategy are listed, and finally in the last two columns the average number of transactions and the average profit of the profit-maximizing strategy are presented.

Table 1. Constants used in the end-users' decisions.

Symbol	Description of the Constants	Value
ε	constant for the inertia to changes	1
x_C^-	accounts for the consumers' inertia to drop cards	0.005
x_C^+	accounts for the consumers' inertia to add cards	2
x_m^-	accounts for the merchant' inertia to drop cards	0.05
x_m^+	accounts for the merchant' inertia to add cards	9
α	accounts for the impact of the publicity cost	0.1
φ	account for the impact of the publicity cost	5

Table 2. Strategy's domains.

Symbol	Domain	Value
$\mathbb{D}_{F_P}$	Consumers Fixed Fee Domain	[0; 10]
$\mathbb{D}_{\Gamma_P}$	Merchants Fixed Fee Domain	[0; 10]
$\mathbb{D}_{b_P}$	Domain of Consumers' Benefits	[–1; 1]
$\mathbb{D}_{\beta_P}$	Domain of Merchants' Benefits	[–1; 1]
$\mathbb{D}_{l_P}$	Publicity Cost's Domain	[0; ∞]

Table 3. Randomly generated strategies.

Id	F_P	Γ_P	β_P	b_P	l_P
1	5.50	0.49	–0.10	–0.79	14.84
2	5.82	0.42	–0.74	0.36	14.24
3	6.62	3.96	0.12	–0.25	0.70
4	0.92	4.49	–0.66	–0.09	5.68
5	6.07	6.17	–0.87	0.04	15.42
6	3.16	3.95	–0.66	–0.88	0.69
7	6.16	1.87	0.48	–0.72	1.92
8	7.80	5.87	–0.15	–0.64	0.51
9	6.31	3.57	–0.07	0.53	8.29
10	6.57	3.93	0.56	–0.47	12.80

Table 4. Profit-maximizing strategies.

Id	F_P	Γ_P	β_P	b_P	l_P
1	7.57	0.00	–1.00	–1.00	11.71
2	5.66	0.00	–1.00	–1.00	10.82
3	5.33	0.00	–1.00	–1.00	7.66
4	6.03	0.00	0.48	–1.00	11.82
5	3.51	0.00	1.00	–1.00	11.81

Table 5. Efficiency test, strategy 1.

Id	$N_{I_p}^{rm}$	Φ_p^{rm}	$N_{I_p}^{gp}$	Φ_p^{gp}
1	293 587	4 183 603	190 045	6 112 638
2	155 146	4 777 710	196 131	6 074 918
3	71 466	3 818 894	205 027	6 127 442
4	109 469	1 267 441	199 041	6 016 778
5	31 243	5 217 924	209 679	6 148 302
6	154 401	3 352 744	197 677	6 045 649
7	190 495	4 020 676	195 060	6 062 770
8	63 695	3 896 739	206 835	6 154 275
9	59 307	3 726 206	205 612	6 053 948
10	56 643	4 061 625	208 539	6 177 387

Table 6. Efficiency test, strategy 2.

Id	$N_{I_p}^{rm}$	Φ_p^{rm}	$N_{I_p}^{gp}$	Φ_p^{gp}
1	270 086	4 028 220	200 859	5 508 231
2	132 753	4 580 050	206 786	5 467 251
3	65 463	3 638 983	215 090	5 531 546
4	89 048	1 195 634	211 330	5 404 561
5	25 704	4 932 694	220 308	5 537 538
6	138 006	3 235 442	208 256	5 429 815
7	173 742	3 837 502	206 364	5 482 818
8	56 985	3 745 939	217 497	5 532 962
9	53 913	3 525 837	215 560	5 458 349
10	51 820	3 860 624	218 343	5 554 859

Table 7. Efficiency test, strategy 3.

Id	$N_{I_p}^{rm}$	Φ_p^{rm}	$N_{I_p}^{gp}$	Φ_p^{gp}
1	264 663	3 990 413	202 948	5 367 858
2	130 425	4 586 414	208 333	5 328 684
3	64 575	3 603 082	216 486	5 394 379
4	87 680	1 187 830	212 724	5 271 941
5	24 624	4 897 958	222 170	5 389 579
6	133 747	3 193 190	210 157	5 291 645
7	173 181	3 793 127	207 436	5 346 346
8	54 461	3 710 916	219 947	5 399 237
9	53 267	3 511 761	217 211	5 342 348
10	50 646	3 832 682	220 048	5 412 604

Table 8. Efficiency test, strategy 4.

Id	$N_{I_p}^{rm}$	Φ_p^{rm}	$N_{I_p}^{gp}$	Φ_p^{gp}
1	158 276	4 051 594	242 115	4 379 904
2	120 743	4 864 813	245 621	4 375 830
3	43 607	3 837 620	249 250	4 415 086
4	70 769	1 225 230	240 926	4 291 554
5	27 929	5 252 087	251 869	4 384 489
6	99 064	3 228 579	244 468	4 357 959
7	129 401	4 051 465	241 096	4 375 036
8	32 625	3 875 549	250 768	4 459 407
9	34 440	3 765 368	250 344	4 412 800
10	45 266	4 122 748	247 065	4 399 842

Table 9. Efficiency test, strategy 5.

Id	$N_{I_p}^{rm}$	Φ_p^{rm}	$N_{I_p}^{gp}$	Φ_p^{gp}
1	115 092	3 711 873	276 873	3 235 839
2	78 536	4 502 427	281 802	3 226 917
3	31 653	3 539 388	285 434	3 273 227
4	47 468	1 130 966	277 161	3 172 209
5	17 945	4 842 936	286 763	3 231 322
6	69 554	3 037 368	279 898	3 224 163
7	73 756	3 711 376	280 569	3 242 955
8	23 724	3 669 594	285 970	3 291 332
9	21 821	3 476 859	286 019	3 264 873
10	27 007	3 807 948	283 580	3 256 304

the ten random strategies used, whereas in the second part we compare the performance between the set of GPBIL strategies against the set of random strategies.

5.1. Individual Performance

In this subsection we make an individual comparison between each of the five profit-maximizing strategy (see **Figures 2-6**) versus the performances of the ten randomly generated strategies[9]. We present the performance of the strategies in two dimensions: profit and number of card transactions.

Observation 1: In the first three exercises the GPBIL-strategies (**Figures 2-4**) have achieved better profit then the ten randomly generated strategies. Further, the evolved strategies report a higher number of card transactions in night of the ten cases tested, *i.e.* only one randomly

[9]Please refer to **Table 3**.

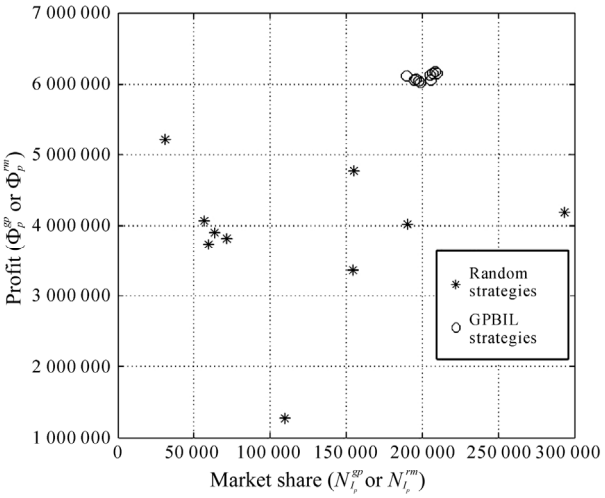

Figure 2. Efficiency test, strategy 1.

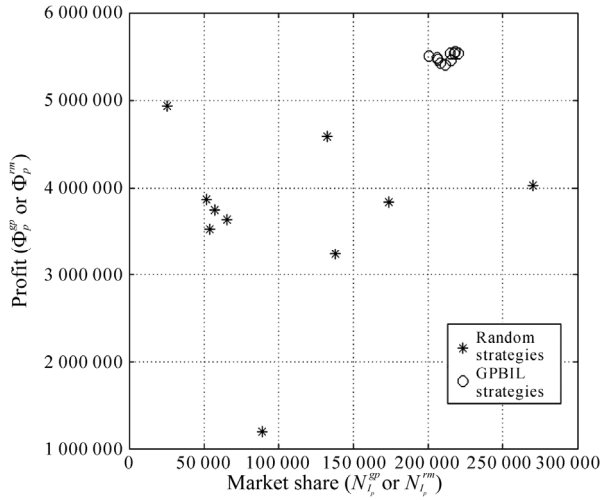

Figure 3. Efficiency test, strategy 2.

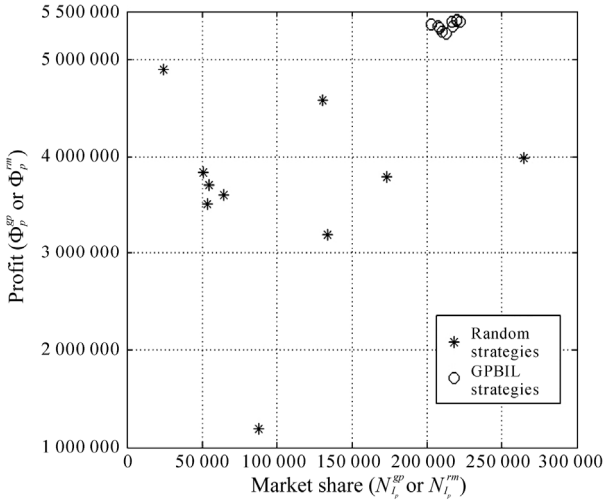

Figure 4. Efficiency test, strategy 3.

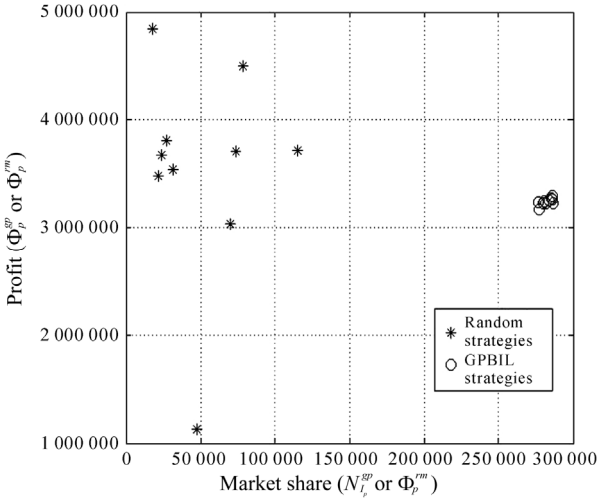

Figure 5. Efficiency test, strategy 4.

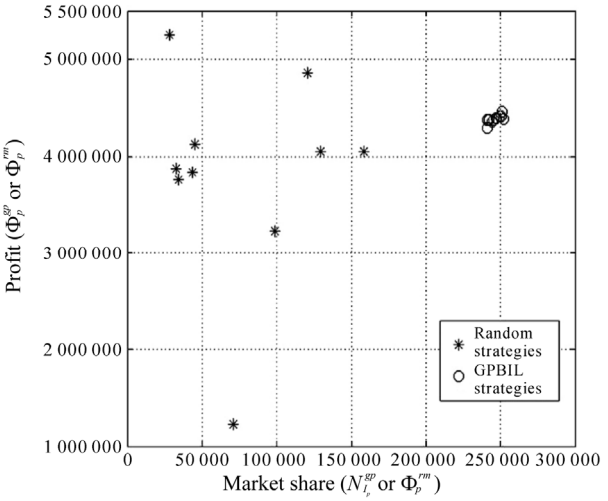

Figure 6. Efficiency test, strategy 5.

generated strategy outrange the three evolved strategies in terms of card transactions, but regarding the profit none of the random strategy could perform better than the GPBIL-strategies 1, 2, and 3. Following, strategy 4 outperforms eight of the ten randomly generated strategies in terms of profit and overpasses all of them in terms of number of card transactions.

Finally, strategy 5 outranges significantly the random strategies in terms of number of card transactions, but in terms of profit was passed in eight of the ten cases tested. Over all, with respect to the profit achieved, the GPBIL-strategies have performed statistically better than the randomly generated strategies. Even more, regarding the number of card transactions reported, the performance of the evolved strategies are better than the randomly generated strategies.

5.2. The Performance of the Set of GPBIL Strategies vs. the Set of Random Strategies

In this subsection we present a second comparison, which is made using the same data presented in **Tables 5-9**. This time we compare on one hand the performance of each GPBIL-strategy among its own records achieved in the ten cases tested and on the other hand the performance of each random generated strategy among its own achievements in the five exercises presented. To that end the performance of each strategy is presented graphically as an area, by plotting together the different performances of the same strategy (either random or GPBIL) resulting from independent instantiations of the model.

More specifically, given that there are five GPBIL strategies tested against ten random strategies, each random strategy has five different performances, whereas each GPBIL strategy has ten performances. In this presentation, if the resulting strategy's area has similar to a rectangle shape, it means that its achievements are similar among the different scenarios and its performance is consistent. On contrary, if the resulting strategy's area has an irregular shape, it means that the performance of the strategy varies among scenarios and it is not consistent.

In **Figures 7** and **8** we plot the random strategies' and GPBIL strategies' areas, resulting from the performance in terms of number of transactions. Next, in **Figures 9** and **10** we present the random strategies' and GPBIL strategies' areas formed according to the obtained profit.

Observation 2: In the second comparison, we observe that the areas formed by the individual performance of profit-maximizing strategies have regular shape in contrast to the areas from by the individual performance of the randomly generated strategies, which is irregular. In particular the evolved strategies' areas in terms of profit and number of card transactions are approaching a rectangular shape, whereas the individual areas of the random strategies are irregular in the both dimensions. Further, a more detailed observation which compare the degree of heterogeneity among the sizes of the areas formed by the individual performance inside the same set, allow us to say that the evolved strategies present similar sizes, whereas the randomly generated strategies have significant differences among the individual performances achieved. This observation is true for the both dimensions studied: profit and card transactions.

Observation 3: Regarding the GPBIL-strategies, in **Figures 8** and **10** an inverse relation is observed between profit and number of card transactions, *i.e.* the strategies that have achieved the highest profit are the strategies

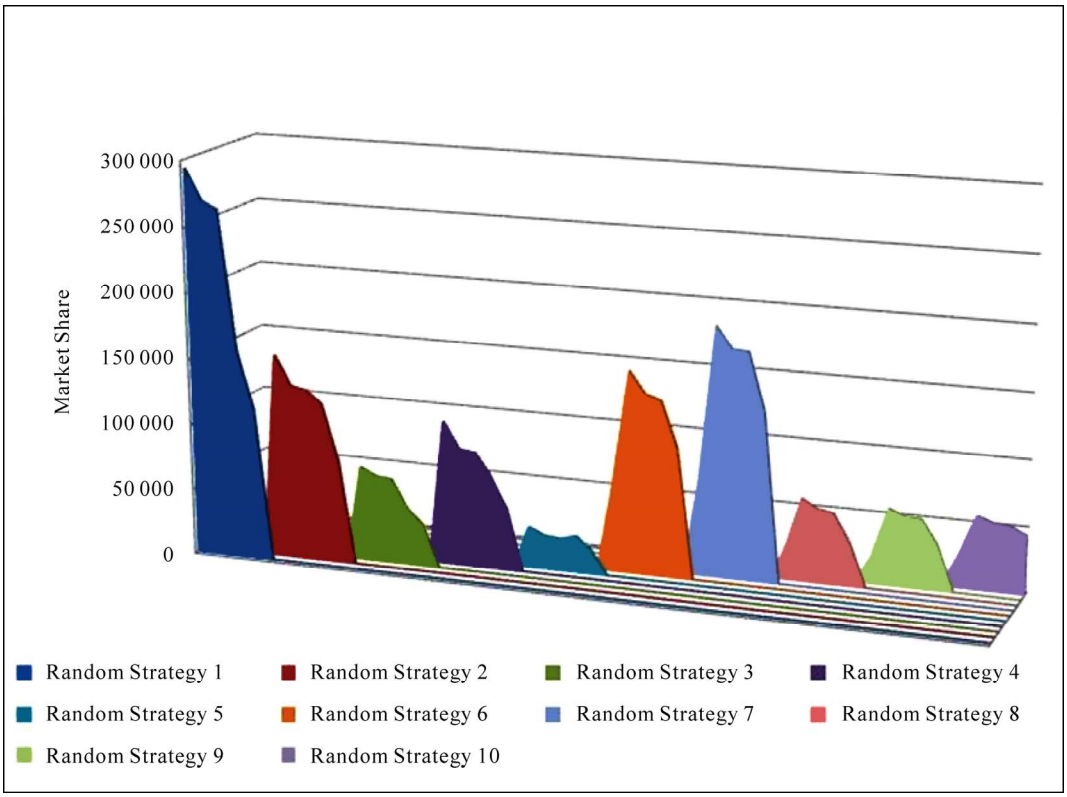

Figure 7. Random strategies performance in terms of number of transactions.

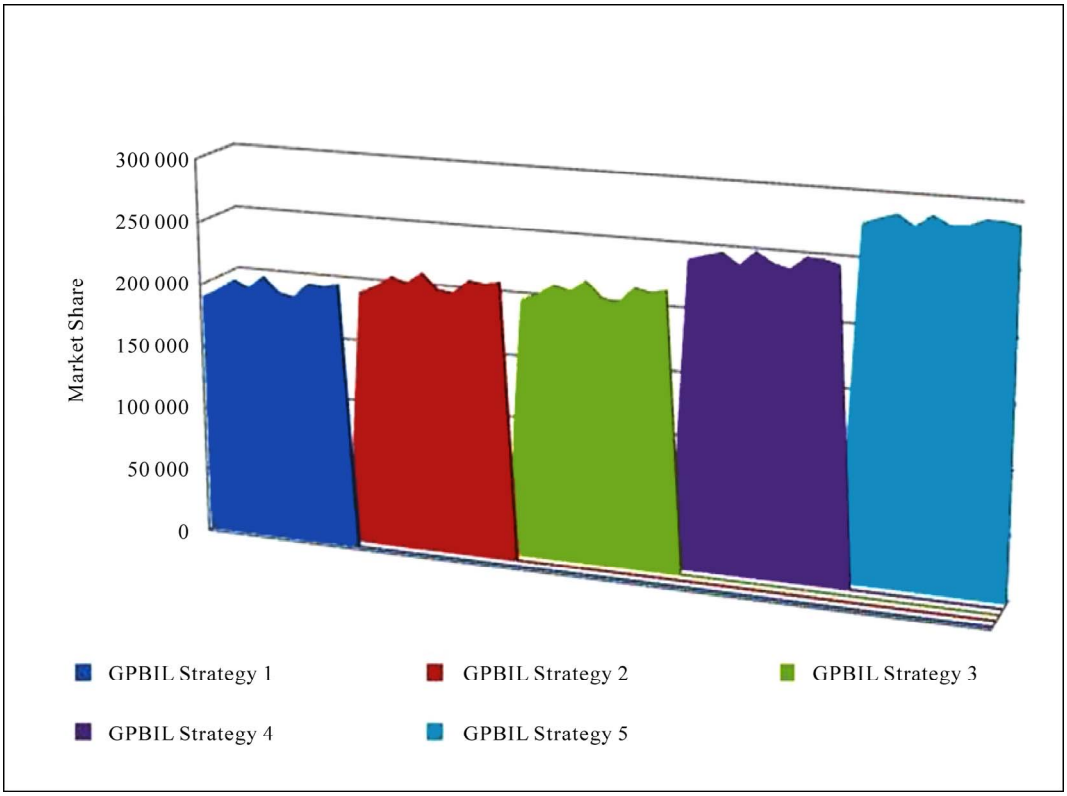

Figure 8. GPBIL strategies performance in terms of number of transactions.

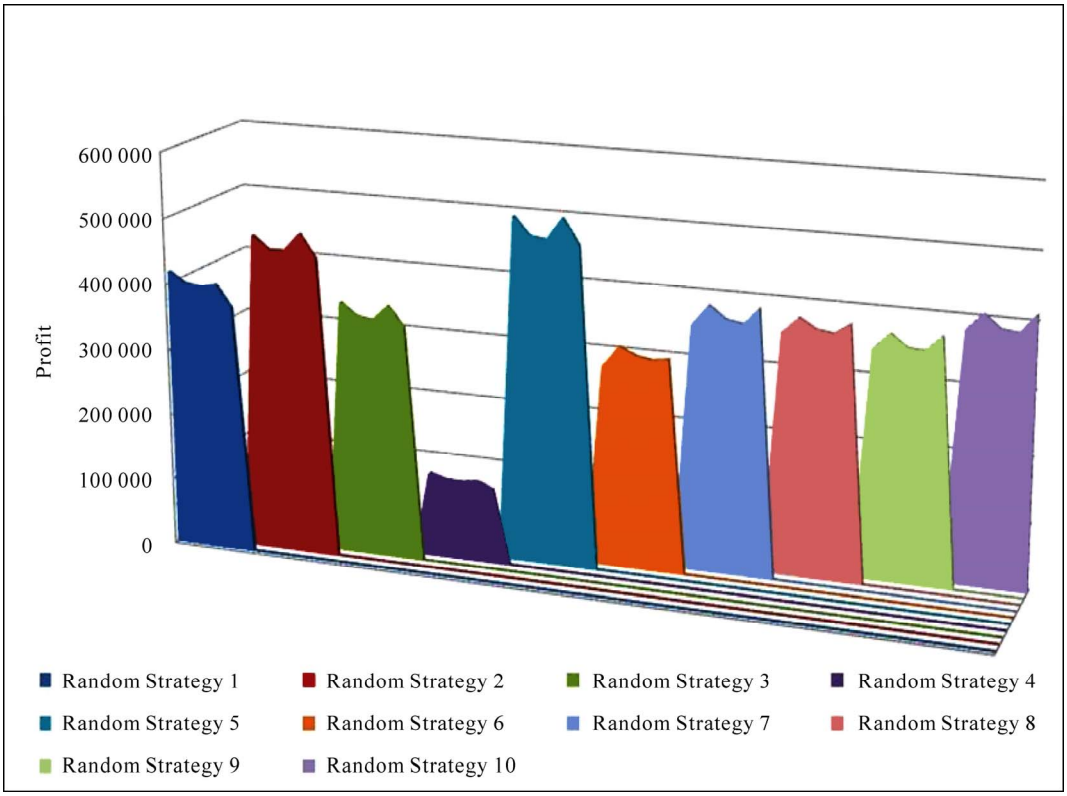

Figure 9. Random strategies performance in terms of profit.

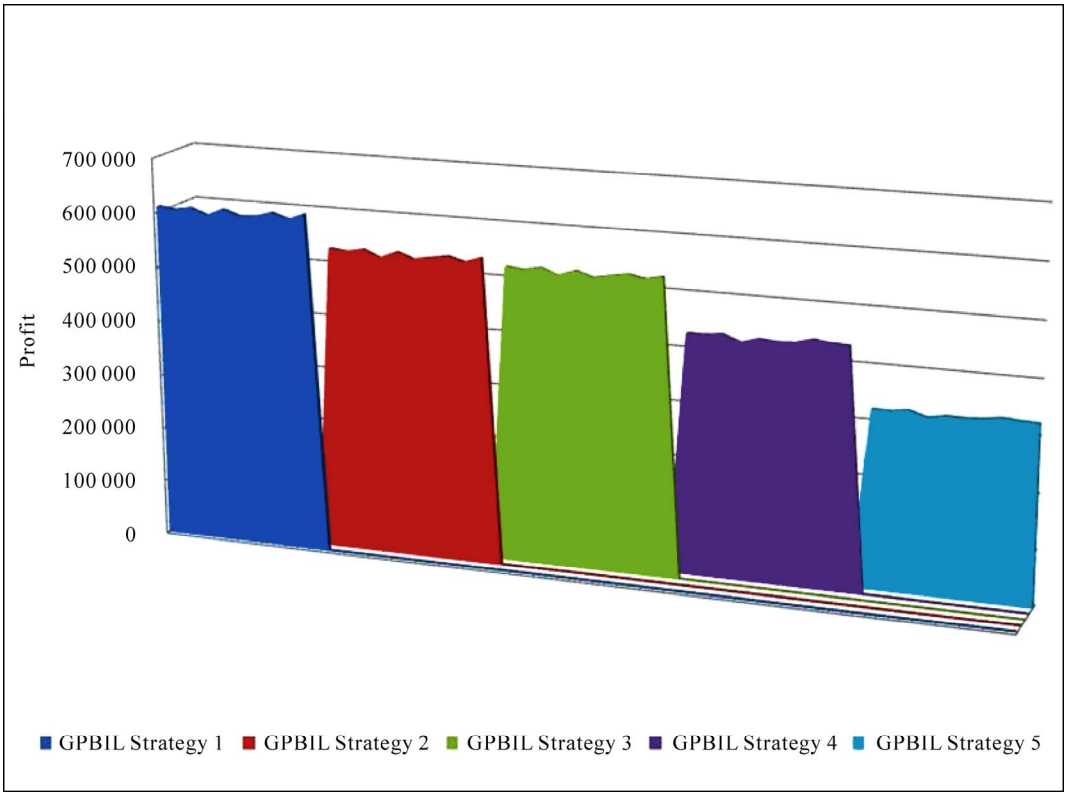

Figure 10. GPBIL strategies performance in terms of profit.

with the lower number of card transactions and vice versa. On the other hand, with respect to the randomly generated strategies the relation between profit and market share observed in **Figures 7** and **9** is different. For instance, random strategy 5, which is the strategy with the highest profit, is also the strategies with the lower number of card transactions, nevertheless random strategy 4, which had achieved the lower profit is not among the three best performance in terms of card transactions.

Conclusions: from *Observations* 1, 2 *and* 3 it follows that the strategies found by the Generalised Population Based Incremental Learning algorithm are efficient strategies in achieving average number of card transactions and maximum profit.

It also follows that with randomly generated strategies is not possible to archive similar performance, therefore the GPBIL-strategies fulfill the purpose they have been designed for.

6. Conclusions

In this section we present general conclusions and we give suggestions for further research. The payment card market is characterized with complex relationships among market participants. The two-sided nature of the market gives rise to network externalities that influences the dynamics of the competition. Therefore in the context of the growing importance of the payment cards, the models of the market aimed to study the competition need to incorporate more realistic features. In this sense we consider that the use of agent-based models will allow us to gain better understanding of the payment card market dynamics. In the current study, given the complex shape of the aggregated end-users' demand for electronic payment methods modelled explicitly at the artificial market, the GPBIL algorithm applied has been able to find a price structure and price level that maximize the profit of the card purveyors and has successfully fulfill the objectives of the search. More the all so, we conclude that applying evolutionary techniques, as the GPBIL, in studying relevant aspects of this market opens new research opportunities, untractable with the analytical models. A possible extension of the model is to convert it to a four party scheme, as Visa and Mastercard organized their business this way. From our observations of the market, we believe that the issuers' and acquirers' decisions affect substantially the market competition. Therefore incorporating into the model the sets of issuers and acquirers and their corresponding decisions will allow us to study which rules governing the participants' behavior, could make the market more efficient. Furthermore, of particular interest could be to understand what conditions are required in order for the competitors to offer better

prices to consumers and merchants.

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